A NEW

TREATISE OF ALGEBRA,

According to the late Improvement

Apply'd to *Numerical Questions* and *Geometry*.

WITH A

CONVERGING SERIES

For all manner of *Affected Equations*.

By RICHARD SAULT,

Of *Magd.* Coll. in *Camb.*

Royal Exchange.

The CONTENTS.

Types of the Sun and Moon's Eclipse according to Ptolomy. 28
A Type of the Sun and Moon's Eclipse according to Copernicus. 29
Of the Passions of the Planets. 30
The Rudiments of Astronomy in plain Rhyme. 32
Of the Constellation of the Fixed Stars; giving an Account of the English, Greek, Hebrew, Latin, Arabick, Caldee, Syriack, Turkish, &c. Names; and of the most Eminent Stars in each of them: With the Poetical Fables; declaring how these Asterisms came to be placed in the Heavens; and also of the Galaxia. 35

TRACT. VI. Horometrical Recreations.

TO make an Horizontal Dial in any Latitude. Pag. 1
Of North and South Dials. 2
Of East, West and Polar Dials. 3
Of Declining Dials. 5
Of Dialling Mechanical. 8
Of Projective Dials. 10
Of Reflected Dials. 12
Mechanically, to inscribe the Equinoctial Tropicks, and other Parallels of Declination, upon all sorts of Sun Dials: And also the Azimuths, Almicantars, &c. 13
Of several Night Dials, how to make and use them. 18
Horometrical Tables of the Sun and Stars. 21
Another Mechanical way to make Reflective Dials. 28

TRACT. VII. Crytographical Recreations.

OF some Ancient ways of Secret Conveyance. Pag. 1
By the 24 Letters of the Common Alphabet variously disposed. 2
By Transposition of the Alphabet. 7
By Astronomical Characters. 9
By Knots tied upon a String. ibid.
By a New Character; easy to Learn, difficult to decipher; and yet Legible by the Motion of the Fingers. 11
Other ways of Secret Writing. 13

TRACT. VIII. Magnetical Recreations.

OF the Magnet, its Name, &c. Pag. 1
That the whole Globe of the Earth hath true Magnetical Virtue. 3
That Magnetical Virtue may be infused into Iron, without the help of the Load-stone. 4
Of the Attractive virtue of the Load-stone. ibid.
Of the Sympathy and Antipathy thereof. 5
Of the Cutting or Dividing of Load-stones. 6
To find the Poles of a Load-stone. ibid.
Of several Attempts that have been made to contrive a Perpetual motion by Magnetical Virtue. 7
Of Magnetical Inclination, and thereby to find the Latitude at Sea, or on the Land. 9

TRACT. IX. Chymical Recreations.

OF Artificial Representations. Pag. 1
Of the Fire of Vessels. 2
Of the Philosopher's Tree: To make it. ibid.
Of the Revivification of Simples. ibid.
Of the representation of the Great World. 3
Of a famous Perpetual motion, invented by Cornelius Drebble; and made for K. James the First of England. 4
Of the making of Gold. ibid.
Of the Nature of Gold. 6
Of Incumbustible Flax: Or a Substance which will not be consumed by Fire. ibid.
Of Subterraneous Lamps. 8

TRACT. X. Automatical Recreations.

OF the Nature, and of the making of Clocks, Watches, and other Movements. Pag. 1
Concerning Pendulums. 2
Of finding out fit Numbers for Wheels and Pinnions. 5
Of giving particular motion to any Movement. 8
To Regulate a true made Pendulum Clock or Watch. ibid.
A Table for the same. 10
Its Use. 11

TRACT. XI. Historical Recreations.

OF the Measures and Proportions of the several Members of Man's Body. Pag. 1
Of Men, or Giants of Prodigious Statures. 3
Of Dwarfs and Pigmies, &c. 4
Of Monsters, &c. 5
Of the Length of Age which Men lived in former Times, shortly after the Creation; and of others of later date. 10
Of the First Authors of divers famous Inventions. 13

THE END.

ERRATA.

In Numerical Recreations.

Page 41. line 20. read doubled.

In Geometrical Recreations.

Pag. 1. l. 20. read As the. p. 41. l. 29. r. points E and T. p. 15. l. 20. r. Marks H and L. p. 17. l. 14. r. 2 c and 2 d. p. 22. l. 27. r. one foot in V. p. 24. l. 5. r. (by Concl. IV.) p. 26. l. 20. r. (by Conclu. III.) p. 26. l. 33. r. a to e. p. 26. in the under Scheme M is wanting. p. 29. l. 16. r. erect a Perpendicular. p. 29. In the under Scheme P is wanting. p. 31. l. 43. r. points P and S. p. 32. In the uppermost Scheme the Letters C and D are in each other's place. p. 37. l. 6, and 7. r. A, B, C and D. p. 38. The Letter M is wanting in the Scheme. p. 39. l. ult. r. 297 4 inches. p. 41. l. 24. r. distance C D. p. 43. l. 32. r. as Figure X. p. 44. l. 3. the line O N. p. 45. l. 5. whereon. p. 45. l. 13. 7 Links. Ibid. in several places for 14 25625. r. 16. 25625. p. 47. in the Scheme 8. 28 is wanting. p. 51. l. 6. r. find. p. 59. l. 25. r. A B C D E F G.

In Mechanical Recreations.

Pag. 14. l. pen r. Wheels were augmented. p. 15. The Schemes belonging to Sect. 4. in that Page is omitted. p. 16. l. 22. r. Automata. p. 28. l. 13. r. The Figure of

In Statical Recreations.

Page 3. line 26. r. Pendency.

In Astronomical Recreations.

Page 35. line 22. read Galaxia.

In Horometrical Recreations.

Page 3. line 8. read (viz. 9, 10, 11 at Night, and 1, 2, 3 in the Morning.) p. 28. l. 14. r. Quadrant, or the like. p. ibid. l. 15. r. Day: And being.

In Magnetical Recreations.

Page 2. line 25. read Cynosura. p. 4. l. 29. r. it will there hold it. p. 9. l. ult. r. in this Table.

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By *RICHARD SAULT,* Math. Prof.

In *Adam*'s Court in *Broadstreet,* near the Royal Exchange. *London.*

A PRE-

PREFACE.

ALGEBRA *depends upon the ſame Definitions and Axioms that a conſiderable part of* Euclid's Elements *do, and may therefore very juſtly be call'd a* New Geometry, *thô it has no ſmall Advantage above that which is commonly called by that Name; ſince the Subſtance of whole Sciences at once are ſometimes comprehended in one ſingle* Algebraic Theorem; *an Inſtance of this the Ingenious Mr.* Hally *gives us in the late* Tranſactions of the Royal Society. *And in many Geometric Conſtructions, which otherwiſe would be deſperate,* Algebra *furniſhes us with ſuch Laws and Rules, as fail not to effect to what we want. So that no one can be a good* Geometrician *that is not a good* Algebraiſt; *by conſequence no tolerable* Mathematician *without it: And ſince Mathematics are now deſervedly grown into that Repute, that they begin to be a neceſſary part of a Gentleman's Education, (as well as a public Encouragement of them, the Intereſt of a Kingdom) therefore* Algebra, *amongſt other Parts (which in ſome ſenſe may be term'd the whole) will certainly come in for a conſiderable Share.*

As to my own Performance herein, I have omitted nothing that I know of which is uſeful, but have run through a whole Syſtem *with what brevity and plainneſs I could, advancing nothing which I have not demonſtrated where it's neceſſary, or ſhew'd the Reaſon of the Proceſs. I have applied* Algebra *to Numeral Queſtions and Geometry, and ſhew'd the Method of Geometric Conſtructions; and each of theſe ſo far, that afterwards the Learner may proceed himſelf to what higher Progreſs his own Induſtry and Sagacity may prompt him.*

As for the Buſineſs of Equations, *I have made ſome little Progreſs, which I have not ſeen in any other. In* Quadratics *I find both the Roots at once; and in* Cubics *I have determin'd all the ſixteen Caſes, and the Qualities of the Roots, as how great (near) if compar'd with each other, how many Poſitive, Negative, or Imaginary; and this by conſidering all the poſſible Combinations of the* Coefficients *ariſing only from three* Geneſes, viz. *from three poſitive Roots, or two Poſitive and one Negative; or, laſtly, from two Negative and one Poſitive.*

I have alſo given an Account of Mr. Raphſon's Converging Series, *with its Demonſtration, and an Application of it to ſeveral Examples, ſufficient to ſhow how all Equations however high or adfected may be eaſily reſolv'd, and have conſider'd as previous to this* Series *the Punctation of Adfected Equations which is very uſeful in Common Practice; thô now there's no abſolute neceſſity of it, for* this *reaches the Caſe of* Anticipations, *a thing troubleſome enough in any* Exegiſis Numeroſa *that has yet been publiſh'd.*

I have yet two other Infinite Approximations *for all Adfected Equations whatever, built upon very different Principles from that of Mr.* Raphſon's, *with new Methods of Extracting Roots, and ſomething elſe of this Nature, but do not think fit to publiſh them yet, eſpecially ſince I'm inform'd that Mr.* Hally *has improv'd* Lagny's *Method for Adfected Equations; which I doubt not but is ſomething worthy the Application of ſo good a Mathematician as Mr.* Hally *is, and therefore will be very welcome to the Public when he pleaſes to communicate it.*

As for the Errata's *in this Folio, in tranſpoſing it to a Quarto, (which I have printed for my own uſe) I have alter'd moſt of them; and if there be any other, which in a curſory reading over have eſcaped me, I will endeavour to ſatisfy any Perſon whatever into whoſe Hands this ſhall fall, if they pleaſe to give themſelves the trouble of writing to me about it.*

An

An Explication of the Characters.

	Signifies
$a+b$	a added to b.
$a-b$	a leſſened by b, or b ſubtracted from a.
$a \times b$	a multiply'd into b.
$\frac{a}{b}$	a divided by b.
$a \backsim b$	The Difference betwixt a and b.
$a.b::c.\frac{bc}{a}$	a is to b, as c to $\frac{bc}{a}$.
$a.b.c \div$	a, b, c, are continued Proportionals.
$a=b$	a is equal to b.
$a \sqsubset b$	a is greater than b.
$a \sqsupset b$	a is leſs than b.
$\sqrt{ab}$	The Root of ab.
$\sqrt{dd+cc}$	The Root of both the Quantities $dd+cc$.
$\sqrt{ddd+\sqrt{bb+q}}$	The Univerſal Root of the Root of $bb+q$ added to ddd.
$\sqrt{(3)a+b}$, or $\overline{a+b}\|^{\frac{1}{3}}$	The Cubic Root of $a+b$, if $\sqrt{(4)a+b}$, it had been the Biquadratic; and ſo of all other Powers.
$\overline{a+b}\|^{\frac{3}{2}}$	The Cube of the Binomial $a+b$.
$\overline{a+b}\|^{n}$	n may ſignify the Index of any Power whatever of $a+b$.
$\overline{a+b}\|^{-1}$	An Unit divided by $a+b$.
$\overline{a+b}\|^{-3}$	An Unit divided 3 times by $a+b$, or by the Cube of $a+b$.
$c \times \overline{a+b}\|^{-\frac{1}{3}}$	c divided by the Cubic Root of $a+b$.
$\overline{a+b}\|^{\frac{3}{4}}$	The Biquadratic Root of the Cube of $a+b$.

ALGEBRA

ALGEBRA *is the Science of Quantity in General.* More Particularly, *'Tis an Art of Reaſoning with unknown Quantities, in order to diſcover their Habitude or Relation to ſuch as are known.*

§. 1. ALGEBRA is term'd *Literal*, or *Speciosa Arithmetic*, in oppoſition to the *Numeral*, where the Figures firſt taken, are loſt or ſwallowed up in others, which by ſeveral Operations are deriv'd from them; but in *Letters* the whole Proceſs appears at firſt ſight, and gives one *general Theorem* for all Queſtions of the ſame Nature.

§. 2. In any Operation where Letters are alike, they are ſuppoſed to be all of the ſame Nature, as a, $\frac{a}{2}$, $3a$, &c. But different Letters ſuppoſe Quantities of different Nature, as b, c, d, &c. unleſs the contrary be expreſt, as b & c, may be given for two Lines that are known Sides in a Triangle, but of different length.

§. 2. The *Roots* of Quantities are diſtinguiſhable by Figures prefix'd, $\sqrt{ab}$, or $\sqrt{(2)}ab$, ſignifies the Square Root of ab. $\sqrt{(3)}ab$, expreſſes the Cubick Root of ab. $\sqrt{(4)}ab$ the Biquadratic Root, &c.

§. 3. *Unity* is ſuppos'd to be prefix'd to every Quantity, as $1a$, or once a, is the ſame as a it ſelf.

§. 4. If a Quantity have no Sign before it, $+$ is ſuppos'd to be prefix'd, as $+a$ is the ſame with a.

§. 5. The Sign $\times$ has reference to the whole Quantity that follows or precedes it, if a Line be drawn over every Member thereof, as $a \times \overline{b+c+d}$, or $\overline{b+c+d} \times a$, where a is ſuppoſed to be multiplied into each ſimple Quantity b, c, d.

§. 6. A Quantity drawn or multiply'd into it ſelf, is a Square as $a \times a = aa$; if multiply'd into its ſelf three times, 'tis a *Cube* or third Power, as $a \times a \times a = aaa$; if four times, a *Biquadrate*, or fourth Power, as $a \times a \times a \times a = aaaa$, &c. And ſo in Figures, 9 is the Square of 3, 64 the Cube of 4, *&c.* Theſe *literal Powers*, when they riſe high, are more commodiouſly expreſs'd by the Index of the *Power* it ſelf ſet over them: inſtead of aaa, we write a^3, and a^4 inſtead of $aaaa$; ſuppoſe the ſame in the *5th*, *6th*, *7th*, and higher *Powers*.

§. 7. Quantities are *Simple* or *Compound*; *Simple*, when there's but one Member, as b, or bcd, or $ddgg$, &c. *Compounds*, when connected by the Signs $+$ or $-$, as $a+b$, or $dd-ee+gg$, &c.

ADDITION *of Algebraic Integers.*

ADDITION *finds the Sum of two or more given Quantities.*

All the poſſible *Caſes* in Addition are *four*.

I. *Where Quantities have the ſame* Sign *prefix'd, and are of the ſame Nature.*

The Reaſon of the Proceſs. It's ſelf-evident from the ordinary way of Notation, that 2 and 3 make 5, whatever the things be that are added, provided they be of like Nature; as two Men, two Lines, two like Poſitions, (or

like Quantities with the Sign + prefix'd) two like Negations, (or like Quantities with the Sign — prefix'd).

To $2a$	To $+cc-d$	To $b-dd+ccd$	To $-10+da$
Add a	Add $+cc-d$	Add $b-4dd+10ccd$	Add $+3da-16$
Sum $2a+a$	Sum $+2cc-2d$	Sum $2b-5dd+11ccd$	Sum $-26+14da$
or $3a$			

II. *Where Quantities are of the same Value and Nature, and have different Signs.*

The Reason of the Process. If *A* has 5000 *l.* and owes 5000 *l.* it's evident that the Sum (or his whole Estate) is 0. So also in *Quantities*, if the Line *A*——*B* be equal to the Line *B*——*C*, and the last be subducted from the first, (or the first lessened by the last) it's manifest that the Whole is taken away, or is equal to 0.

To $2a$	To $-dd-ee-f$	To $13d+25+\frac{1}{2}+qaa+c^3$
Ad. $-2a$	Add $dd+ee+f$	Add $-13d-25-\frac{1}{2}-qaa-c^3$
Sum $2a-2a$	Sum 0 . 0 . 0	Sum 0 . 0 . 0 . 0 . 0
or 0		

III. *Where Quantities have not the same Value, yet are of like Nature, and have different Signs.* Which is very little different from the former Case.

The Reason of the Process. If *A* has 5000 *l.* and owes 500 *l.* the Whole of his Estate is only lessen'd by so much as the Debt, that is 5000—500, or 4500; but if the Debt had been greater, his Estate had been —4500, or 4500 worse than nothing. Suppose the same in Quantities.

To $3a$	To $3bb+19x-dd-ef$	To $29bb-c^3+d^3-16q^{\frac{1}{3}}+17$
Add $-2a$	Add $-bb-36c+2dd+13ef$	Add $-13bb+2c^3-10d^3+17q^{\frac{1}{3}}-13$
Tot. $3a-2a$	Sum $2bb-17c+dd+12ef$	Sum $16bb+c^3-9d^3+q^{\frac{1}{3}}+4$
or a		

IV. *Where Quantities are of a different Nature.*

The Reason of the Process. Where Quantities are of different Species, (whether the Signs be like or unlike) they are incapable of further Connection, and must therefore be set down with their own Signs prefix'd; two Men and three Horses, make not five Men or five Horses.

To $bb-cc$	To $d^3+qq+dd-h$	To $13+d$
Add $q+d$	Add $rr+ss$	Add $ggg-bb$
Sum $bb-cc+q+d$	Sum $ddd+qq+dd-h+rr+s$	Sum $13+d+ggg-bb$

SUBTRACTION *in Algebraic Integers.*

SUBTRACTION finds the Difference (whether Defect or Excess) between any two propos'd Quantities.

The three Cases that occur in this Rule, are, Subducting Quantities, whose prefix'd Signs are, (1.) + and +. (2.) + and —. (3.) — and —.

The Reaſon of the Proceſs in each. I. To take a Poſitive Quantity out of a Poſitive One, is the ſame thing as to ſubjoin the Defect of the Quantity taken; thus 3 taken out of 5, leaves 5—3, or 2.

II. To ſubtract a Negative Quantity out of a Poſitive One, is the ſame thing as to ſubtract the Subtraction, or take away the Defect of the Quantity taken; and to take away the Defect of a Quantity, is to put it in a poſitive State: Thus—3 taken out of 5, gives 5+3, or 8.

III. To take a Negative Quantity out of a Negative, is (as before) to take away the Negation of ſuch Quantity. Thus—3 taken out of—5, gives—5+3, or—2.

Hence ariſes this general Rule for *Subtraction*, both in whole Numbers and Fractions.

Rule. *Change the Sign or Signs of the Quantity ſubtracting.*

From $5a$	From $3bb$	From $-3cd$	From $-dd+ee-gg$
Take $2a$	Take $-2bb$	Take bb	Take $-dd+gg$
Rem. $5a-2a$	Rem. $3bb+2bb$	Rem. $-3cd-bb$	Rem. $-dd+dd+ee-gg-gg$
or $3a$	or $5bb$		or $ee-gg-gg$

Note. That when it's doubtful whether Quantity is greater, the Difference of them is uſually expreſſed by this Character $a \sim b$, and $ab-dc \sim dd-e$.

MULTIPLICATION *in Algebraic Integers.*

MULTIPLICATION finds the Product or Rectangle of any two propos'd Quantities.

Since Multiplication is nothing elſe but a taking the Multiplicand, ſo often as there are Unites in the Multiplier. Therefore,

As 1 to the Multiplier ∴, ſo the Multiplicand to the Product.

Here alſo occur three Caſes. (1.) When + multiplies +. (2.) When + multiplies —, or the contrary. (3.) When — multiplies —.

The Reaſon of the Proceſs in each. 1. In multiplying + 4 into + 5, I put or repeat the Poſition of 5, ſo often as there are Unites in the Multiplier; therefore the Total will be poſitive, *viz.* + 20.

2. In multiplying + 4 into — 5, I put the Negation of 5 four times; therefore $4 \times -5 = -20$.

And in the Converſe, in multiplying — 4 into + 5, I deny the Poſition of 5 four times; therefore $-4 \times 5 = -20$.

3. In multiplying — 4 into — 5, I deny the Negation of 5 four times; therefore it's made Poſitive four times, *viz.* + 20.

4. Multiplication is either of Simple Quantities into Simple Quantities, or Simple Quantities into Compound ones; or, laſtly, Compound Quantities into Compound.

1. *Simple Quantities* are immediately connected.

Multip. a	Multip. $3aa$	Multip. $-3bb$	Multip. 18
into b	into $-a$	into $-deg$	into ddo
Prod. ba	Prod. $-a^3$	Prod. $+3degbb$	Prod. $18ddo$

2. If one Number multiplies another, the Product ſhall be equal to two Products made of the Multiplication of the firſt Number, into two parts of the ſecond divided at pleaſure: Let 4 multiply 7 for a Product, divide 7 into any two parts, 5 + 2; and multiply each part by 4 for two more Products; Then,

$7 = 5+2$
$4 \quad\quad 4$
$28 = 20+8$

Hence arises the Method of multiplying a Simple Quantity into every Member of a Compound.

Multip. $a+b$ by c — Prod. $ca+cb$

Multip. $dd-16$ by 4 — Prod. $4dd-64$

Multip. $8-3$ by 6 — Prod. $48-18$, or $+30$

3. If two Numbers be divided, each into two parts, the Product of the Whole shall be equal to the several Products arising by the Multiplication of all the Parts of the one into all the Parts of the other. Let 7 multiply 8 for a Product, divide 8 into $6+2$, and 7 into $4+3$. Then,

$8 = 6+2$
$7 = 4+3$
$56 = 24+8+18+6$

Hence arises the Method of multiplying every Member of one Compound Quantity into every Member of another.

Mult. $a+b$ by $a+b$
$aa+ab$
$\quad ab+bb$
Prod. $aa+2ab+cb$

Mult. $aa+cc-dd$ by $aa-cc$
$a^4+aacc-aadd$
$\quad -aacc-c^4+ccdd$
Prod. $a^4-aadd-c^4+ccdd$

Mult. $bcd-16$ by $4+cc$
$4bcd-64$
$\quad -c^3bd-16cc$
Prod. $4bcd-64-c^3bd-16cc$

The beginning of the second Book of *Euclid*'s *Elements* demonstrates this by Lines, which I would not have omitted, but that I presum'd the Process here made use of is very evident without it.

Note. That since there's no such thing as carrying of Tens, Hundreds, *&c.* to the next place, as in Vulgar Arithmetic, we may begin to multiply either, at the right or left Hand, and set the Results where we please, only by observing the preceding Method, like Quantities usually happen to stand over one another, for the greater conveniency of Addition; as in the first Example, ab and ab make $2ab$; in the second, $aacc$, and $-aacc$ destroy each other.

Division *in Algebraic Integers.*

Division teaches how to divide a propos'd Quantity into any assign'd Number of equal Parts.

In common Division 'tis the same thing for Value, whether in dividing 12 by 3, I give 4, or $\frac{12}{3}$ for the Quote. And hence arises the General Rule for Algebraic Division.

Rule. Set the Divisor under the Dividend Fraction-wise.

Divide bc by d } the Quote $\frac{bc}{d}$

Divide $aa+bc$ by $de+10$ } the Quote $\frac{aa+bc}{de+10}$

It frequently happens that these Quotes may be reduc'd into lower Terms by the Methods hereafter mention'd; and particularly where the Terms are great, by a Division much like that in Vulgar Arithmetic; only here as in Multiplication, it's necessary to take notice, that if $+$ divides $+$, it gives $+$ in the Quote; if $+$ divides $-$, (or the contrary) there arises $-$; if $-$ divides $-$, the Result is $+$: And because this depends upon the same Reasoning as that in Multiplication, I presume there's no need of Repetition.

Suppose

Suppoſe I would divide *aaa–eee*, by *a–e*.

$$\begin{array}{l|l} a-e & aaa-eee \;\big(\; aa+ae+ee \\ & \underline{aaa-aae} \\ & \quad aae-eee \\ & \quad \underline{aae-aee} \\ & \quad\quad aee-eee \\ & \quad\quad \underline{aee-eee} \end{array}$$

I firſt enquire what Quantity multiply'd into *a*, (the firſt Member of my Diviſor) will give *aaa*, (the firſt Member in my Dividend) and find *aa*; which ſet in the Quotient, and multiplied into *a–e*, (all the Members of my Diviſor) gives *aaa–aae*; which ſubtracted from the Homologous Term or Terms in the Dividend, leaves *aae–eee* for a new Dividend: Then I enquire again, what Quantity multiplied into *a* will give *aae*, and find *ae*; which ſet in the Quote, multiply'd and ſubducted as before, leaves *aee–eee*. Again, I enquire what Quantity multiply'd into *a* will give *aee*, and find *ee*; which alſo ſet in the Quote, multiply'd and ſubducted as before, leaves nothing. More Examples may be theſe.

$$\begin{array}{l|l} a+b & aa+2ab+bb \;\big(\; a+b \\ & \underline{aa+\;ab} \\ & \quad ab+bb \\ & \quad \underline{ab+bb} \end{array}$$

$$\begin{array}{l|l} a-d & aaa-ddd+b \;\big(\; aa+ad+dd+\frac{b}{a-d} \\ & \underline{aaa-aad} \\ & \quad aad-ddd \\ & \quad \underline{aad-add} \\ & \quad\quad add-ddd \\ & \quad\quad \underline{add-ddd} \\ & \quad\quad\quad +b \\ & \quad\quad\quad \underline{\quad} \\ & \quad\quad\quad a-d \end{array}$$

If (as in the laſt Example) there be a Remainder when the Diviſion is finiſh'd, ſet it Fraction-wiſe as in Vulgar Arithmetic.

Note. Sometimes the Diviſion may not be performed the neareſt way, yet the Quotient will be of the ſame Value, and may be abbreviated.

Of the Definition, Nature and Reduction of FRACTIONS.

A *FRACTION is a Part or Parts of an Unit, repreſenting ſome Diviſible Integer.*

§. 1. The Denominator expreſſes how many Parts *Unity* is divided into, and the Numerator ſhews how many of thoſe Parts are to be taken; as in $\frac{3}{4}$, *Unity* is divided into four Parts, and three of theſe four are the Value of the Fraction $\frac{3}{4}$. Hence,

1. When the Numerator is greater than the Denominator, the Fraction is greater than *Unity*, as $\frac{4}{3} = 1 + \frac{1}{3}$.

2. When the Numerator and Denominator are equal, the Fraction is equal to *Unity*, $\frac{3}{3} = 1$.

3. When the Numerator is leſs than the Denominator, the Fraction is leſs than *Unity*; in the Fraction $\frac{3}{4}$, 4 is not contain'd once in 3.

In the firſt caſe $\frac{4}{3}$ is call'd an *improper Fraction*; and if actually divided into $1 + \frac{1}{3}$, it is term'd a *mix'd Number*, as being part Integer and part Fraction; the laſt Caſe, *viz.* $\frac{3}{4}$ is call'd a *proper Fraction.*

§. 2 The Ratio of two Numbers, is the Quote of the Antecedent divided by the Consequent.

As the Ratio of 8 to 4 is ($\frac{8}{4}$ or) Duple, the Ratio of 4 to 8 is ($\frac{4}{8}$ or) Subduple.

Hence, if the Numerator of one Fraction be to its Denominator, as the Numerator of another Fraction to its Denominator, (that is, if their Quotes are equal) then these Fractions are equal, $\frac{8}{4} = 2 = \frac{6}{3}$.

LEMMA I.

If a Number multiply two Numbers, the Products arising are in the same proportion to each other as the Numbers multiplied. Let 2 and 4 be respectively multiply'd by 3. I say (a) 2. 4 :: (3×2. 3×4) 6. 12, for by §. II. $\frac{2}{4} = \frac{6}{12}$.

(a) 15. *Prop.* 7. *Elect. Coroll.* *id.*

LEMMA II.

If a Number divide two Numbers, the Quotes are in the same proportion to each other as the Numbers divided, let 6 and 8 be respectively divided by 2. I say, that 6. 8 :: $\frac{1}{2}$6. $\frac{1}{2}$8. that is 6. 8 :: ($\frac{1}{2}$ 6 to $\frac{1}{2}$ 8) 3. 4 for $\frac{6}{8} = \frac{3}{4}$.

§. III. *To reduce improper Fractions to Integers.*

Since any Fraction, suppose $\frac{6}{3}$, does not only signify six Thirds of Unity, but also the Third of six Unities: Therefore divide the Numerator by the Denominator, and the Quote shall be an Integer.

Divide $\frac{12}{4}$ by $\frac{4}{1}$ $\Big($ $= 3$ Div. $\frac{ab}{a}$ by a $\Big($ $= b$ Div. $\frac{xx+2xb+bb}{x+b}$ by $x+b$ $\Big($ $= x+b$.

§. IV. *To reduce Integers to Fractions of the same Value.*

If no Denominator is assign'd, let Unity supply the place, and it's evident the Value is unalter'd.

Thus 13 is made $\frac{13}{1}$ $ab = \frac{ab}{1}$ $xa + cx = \frac{xa+cx}{1}$.

But if the Denominator be assign'd, first imagine the Integer made a Fraction by Unity as before; then multiply the assign'd Denominator into both the Numerator and Denominator, the Fraction arising will keep its Value by *Lemma* I.

Let a have b for its Denominator, then $\frac{a}{1} = \frac{ba}{b}$. Let $bb+cc$, have $e+l$ for a Denominator. Then $\frac{bb+cc}{1} = \frac{ebb+ecc+lbb+lcc}{e+l}$; and so of all others.

From the Converse of this arises the following Case.

§. V. *To reduce Fractions into lesser Terms (when possible) of the same Value.*

You may, by inspection, discover what Quantity is multiply'd into both the Numerator and Denominator of the given Fraction, divide each Part by that Quantity, and the Quotes will make a new Fraction of the same Value, by *Lemma* II.

Thus $\frac{bs}{b}$ divided by the common Multiplex b produces $\frac{s}{1}$ or s; and $\frac{bbe+ecc+bbl+ccl}{e+l}$ divided by the common Multiplex $e+l$, is reduced to $\frac{bb+cc}{1}$, or $bb+cc$ of the same Value.

$\frac{abdd}{bbd}$ { Div. by bd } $= \frac{ad}{b}$ $\frac{aa+2ab+bb}{a+b}$ { Div. by $a+b$ } $= \frac{a+b}{1}$, or $a+b$. $\frac{d+e}{dd+e} = 1$;

§. VI.

§. VI. *To reduce mix'd Numbers to Fractions of the same Value.*

First (by §. IV.) make the Integral Part a Fraction, whose Denominator shall be the same with that of the Fractional Part given; and to the Numerator of this new Fraction, add the first Numerator.

Thus $c+\frac{da}{a}=\frac{ca+da}{a}$. $b+c+\frac{bb+cc}{b-c}=\frac{bb-cc+bb+cc}{b-c}$ or $\frac{2bb}{b-c}$

§. VII. *To reduce two Fractions to two Integers, which shall have the same Ratio to each other as Fractions given.*

Multiply alternately the Numerator of the first by the Denominator of the second, and the Numerator of the second by the Denominator of the first, and the two Products shall be the Integers sought.

Thus $\frac{a}{b}$ and $\frac{b}{c}$ are in proportion to each other, as ac and bb. For (by *Lemma* I)

$\frac{a}{b}=\frac{ca}{cb}$ also $\frac{b}{c}=\frac{bb}{cb}$ therefore instead of using the Ratio of $\frac{a}{b}$ to $\frac{b}{c}$ I may use that of $\frac{ac}{cb}$ to $\frac{bb}{cb}$ which is equal thereto: But (by *Lemma* II.) $\frac{ac}{cb}$. $\frac{bb}{cb}$ :: ac bb. *w.w.*&c.

Thus $\frac{2m+ab}{d}$ $\frac{aa-dx}{mm}$ :: $2mm+abmm$. $daa-ddx$.

Thus also $\frac{a+b}{c-e}$ $\frac{c+rr}{p+q}$ :: $pa+bb+qa+qb$. $cc+crr-ec-err$.

And thus, if $\frac{a+b}{e}\cdot=\frac{dd}{rr}$, then also $arr+rrb=cdd$, which will be of frequent use hereafter in the Reduction of Æquations.

§. VIII. *To reduce Fractions to a Common Denominator, retaining the first Value.*

Multiply alternately the Denominator of the last Fraction, into both the Numerator and Denominator of the first, and the Denominator of the first into the Numerator and Denominator of the last, the two new Fractions will be of the same Value by *Lemma* I.

Thus $\frac{a}{b}+\frac{c}{d}$ are reduced to $\frac{da}{db}+\frac{bc}{bd}$ that is $\frac{da+bc}{db}$

Thus also $\frac{aa+cc}{rr}+\frac{dg-gg}{b}=\frac{baa+bcc+dgrr-ggrr}{rrb}$.

If there be more than two Fractions to be reduc'd to a common Denominator, first reduce two of them, as above, then there will be one less than before; and so on till they are all reduc'd to one: or which is the same thing, and perhaps shorter, multiply the Numerator of each Fraction into all the Denominations of the others, for new Numerators; then multiply all the Denominators together for a new Denominator.

$\frac{a}{b}+\frac{c}{d}+\frac{e}{f}=\frac{adf+cbf+cbd}{bdf}$. And so also in Compound Quantities.

§. IX. *To reduce Fractions to a Common Denominator of the same Value, and in the least Terms, when it may be done.*

(1.) If possible, reduce the two Fractions propos'd into the least Term, by the greatest Common Divisor, (*per Lemma* II.) Then multiply alternately as before, (§. VIII.) (2.) If this is not to be done; (1.) Divide the two Denominators by the greatest Common Divisor. (2.) Multiply the Alternate Numerators by these Quotes for new Numerators. (3.) Multiply either of the Denominators by its Alternate Quote for a Common Denominator.

The Abbreviation here (which is of very great uſe) depends on this, That every Member of the New Fraction, is the Quote of every Member of the old One, (when reduc'd as above) divided by the Common Diviſor of the old Denominator, *viz.* *a*; and therefore this Fraction is equal to what it would have been by §. VIII. *viz.* $\frac{nrsa+nsd+atqq-atm}{astn}$ as is evident by *Lemma* II.

Let $\frac{rt+d}{ast}$ and $\frac{qq-m}{ns}$ be propos'd.
a the Common Diviſor.
at and *n* the Quotes.
$\frac{nrt+nd+atqq-atm}{astn}$ the New Fraction.

Theſe 9 Sections well underſtood, there is no difficulty in *Addition, Subtraction*, &c.

Addition *and* Subtraction *of Algebraic Fractions.*

FRACTIONS *(if need be) being firſt reduc'd to a Common Denominator, (by* Lemma I.*) add or ſubtract the Numerators as in Integers, and ſet the Sum or Remainder over the common Denominator.* For from the common way of Notation $\frac{1}{4}+\frac{2}{4}$ of the ſame thing, make $\frac{3}{4}$; alſo $\frac{3}{4}-\frac{1}{4}$ of the ſame thing, is $\frac{2}{4}$ or $\frac{1}{2}$.

Examp. 1. To $\frac{3}{4}$ Add $\frac{3}{7}$ — Sum $\frac{6}{4}$

To $\frac{2}{5}$ Add $\frac{3}{4}$ — Sum $\frac{17}{12}$ when reduc'd.

To $\frac{al+cd}{de}$ Add $\frac{3gg+xx}{de}$ — Sum is $\frac{al+cd+3gg+xx}{de}$

** Remember as in whole Numbers, to change the Sign or Signs of the Quantity ſubtracting.*

Examp. 2. From $\frac{3}{8}$ Take $\frac{2}{8}$ — Rem. $\frac{1}{8}$

From $\frac{2ab-cc}{gg}$ Take $\frac{ab+ec}{gg}$ — Rem. is $\frac{*2ab-cc-ab-ec}{gg}$ or $\frac{ab-2cc}{gg}$

Multiplication *of Algebraic Fractions.*

§. 1. IN Multiplication of *proper* Fractions, the Product is leſs than either of the Factors.

Demonſtr. I would multiply $\frac{2}{3}$ into $\frac{3}{4}$. From the Definition of Multiplication,

1. $\frac{2}{3}$ :: $\frac{3}{4}$ to a fourth Proportional. But 1 ⊏ $\frac{2}{3}$; therefore $\frac{3}{4}$ ⊏ Product; or alternately.

1. $\frac{3}{4}$ :: $\frac{2}{3}$ to a fourth Proportional. But 1 ⊐ $\frac{3}{4}$; therefore $\frac{2}{3}$ ⊏ Product. *W. was to be dem.*

§. 2. Fractions (if poſſible) being firſt reduc'd into lower Terms, multiply the Numerators together for a Common Numerator, and the Denominators for a Common Denominator.

$\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}$ or $\frac{1}{2}$ in the loweſt Terms. $\frac{a}{b}\times\frac{d}{e}=\frac{ad}{be}$.

Multip. $\frac{bl+2cd}{1-t}$ into $\frac{2dr-2rd}{2r}$ — The Product ariſing is $\frac{3drbb+bcddr-2cdbb-4ccdd}{2rr-2rt}$.

To

To demonstrate the Truth of the preceeding Rule, *viz.* $\frac{2}{3} \times \frac{1}{4} = \frac{6}{12}$. From the *defin.* of Multiplication, 1. $\frac{2}{3} :: \frac{1}{4}$ to a fourth Proportional; I am therefore to prove that 1. $\frac{2}{3} :: \frac{1}{4} . \frac{6}{12}$: Or in Letters, 1 $\frac{b}{a} :: \frac{d}{c}\left(\frac{ab}{ac}\right) \frac{b}{c}$ instead of 1 put $\frac{a}{a}$ equal to it, (to bring it in the same denomination with the next Term); then $\frac{a}{a} \cdot \frac{b}{a} :: \frac{a}{c} \cdot \frac{b}{c}$, that is by *Lemma* II. $a . b :: a . b$, which is evidently true; therefore by equality of Proportion, 1. $\frac{b}{a} :: \frac{a}{c}\left(\frac{ab}{ac}\right)\frac{b}{d}$; or 1. $\frac{2}{3} :: \frac{1}{4}, \frac{6}{12}$, *which was to be demonstrated,*

Division *of Algebraic Fractions.*

§. I. IN the division of proper Fractions, the Quote is ⊏ than either the Divisor or Dividend.

Demonst. I would divide $\frac{2}{3}$ by $\frac{1}{4}$, or the contrary: from the Definition of Division; As Divisor to Unity : : so Dividend to the Quote. Then $\frac{2}{3} . 1 :: \frac{1}{4}$ to a fourth Proportional; which must be ⊏ than $\frac{1}{4}$, because $1 ⊏ \frac{2}{3}$. and alternately, $\frac{1}{4} . 1 :: \frac{2}{3}$ to a fourth Proportional, which must be ⊏ than $\frac{2}{3}$, because $1 ⊏ \frac{1}{4}$; *which was to be demonstrated.*

§. II. Fractions (if need be) being first reduc'd to a Common Denominator in the lowest Terms, cast away such Common Denominator, and divide the Numerator of the Dividend by the Numerator of the Divisor.

Divide $\frac{5}{4}$ by $\frac{3}{4}$ } the Quote is $\frac{5}{3}$ For by *Lemma* II. $\frac{5}{4} . \frac{3}{4} :: 5 . 3$.

Divide $\frac{aa+2ab}{3d}$ by $\frac{bb-2ab}{3d}$ } Quote $\frac{aa+2ab}{bb-2ab}$

And so of all others, whether Fractions proper, improper, or mix'd Numbers when reduc'd to a Common Denominator.

Of the Genesis *and* Analysis *of* Powers.

§. I. BY the word *Powers*, is meant *Squares*, *Cubes*, *Biquadrates*, &c. as before. By the *Genesis of Powers*, is understood a Procreation of them, arising by the continual Multiplication of some *Root* into it self; as if a be the Root, then $a \times a$ or a^2 is the second *Power*, $a \times a \times a$, or a^3 is the third *Power*, and so on: 'Tis the same if the Root be a *Binomial*, $a + b$; a *Residual*, as $a - b$; a *Trinomial*, as $a + b + c$; a *Quadrinomial*, as $a + b + c + d$, *&c.*

But the *Power* of a *Binomial*, being the most proper and easy to be resolv'd again into its Root, (which Resolution is what is meant by *Analysis*) I shall only prosecute the Generation of *Powers* from it. *Ex. Gr.*

$a+b$, the Root.

* $\overline{a+b}|^2$, or $aa+2ab+bb$, the Square.

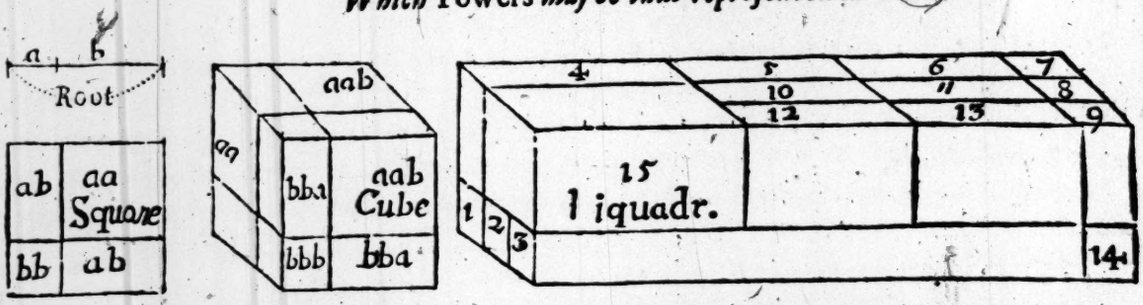
$\overline{a+b}|^3$, or $aaa+3aab+3abb+bbb$, the Cube.

* *See the Characters at the beginning.*

$\overline{a+b}|^{4}_{1}$, or $a^4+4a^3b+6aabb+4ab^3+b^4$, the Biquadrate.

$\overline{a+b}|^{5}_{1}$, or $a^5+5a^4b+10a^3bb+10aab^3+5ab^4+b^5$, the Cubiquadrate, *&c.*

Which Powers *may be thus repreſented in* Plano.



The Parts of the Square are all apparent.

The Parts of the Cube are all viſible except *aaa*, which lies under the higheſt *aab*.

In the Biquadrate the Parts lie thus.

1. a^3b	4. $abbb$	7. a^2b^2	10. a^2b^2	13. ab^3
2. a^3b	5. a^4	8. a^2b^2	11. a^2b^2	14. ab^3
3. a^3b	6. a^3b	9. a^2b^2	12. a^2b^2	15. ab^3

16. $bbbb$, lies under 13, ab^3. all the reſt are viſible.

§. II. All Higher Powers may be repreſented in *Plano*, and will be Parallelipipedons, which I have not found any one yet take notice of, only the length of them will depend upon the Value of their *Binomial Root*, and where we come to have a higher Power than a Cube. Suppoſe *bbba*, it's but multiplying *b* into its ſelf, and let the Product extend in length; and then the two other parts multiply'd into that length, ſhall make up a Parallelipipedon.

§. III. From the Proceſs in the preceding *Geneſis*, which is our Exemplar in all Caſes that can happen, we have a Method of generating Powers from any given *Root*.

Suppoſe I would have the Square of $\overline{dd+rr}|^{2}_{1}$; I muſt make it to anſwer the reſpective Parts of the Square of $a+b$. Thus,

aa	d^4
$2ab$	$2d^2r^2$
bb	r^4

} the Square ſought.

§. IV. If the Root be a *Trinomial*, or have yet more Members, the Caſe is ſtill the ſame, only $aa+2ab+bb$, muſt be repeated ſo many times ſave one as there are Members in the given Root. For Inſtance, if the Root be a *Trinomial*, there will be two Operations.

Suppoſe $\overline{dd-2de+qq}|^{2}_{1}$ was demanded; (1.) Square the two firſt Members as in the Margin. (2.) Let the two firſt Members be *a*, and the next be *b*, then will *aa* be equal to the Square of the two firſt Members, and the ſecond Operation will be thus.

1ſt.

aa	$+dddd$
$2ab$	$-4d^3e.$
bb	$+4dde^2$

} The Square of the firſt two Members.

2d.

aa	$d^4-4d^3e+4dde^2$
$2ab$	$2ddqq-4deqq$
bb	$qqqq$

The whole Square ſought. $d^4-4d^3e+4dde^2+2d^2q^2-4deq^2+q^4$.

This might have been perform'd, by ſquaring the whole Quantity at once; but it's done *gradatim*, for a plainer inſight in the Extraction of Roots, which is only the reverſe hereof, and muſt (ordinarily) be performed *quadratim*. But to apply this to Numbers.

Suppoſe

Suppoſe $\overline{2612}|^{\frac{1}{2}}$ was demanded, we are here to uſe the ſame Proceſs as in the laſt Example, only 'tis neceſſary to have reſpect to the number of Places, as the Square of the firſt Figure 2, or 2000, will have ſix Ciphers after it; the Rectangle of 2 into 6, or 2000 × 600, will have Ciphers. The Square of 6, or 600, is 36 with four Ciphers; and ſo of all others.

aa	4.....	*a*=2
2*ab*	24.....	*b*=6
bb	36....	
aa	676....	*a*=26
2*ab*	52...	*b*=1
bb	1..	
aa	68121..	*a*=261
2*ab*	1044.	*b*=2
bb	4	

6822544 the whole Square of the given *Root*, *viz.* 6212.

§. VI. The Reverſe of this Procedure furniſhes us with a Method for the *Analyſis* or Extraction of Square Roots.

Suppoſe $\overline{aa+2ab+bb}|^{\frac{1}{2}}$ was demanded, (1.) I extract the Root of *aa*, which ſquar'd ſet in the Quote, and ſubducted from *aa*, leaves nothing.

$aa+2ab+bb\,(a+b$
aa
$2ab+bb$
$2ab+bb$

(2.) To find the other Member of the Binomial, *viz.* *b*, I have the double Rectangle of 2*ab*; *a* is already found, by conſequence 2*a* one of the Factors; but if a Rectangle be given, and one of the Factors, the other Factor is given by Diviſion, *viz.* $\frac{2ab}{2a}(b.$ having found the other Member, ſquare it, multiply it into the Diviſor, and ſubduct the Products from 2*ab*+*bb*, (which muſt be firſt brought down as in common Diviſion) and there remains nothing.

1. Hence the in extracting Square Roots, the Diviſor is always twice the Quotient.

2. The *Reſolvend* does always contain, or at leaſt is equal to (2*ab*+*bb*, or) the Square of the Figure laſt ſet in the Quote, more two Rectangles of the laſt Figure into all the Quote.

3. The *Ablatitium* or Number ſubtracting, is always 2*ab*+*bb* exactly.

4. From the Order and Nature of places Unity in the Diviſor, will ſtand under Tens in the Reſolvend.

But in order to diſcover the Square, Cubic, or Biquadratic Root of the firſt Punctation (as alſo further if need be) in Numeral Extractions, the following Table may properly intervene here, whoſe Uſe is briefly thus.

What's the neareſt Square Root of 6; this lies betwixt 4 and 9 in the Column of Squares, and therefore the next Square Root under the Truth is 2 oppoſite to 4. Thus the next Square Root of 79, is that of 64, *viz.* 8. The next Cube Root of 187, is that of 125, *viz.* 5. The next Biquadrate Root of 4563, is that of 4096, *viz.* 8, *&c.*

A Table of Numeral Powers.

√	Sq.	√	Sq.	√	Sq.	√	Sq.	√	Sq.
1	1	21	441	41	1681	61	3721	81	6561
2	4	22	484	42	1764	62	3844	82	6724
3	9	23	529	43	1849	63	3969	83	6889
4	16	24	576	44	1936	64	4096	84	7056
5	25	25	625	45	2025	65	4225	85	7225
6	36	26	676	46	2116	66	4356	86	7396
7	49	27	729	47	2209	67	4489	87	7569
8	64	28	784	48	2304	68	4624	88	7744
9	81	29	841	49	2401	69	4761	89	7921
10	100	30	900	50	2500	70	4900	90	8100
11	121	31	961	51	2601	71	5041	91	8281
12	144	32	1024	52	2704	72	5184	92	8464
13	169	33	1089	53	2809	73	5329	93	8649
14	196	34	1156	54	2916	74	5476	94	8836
15	225	35	1225	55	3025	75	5625	95	9025
16	256	36	1296	56	3136	76	5776	96	9216
17	289	37	1369	57	3249	77	5929	97	9409
18	324	38	1444	58	3364	78	6084	98	9604
19	361	39	1521	59	3481	79	6241	99	9801
20	400	40	1600	60	3600	80	6400	100	10000
√3.	Cub.	√3.	Cub.	√3.	Cub.	√4.	Biqua.	√4.	Biquad.
1	1	11	1331	21	9261	1	1	11	14641
2	8	12	1728	22	10648	2	16	12	20736
3	27	13	2197	23	12167	3	81	13	28561
4	64	14	2744	24	13824	4	256	14	38416
5	125	15	3357	25	15625	5	625	15	50625
6	216	16	4096	26	17576	6	1296	16	65536
7	343	17	4913	27	19683	7	2401	17	83521
8	512	18	5832	28	21952	8	4096	18	104976
9	729	19	6859	29	24389	9	6561	19	130321
10	1000	20	8000	30	27000	10	10000	20	160000

§. VII. To apply the preceding Extraction to Numbers.

Suppoſe $\sqrt{6822544}$ was demanded. After pointing of every other Figure, beginning at the right Hand, the neareſt Root (under truth) of the firſt Punctation, is 2; which ſquared, ſet in the Quote, and ſubducted from 6, leaves 2. Bring down the next Punctation 82, then is 282 (the Reſolvend) $= 2ab + bb$; therefore double the Quote, *viz.* $= 2a$, and thereby divide only the two firſt Figures of 282, to leave room for the Square of the Quote, which is part of the Ablatitium, and there ariſes 6; which ſquar'd, multiply'd by the Diviſor, and the Sum ſubducted from 282, leaves 6.

3. Let $a = 26$, bring down the next Punctat. 25, then is $625 = 2ab + bb$. To find

```
. . . .  (
6822544 (2612 Root.
4 . . . . Ablatitium.
2a..4 )282 .... Reſolvend.
      36 == bb
      24 == 2ab
     276 ..... Ablatit.
2a..52 )625 ... Reſolv.
       1 == bb
      52 == 2ab
     521 ... Ablatit.
2a.522 )10444 . Reſolv.
       4 == bb
    1044 == 2ab
   10444 Ablat:
       0.
```

find *b*, ſquare the Quote, &c. continuing the ſame Operation as before through all the Punctations, and the Proceſs will appear as in the Margin.

But ſince the Tens reſulting in ſquaring 6, (the ſecond Figure in the Quote) may be carried to the Tens in $2ab$, (the other part of the Ablatitium) therefore the Operation may be abbreviated here, though it can't (ordinarily) in higher Powers. Thus.

```
. . . . . 6822544(2612 Root.
                4
              ----
4)282
  276 = 2ab + bb
              ----
52)625
   521
              ----
522)10444
    10444
```

The proof of a Square Extraction. If there be any Remainder, ſubtract it from the Sum propos'd to be extracted, and the Remainder ſhall be equal to the Square of the Quote or Root found, if true, *Ex. gr.* the Root of 6094 is 78, and there remains 10 after the Extraction; extract this 10 from 6094, and there reſts 6084 equal to the Square of Quote or Root 78. If there's no Remainder, the Quote ſquar'd will give the Sum again, as the ſquare Root of 5625 is 75 without a Remainder, for the Square of 75=5615.

§. VIII. If the Quantity propos'd be not a perfect Square, it may nevertheleſs be continued after the ſame manner, by a *Converging Series*, nearer the Truth than any aſſign'd Quantity.

Suppoſe $\overline{aa+b}$; was demanded:

$$\begin{array}{r|l} aa+b & a+\frac{b}{2a}-\frac{bb}{8a^3+4ab}, \text{ &c.} \\ \hline aa & \\ \hline 2a)+b & \\ \quad b+\frac{bb}{4aa} & \\ \hline 2aa+b) & -\frac{bb}{4aa} \\ a & \end{array}$$

If the Quote be continued infinitely, the ſame Method of procedure will continue infinitely: *viz. Always ſquare the Quantity laſt ſet in the Quote, and divide the Product by twice the whole Quote:* After the two firſt Quantities, this will be always a Negative Series. From hence we have a new Method of extracting Square Roots, to an hundred Figures or more in an hour or two's Time, which by the common Methods can't be done in ſo many Years, perhaps not in an Age.

I have ſeen a ſmall Abſtract of *Father Lagny's Nouvelles Methodes*, &c. which is built (I preſume) upon this Principle, and has the ſame Pretenſions, though the Abſtract was too ſhort and imperfect to find the Reaſon of his Proceſs. Alſo our own Ingenious Mr. *Raphſon* was pleaſed to ſhew me the ſame *Series* himſelf; and I doubt not but if he would allow himſelf Time to proſecute it, he might make the ſame Application, and perhaps better than I have done. Mine was this;

1. I conſider'd, that in extracting the Square Root of any Figure, ſuppoſe 2, to any Number of Decimal Places, ſuppoſe 27, at the ſame time I extracted the Root of *two hundred Millions of Millions of Millions of Millions*.

2. That in extracting the Root of four Figures, I ſhould not miſs Unity, if I extracted only the Root of the two firſt with two Ciphers; or if of eight Figures, the Root of 4 with four Ciphers; if of ſixteen Figures, the Root of 8 with eight Ciphers: and ſo on.

3. That in the Operation, great care ought to be taken, that the Surpluſage after Extraction be added in its proper Place; and that alſo in ſubtracting the Square of the true Root from its correſpondent Place, the ſame Caution be obſerved; which perhaps is eaſieſt done, ſuppoſing all to be Decimals after the firſt Punctation, till the Operation be finiſh'd.

From the preceding Conſiderations.

Suppoſe $\overline{2000000000000000000000000}|^{\frac{1}{3}}$ were demanded.

A B C D E

2000000000000000000000000

2) 10 (5
..... Rem.

3) −25 (83
10

−1000 Reſolv. = Rem. + *C*.
−6889 Ablat. = $83|^{\frac{2}{1}}$
$-7889 \times 2834|^{-1}$ (= −2783
−1978000 Reſolv. = Rem. + *D*.
−7745089 Ablat. = $2783|^{\frac{2}{1}}$
$-9723089 \times 2828434|^{-1}$ (= −34376220.

(15 the Root of *A* + *B*
−83 Root of *C*
1417 the Sum.
−2783 Root of *D*
1414217 the Sum.
−34376220 Root of *E*
1414213562380 Root total.

If there had been 32 Figures more of any ſort, Ciphers or not Ciphers, in the Number propos'd, one bare Diviſion more would have given their ſixteen radical Figures; which is an immenſe abbreviation, and perhaps incredible to ſuch as do not apprehend the Reaſon of it.

Here alſo might be ſhew'd from the ſame Principle, a prodigious ſhort way of ſquaring any large Number near, but this would be too great a Digreſſion in this place, and foreign to my purpoſe at preſent.

This I believe is as ſhort a Method as is yet publiſh'd, and I am ſure not yet commonly known; though I have two others, one built upon Mr. *Raphſon's Numeral Series*, conſiderably ſhorter in practice than this: The other much ſhorter than either, which gives the Radical Integers of the whole Sum propos'd only by Inſpection, or at leaſt by one Diviſion, and converges as faſt in Decimals as either of the other; but this I do not yet think fit to publiſh, becauſe I hope to make a farther progreſs in it as to its application to all ſorts of adfected Æquations.

§. X. From the *Binomial* $a+b$, is generated (as before) $\overline{a+b}|^{\frac{3}{1}}$, or $a^3+3aab+3abb+b^3$, which is a ſtanding Original for the Geneſis of a Cube from any given Root.

Let $d-13|^{\frac{3}{1}}$ be propos'd, and the Members reſulting will anſwer the reſpective Parts in the Original, as in the Margin. Suppoſe the ſame in a *Trinomial*, *Quadrinomial*, &c.

aaa	*ddd*
3*aab*	−39*dd*
3*abb*	+507*d*
bbb	−2197

Let $\overline{57209}|^{\frac{3}{1}}$ be demanded.

As in the Square, ſo here we muſt obſerve the Order of Places, the Cube of 5 having twelve Places or Ciphers after it, &c. Alſo *a* is firſt equal to 5, and *b* = 7; then *a* = 57, and *b* = 2. Again, *a* = 572, and *b* = 0. Laſtly, *a* = 5720, and *b* = 9.

a^3	125.....
$3a^2b$	525.....
$3ab^2$	735.....
b^3	343.....
a^3	185193.....
$3a^2b$	19494.....
$3ab^2$	684.....
b^3	8.....
a^3	187149248.....
$3a^2b$	8833968....
$3ab^2$	138996..
b^3	729
	187237601580329

§. XI. As for the Analyſis or Reſolution of the Cube into its Root, we have no more to do than to find the Members that compoſe it, (ſuppoſe it to be a *Binomial*) one whereof is diſcover'd by inſpection, and the other as in the ſubſequent Operation.

Let

Let $\overline{a+b}|^{\frac{1}{3}}$ be demanded.

1. Set the Root of the firſt Member aaa in the Quote; Cube it, and ſubtract from aaa, there reſts nothing.

$$\begin{array}{l|l} aaa+3aab+3abb+bbb & (a+b \\ aaa & \\ \hline & \\ 3aa\,) \; 3aab+3abb+bbb & \text{Reſolvend.} \\ \quad\; 3aab+3abb+bbb & \text{Ablatitium.} \end{array}$$

2. To find the other Member b, we have $3aab$; a is found already, and by conſequence $3aa$; but having the whole Product $3aab$, and one of the Factors, *viz.* $3aa$, the other alſo is found by Diviſion $\frac{3aab}{3aa} = b$.

Set b in the Quote, Cube it, multiply the tripple Square thereof into all the Quote, (which here is only a); multiply alſo the tripple thereof into the Quote (a) Squar'd; which ſeveral Products ſubtracted from the reſpective Members in the Reſolvend, leaves nothing.

COROLLARIES for Cubic Extractions.

I. Hence in extracting Cube Roots, the Triple Square of the whole Quote is always the Diviſor.

II. The Reſolvend does always contain, or at leaſt is $3aab+3abb+bbb$, or the Values expreſs'd by theſe Letters.

III. The *Ablatitium* is always exactly $3aab+3abb+bbb$.

IV. From the Nature of the Punctations, and Order of Places, Unity in the Diviſor will always ſtand under the Place of Hundreds in the Reſolvend. I make uſe of no more for my Diviſor than $3aa$, and thereby ſave ſome Labour; though all I ever yet ſaw add $3aa+3a$; but it comes all to one End.

Let $\overline{187237601}|^{\frac{1}{3}}$ be requir'd.

After pointing every third Figure from the left Hand, take the neareſt Cube Root of the firſt Punctation, by the preceding Table, *viz.* 5. which ſet in the Quote, cub'd and ſubducted from 187, leaves 62; to which bring down 237, it makes 62237 for the Reſolvend.

2. Triple ſquare the Quote for a Diviſor, *viz.* 5, which is 75. 5 will ſtand under 2, the place of Hundreds: So that 75 divides 622, and gives 7 for the Quote. Which cub'd, ſquar'd and multiply'd into the Triple of a, alſo multiply'd into the triple Square of a, gives the three Products, 343, 735, 525; which ſet in their proper places and added, give 60193, the Ablatitium; to be taken out of the Reſolvend 62237, and there reſts 2044; to which bring the next Punctation 601, it makes 2044601, a new Reſolvend.

$$\begin{array}{rrl} & 187237601 & (572 \\ & 125 & \\ \hline 3aa\,.\,75) & 62237 & \text{Reſolvend.} \\ & 343 \;\ldots & bbb \\ & 735 \;\ldots & 3abb \\ & 525 \;\ldots & 3aab \\ \hline & 60193 \;. & \text{Ablatit.} \\ \hline 3aa\,..\,9747) & 2044601 & \text{Reſolv.} \\ & 8 & bbb \\ & 684 & 3abb \\ & 19494 & 3aab \\ \hline & 1956248 & \text{Ablat.} \\ \hline & 88353 & \text{Remaind.} \end{array}$$

3. Triple ſquare the whole Quote 57, which makes 9747 for a Diviſor to the new Reſolvend as far as Hundreds; proceed as before, and there reſts 88353, which ſhows that 187237601 contains a perfect Cube, and 88353 over and above: The Truth might be proſecuted in Decimals near, by adding Ternaries of Ciphers, *viz.* 000, and proceeding after the ſame manner for as many places as is required.

The proof of a Cubic Extraction is this, Take the Remainder, if any, and ſubtract it out of the Sum propoſed to be extracted; then Cube the Quote, which will be equal to that Subtraction if true.

Example. The Cubic Root of 24489 is 29, and there remains 100; take this 100 out of 24489, there reſts 24389 equal to the Cube of 29.

This Method of adding Decimals, gives but one Figure at a time in the Quote; and therefore may be much abridg'd by a *Converging Series*, which throws in 3, 6, or 12, at a time, if need be: The Method of which depends upon the Series in the following Extraction.

From the preceding Conſiderations.

Suppoſe $\overline{20000000000000000000000000}|^{\frac{1}{5}}$ were demanded.

$A\ B\ C\ D\ E$

$\overline{20000000000000000000000000}$ (15 the Root of $A+B$
2) 10 (5 -83 Root of C
..... Rem. 1417 the Sum.
3) -25 (83 -2783 Root of D
10 1414217 the Sum.
-34376220 Root of E
-1000 Reſolv. $=$ Rem. $+C$. 1414213623780 Root total.
-6889 Ablat. $=83|^{2}_{1}$
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Let

Let $\overline{a+b}|^{1}_{3}$ be demanded.

1. Set the Root of the firſt Member aaa in the Quote; Cube it, and ſubtract from aaa, there reſts nothing.

$$\begin{array}{l} aaa+3aab+3abb+bbb\,(a+b \\ aaa \\ \hline\hline 3aa\,)\,3aab+3abb+bbb \text{ Reſolvend.} \\ \quad\;\; 3aab+3abb+bbb \text{ Ablatitium.} \end{array}$$

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$$\begin{array}{rl} & \dot{1}\dot{8}\dot{7}\dot{2}3\dot{7}\dot{6}01\,(572 \\ & 125 \\ 3aa\,\dot{:}\,75\,) & 62237 \quad \text{Reſolvend.} \\ & 343 \ldots\ldots bbb \\ & 735 \ldots\ldots 3abb \\ & 525 \ldots\ldots 3aab \\ & 60193 \ldots \text{Ablat.} \\ 3aa\,\dot{:}\,9747) & 2044601 \quad \text{Reſolv.} \\ & 8 \quad bbb \\ & 684 \quad 3abb \\ & 19494 \quad 3aab \\ & 1956248 \quad \text{Ablat.} \\ & 88353 \quad \text{Remaind.} \end{array}$$

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Example. The Cubic Root of 24489 is 29, and there remains 100; take this 100 out of 24489, there reſts 24389 equal to the Cube of 29.

This Method of adding Decimals, gives but one Figure at a time in the Quote; and therefore may be much abridg'd by a *Converging Series*, which throws in 3, 6, or 12, at a time, if need be: The Method of which depends upon the Series in the following Extraction.

Let $\sqrt{aaa+b}$ be demanded.

$$\begin{array}{r|l} aaa+b & a+\frac{b}{3aa}-\&c. \\ aaa & \\ \hline & 3aa\left(b+\frac{bb}{3a^3}+\frac{b^3}{27a^6}\right. \\ \hline & 3\times a+\frac{b}{3aa}\ \text{Sq.}\ \Big)-\frac{bb}{3a^3}-\frac{b^3}{27a^6} \end{array}$$

Let a ſignify the neareſt Root of the firſt Punctation, and b the Remainder; Then this Series may be expreſs'd in words thus. *If to the Cube Root (near) of the firſt Punctation be added, the Quote of the Remainder divided by the Triple Square of the firſt Quote, leſſen'd by the Quote of the Sum of the Cube of the Quantity laſt placed in the Quote + 3 Parallelipipedons, betwixt the Square of this laſt Quantity into Triple the reſt of the Quote divided by the Triple Square of the whole Quote, repeated at pleaſure, the Series ariſing ſhall, if infinitely continued, infinitely approach the true Value of any Cube Number.*

In the application of this to Numbers, great Care muſt be taken (as in the preceding Quadratic Series) of the Value of Places, becauſe of adding and ſubtracting, and conſequently finding the Place of each Figure ariſing in the Quote.

Note. This Method for Cubic Extractions is of little uſe, but when the Number given to be extracted is very large. I ſhall only give as ſmall an Example as is practicable in this Method.

Let the *Cubic Root* of the mix'd Number 37.945 be demanded to four Places in Decimals, and the Operation will be as follows.

Such as pleaſe may, with a little more labour, throw yet more Decimals at a time in the Quote, if inſtead of the aforeſaid Quadratic Series, they uſe $a+\frac{ab}{2aa+b}+$, *&c.* And if inſtead of the Cubic they uſe this $a+\frac{ab}{3aa+b}+$, *&c.* Which two are found by taking as much of the ſecond and third ſtep in each Series together as may be conveniently done.

A B C

37 245 000 000 000 000 (3 . . . . . a
3.7 . . . . . b
3.37
— 00961 . . . c

27
27) 10.0 (.37 = b
81
190
189
1 = Rem.
.955000 = Rem. + B
.050653 = Cube of Quantity laſt placed in the Quote.
1.2321 . . = to its triple Square × a.
Triple the wh. Quote — 1.282753 Ablatitium.
34.0707) — .0277530 (— 00961 = c
3066363
2111670
2044242
67428 = Rem.

All the Qu. 33.6039—

Another Operation would give all the Radical Figures, if the Sum propos'd was continued to eighteen more Places.

And in like manner the *Biquadratic, Cubiquadratick,* &c. *Roots* may be very briefly extracted.

I ſhall paſs over the Buſineſs of mix'd or affected Extractions, ſince that is much more conveniently perform'd by *Converging Series* as hereafter.

Re-

Reduction of Simple Equations.

§. 1. AMongſt Quantities of different Values, there may be yet an Equality, as 40 *s.* = 2 *l.* which may be repreſented in Letters at pleaſure; But becauſe in Algebraic Operations, known and unknown Quantities will be indifferently mix'd and complicated, therefore it will be convenient, for an eaſier diſtinction of them, to uſe different Characters, as *a*, *b*, *c*, *d*, &c. for ſuch as are known; and *x*, *y*, *z*, for ſuch as are unknown.

§. 2. When Quantities are thus mix'd, it's neceſſary ſo to *reduce* them, as that the *known* may poſſeſs one Side of the Equation, and the *unknown* the other. *Ex. gr.* $\frac{ab}{c} = x$; whence by comparing them together, the Value of the *unknown* may be diſcovered.

§. 3. Equation are of different Kinds, according as the Root of the reſpective Powers of the *unknown Quantity* is compounded, *viz. Simple* (or Lateral) *Quadratic*, *Cubic*, *Biquadratic*, or yet higher.

§. 4. In the Reſolution of *Simple* and *Quadratic Equations*, I ſhall make uſe of the common Methods, unleſs where the *Homogeneum Comparationis*, or *abſolutely known Quantity* is very great; in which Caſe (as well as in *Cubics*, *Biquadratics* and *higher Powers*) the new *Converging Series* is much preferable.

The *Reduction* of Simple Equations, which is firſt in order, may be effected, either

By Addition.

§. 5. IF *equal Things be added to Equal, the Totals will be Equal.* Wherefore to get the unknown Quautity *x* by its ſelf one ſide of the Equation,

Let there be propos'd,
- $x - a = b + c$ Add $a \ldots a$ gives $x = b + c + a$
- $c - x = 0$ Add $x \ldots x$ gives $c = x$
- $a + b - x = -gg$ Add $x \ldots . x$, $gg \ldots . gg$ gives $a + b + gg = x$

By Subtraction.

IF *equal things be taken from Equal, the Remainder will be Equal.* Ex. gr.

- $x + a = b + c$ Sub. $-a \ldots . -a$ gives $x = b + c - a$
- $dd + cc = qq + 10 + x + 10d + \frac{1}{7}$ Sub. $-q^{\circ} - 10 - 10d - \frac{1}{7} \ldots - q^{\circ} - 10 - 10d + \frac{1}{7}$ gives $dd + cc - q^{\circ} - 10 - 10d - \frac{1}{7} = x$

Note. Theſe two ways of Reduction, *viz.* by *Addition* and *Subtraction*, are compendiouſly perform'd by tranſpoſition of Quantities to the other Side of the Equation with contrary Signs; as in the preceding Examples.

Then, $x - a = b + c$ Then, $x = b + c + a$

Then, $ab - x = -gg$ Then, $ab + gg = x$

Then, $dd + cc = qq - 10 + x + 10d + \frac{1}{7}$ Then, $dd + cc - qq - 100 - 10d - \frac{1}{7} = x$

By Multiplication.

If equal Quantities be multiply'd by equal Quantities, the Products will be equal.

Propose $\frac{x}{a}=b$ $\frac{x}{3d}=c$ $\frac{x}{bb+c}=rr$

Mult. by a. $\frac{ax}{a}=ab$ Mult. by $3d$. $\frac{3dx}{3d}=3dc$ Mult. by $bb+c$. $\frac{bbx+cx}{bb+c}=bbrr+crr$

That is $x=ab$ That is $x=3dc$ That is $x=bbrr+crr$

But in Practice only multiply the Denominator of the Fraction by the whole Number on the other side the Equation.

Ex. gr. $\frac{x}{3d}=c$, or $x=3dc$; And so of all others.

By Division.

If equal Quantities be divided by equal Quantities, the Quotes will be equal.

Propose $xx.=6x$ $dx+cx=qq$ $16x-10=100$

That is $16x=90$

Div. by x. $\frac{xx}{x}=\frac{6x}{x}$ Div. by $d+c$. $\frac{dx+cx}{d+c}=\frac{qq}{d+c}$ Div. by 16. $\frac{16x}{16}=\frac{90}{16}$

Or $x=6$ Or $x=\frac{qq}{d+c}$ Or $x=\frac{90}{16}$

Abbridg the Operation thus; Take all the Quantities, how many soever, that are multiply'd into the unknown Quantity (here) x, and by them divide the other side of the Equation.

Ex. gr. $dx+cx=qq$. Thus, $x=\frac{qq}{d+c}$. And so of all others.

In Simple Equations, the unknown Term x is always clear'd by a contrary Operation: Therefore if it be multiply'd into a Quantity, divide both Parts of the Equation by that Quantity; If it be divided by a Quantity, multiply, *&c.*

Reduction of Equations *to Proportional Terms.*

Since if four Quantities are proportional, the Product of the *extream Terms* is equal to that of the *middle*: From the Reverse hereof 'tis evident, that if an Equation can be so divided into four Parts, that the Product of the *Means* shall be equal to the Products of the Extreams, then such Parts are proportional Terms; wherein to make the thing useful, it will be convenient that the last Part (or fourth Term) be the unknown Number.

1. Observe, that if the Equation consist of whole Numbers, let that Quantity which is multiply'd into x, be the first Term.

Ex. $3abx=qq$
Then $3ab.q::q.x$.

If $dx+cx-bx=rrd$
Then $d+c+b\left\{\begin{matrix}rr\\r\end{matrix}\right\}::\left\{\begin{matrix}d\\rd\end{matrix}\right\}x$

2. Unity

2. Unity will be one of the Terms, if nothing else can be found.

Ex. $ax+b=r$. That is $ax=r-b$; then $a.1::r-b.x$

In affected Equations $xx+bx=d$. or $xx-d=bx$. then $b.xx-d::1.x$

3. In Fractions; If no other way presents it self, reduce them to Integers, and then work as before.

Ex. $\frac{xb}{c}=dd$. $\frac{b}{c}.d::d.x$. If it had been $\frac{xb}{c}=d$, then

$xb=cd$. Whence again $b.c::d.x$.

These Methods are of great Use for expressing Algebraic Canons in Proportional Terms, and consequently for turning such Canons into Geometric Effections (if need be); also in finding the Tangents of Curves, and other useful Applications.

The Resolution of Simple Equations.

WHEN a Question is propounded, put x for the unknown Number or Quantity sought, and work with it according to the Terms of the Question as if it were known, till an Equation be made; and then by some (or all) of the preceding Cases in Reduction, x may be clear'd and possess only one Side of the Equation; and thus being equal to something that is known, its Value is also discover'd.

Quest. 1. The Sum of two Numbers is s ($=19$) their Difference is d ($=9$); What are the Numbers?

1. Put x for the greater Number ——	x
2. Then the Lesser will be $s-x$ ——	$s-x$
3. Subtract the Lesser from the Greater to find their Difference, which is equal to d. ——	$-s+2x$ Or $2x-s=d$
4. Transpose s ——	$2x=d+s$
5. Clear x by Division ——	$x=\frac{d+s}{2}$

Whence this CANON, *Half the Sum and Difference of any two Numbers, is the greater Number.*

Examen. In Numbers $d=9$, and $s=19$ by Position; Then $\frac{9+19}{2}=14=x$

Therefore also the lesser Number is $19-14$, or 5.

But it's evident that 14 and 5 do satisfy the Question, for their Sum is 19, and their Difference is 9.

Quest. 2. The Product of two Numbers is p ($=100$) their Difference is the greater Number lessen'd by 2. What are the Numbers?

The greater	x
The leſſer	$\frac{p}{x}$
Their Difference is	$x - \frac{p}{x}$ or $\frac{xx-p}{x}$
The Equation	$\frac{xx-p}{x} = x-2$
Which reduc'd to Integers, is	$xx-p = xx-2x$
From each part ſubtract xx	$-p = -2x$
Change the Signs, (the Equality will yet remain) and clear x by Diviſion	$x = \frac{p}{2} = 50$

Then the leſſer will be p divided by $\frac{p}{2}$, *viz.* 2.

The Numbers are 50 and 2, whoſe Product is 100, and whoſe Difference is $50-2$, or 48.

Queſt. 3. The Ratio of two Numbers is r to s, (3 to 2) their Difference is d (8.) What are the Numbers.

The firſt Number	x
The ſecond. $(r.s :: x. \frac{sx}{r})$	$\frac{sx}{r}$
Their Difference is $x - \frac{sx}{r}$ or	$\frac{rx-sx}{r}$
The Equation	$\frac{rx-sx}{r} = d$
Multiply each Part by r	$rx-sx = rd$
Divide each part by $r-s$	$x = \frac{rd}{r-s}$

In Numbers $x = \frac{24}{3-2} = 24$ the firſt Number.

For the ſecond, $3.2 :: 24. \left(\frac{48}{3}\right) 16$; the ſecond Number: Which two will ſatisfy the Queſtion.

A Collection of Queſtions for Exerciſing the preceding Rules.

Q. 4. A Son asks his Father how old he was? Who anſwered, If from my Years thou takeſt 5, and divideſt the Remainder by 8, the Quote will be $\frac{1}{3}$ of thy Years. But if to thy Years thou add 72, and multiply the Sum by 3, taking 7 from the Product, the Remainder is the Sum of my Years. How old was Father and Son?

Q. 5. Two Footmen travel, the firſt goes only ſix Miles every day, the other goes 10, but ſets not out till eight Days after the firſt. In how many Days will the laſt overtake the firſt?

Q. 6. From *Norimberg* to *Rome* are 140 Miles. Two Travellers ſet out at the ſame Time from theſe two Cities, one travelling eight Miles every Day, the other Six; How long will they be before they meet; and how many Miles ſhall each have travelled?

Q. 7. A Captain ſends forth $\frac{1}{3}$ of his Souldiers $+ 10$, and there remains with him $\frac{1}{3} + 15$; How many Souldiers has he in all?

Q. 8. To find the Side of a Square, whoſe *Area* ſhall be to the Sum of the Sides in a given Reaſon. Suppoſe r to s, (45 to 12.)

Q. 9. To find the Side of a Cube, whoſe Surface ſhall be to its Solidity in a given Reaſon. Suppoſe r to s, (6 to 12.)

Q. 10. A certain Vintner has two ſorts of Wines, *A* and *B*: An equal Mixture of both is worth 15 *d.* a Bottle: But if they be ſo mix'd that three times the Quantity

tity of *A* be taken for two times the Quantity of *B*, the mixture will be worth 14 *d.* a Bottle.

Of Questions by Diverse Positions.

WHere Questions are answered by diverse Positions, so many unknown Quantities must be us'd as the Nature and Variety of the Question calls for. But before the Question can be resolv'd, all the unknown Terms must be destroy'd except one; Thus, Equate some one of the unknown Quantities to all the rest by Transposition; then instead of that Quantity, substitute what's equal to it in the following Equation, whereby it will vanish, and so destroy one by one, till only one remains. *Ex. gr.*

Quest. 1. To divide *A* into three parts x, y, z; So that $\begin{cases} 1.\ x+y=a \\ 2.\ y+z=b \\ 3.\ z+x=c \end{cases}$

Solution, 1. $x+y=a$. Whence $y=a-x$

By Subtract. 2. $a-x+z=b$. Whence $z=b-a+x$

3. $b-a+2x=c$. Whence $x=\frac{c+a-b}{2}$ Sought.

Again, 1. $\frac{c+a-b}{2}+y=a$ Whence $y=\frac{a+b-c}{2}$ Sought.

2. $\frac{a+b-c}{2}+z=b$. Whence $z=\frac{b+c-a}{2}$ Sought.

Otherwise (and sometimes more commodiously) the unknown Terms may be destroy'd by Addition or Subtraction. Let the same Question be repeated.

$\left.\begin{array}{l} 1.\ x+y=a \\ 2.\ y+z=b \\ 3.\ z+x=c \end{array}\right\}$ Sub. $\begin{array}{l} 1.\ x+y=a \\ 2.\ y+z=b \\ \hline x-z=a-b \end{array}$ x being found, the other are easily found as before.

Add 3. $z+x=c$

$2x=a+c-b$

Or, $x=\frac{a+c-b}{2}$ Sought.

Multiplication or Division may help sometimes to destroy the unknown Terms, but very rarely.

There's yet another Method frequently made use of by Mr. *Oughtred*, in his *Opuscula Mathematica*, *viz.* putting one unknown Quantity for the Sum of all, which when discover'd, the parts are readily found out. Let the same Question be still propos'd.

$\left.\begin{array}{l} 1.\ x+y=a \\ 2.\ y+z=b \\ 3.\ z+x=c \end{array}\right\}$ & $u=x+y+z$ $\begin{cases} z=u-a \\ x=u-b \\ y=u-c \\ \hline u=3u-a-b-c \end{cases}$

And after Reduction $2u=a+b+c$. Or $u=\frac{a+b+c}{2}$ Sought.

Some of these Methods will never fail in all Cases that can happen in Questions of this Nature. I shall only add three or four for further Exercise.

Q. 2. The Husband, Son and Wife, make 96 Years: Those of the Husband with his Son's are equal to the Wive's + 15. Those of the Wife with the Son's, make the Husband's + 2. The Age of each is demanded?

Q. 3. To find three Numbers; So that the first with $\frac{1}{2}$ of the Remainder, the second with $\frac{1}{3}$ of the Remainder; the third with $\frac{1}{4}$ of the Remainder, shall always make 34.

Q. 4. To find four Numbers, u, x, y, z. So that $u+x+y=z+b$. $x+y+z=u+c$. $y+z+u=x+d$.

Q. 5. To find three Numbers, x, y, z. So that $\frac{1}{4}x+\frac{1}{3}y=\frac{1}{3}y+\frac{1}{4}z=\frac{1}{4}z+\frac{1}{3}x$.

☞ *Note.* In this and the like Questions, it will much shorten the Work and ease the Difficulty, to resolve all the Numbers into x, according to the Tenour of the Question, and there will arise x, $\frac{1}{3}x=y$. and $\frac{1}{4}x=z$: Then take any Number for x, which may be divided by the Denominators of the Numbers resolved in to x, *viz.* 6 and 3, and the Business is done. Let x be 6, then is y 5, and z 4.

Q. 6. To find three Numbers x, y, z. So that $x+\frac{1}{4}y+\frac{1}{3}z=y+\frac{1}{4}z+\frac{1}{3}x=z+\frac{1}{3}x+\frac{1}{4}y$.

Q. 7. Two Numbers being given b and c, to find a third x. So that $\overline{c+x}\times b$. $\overline{x+b}\times c$, and $\overline{b+c}\times a$, may decrease by an equal Interval.

Of Indetermin'd Questions.

IF a Question does not appear to be indetermined at first sight, (which very seldom misses when 'tis such) yet 'tis discoverable by this, if the Equation or Equations can be so transpos'd and varied, that the known Terms may be equated amongst themselves, (considering them as universal Quantities and not particular Numbers) then such Questions are indetermin'd. For instance; Let

1. $z+y=a$. Whence $y=a-z$
2. $y+x=b$. Whence $y=b-x$ } Then $x=z-a+b$.
3. $x+u=c$. Whence $x=c-u=z-a+b$, & $z=a-b+c-u$.
4. $u+z=d$. Whence $z=d-u=a-b+c-u$, & $d=a-b+c$, all known.

But universal Quantities cannot be equal among themselves, unless every Number was equal to every other Number, which is absurd.

2. A Question is also indetermin'd, which, when the Conditions are satisfied, does include two or more unknown Terms in the Equation, $ay=xx$: There may be innumerable Values of y and x, and the Equation yet be true.

3. When there is not a mutual or reciprocal Habitude betwixt the two Terms in a Question, such Question is undetermin'd. For Instance, *There are two Numbers, three times the greater is equal to the triple Square of the less.* If the Question stays here 'tis indetermin'd, but if the second Number does reciprocally affect the first, thus; *And twice the less is equal to half the greater*; or, *and the Sum, Difference*, &c. *of both is so*; then the Question is determin'd, and admits but of one Answer. There might be other Rules laid down, but none (for any thing I yet see) which are not reducible to one of these Heads.

Diophantus, who by his prodigious Sagacity and Artifices in resolving Questions of this Nature, has deservedly perpetuated his Name, may be our Pattern in some Examples.

8. *c.* 2. *Dioph.* Q. 1. To divide a given Square aa into two other Squares.

Let one of the sought Squares be xx, (whose Side is x) then the other will be $aa-xx$, whose Side (any Arbitrary Number being taken, as b) let be $bx-a$; the Square of which is $bbxx-2bxa+aa=aa-xx$; or $bbx-2ba=-x$, or $bba+x=2ba$:

Which

Which reduc'd, gives $x=\frac{2ba}{bb+1}$ the Side of one of the Squares ſought. Which ſquar'd and ſubducted from the firſt given Square, leaves the other Square ſought. This Reſolution is of great uſe, for upon this depends the Invention of innumerable Sides and Varieties in a Right-angled Triangle.

10. *e.* 2. *Q.* 2. To divide $aa+bb$ into two other Squares, Let $a=b$. Aſſuming two Numbers at pleaſure, $c=d$. Let the ſide of the firſt Square be $ax-a$, and the ſide of the other $cx-b$; then proceed as in the laſt.

11. *e.* 2.] *Q.* 3. To find two ſquare Numbers, whoſe Difference is d.

12. *e.* 2.] *Q.* 4. To find a Number, which added to b and to c, ſhall make two Squares.

15. *e.* 2.] *Q.* 5. To divide a into two ſuch Parts, that if aa be added to each, the Sums will be Squares.

21. *e.* 2.] *Q.* 6. To find two Numbers x and y, ſo that $xx+y$, and $yy+x$ ſhall be ſquare Numbers.

24. *e.* 2.] *Q.* 7. To find two Numbers x, y, ſo that $xx-x-y$, and $yy-y-x$, ſhall be ſquare Numbers.

33. *e.* 2.] *Q.* 8. To find three Numbers, x, y, z. So that $xx+y$, and $yy+z$, and $zz+x$ ſhall be Squares.

1. *e.* 3. *Dioph.*] *Q.* 9. To find three Numbers, x, y, z. So that $x+y+z-xx$, and $x+y+z-yy$, and $x+y+z-zz$, ſhall be ſquare Numbers.

9. *e.* 3.] *Q.* 10. To find three Numbers, x, y, z. So that both $x-y=y-z$; and yet the Sums of the three Numbers be all Squares.

18. *e.* 3.] *Q.* 11. To find three Numbers, x, y, z. So that both $xy+x+y$, and $x+y+z$, and $zx+z+x$ ſhall be Squares.

19. *e.* 3.] *Q.* 12. To find three Numbers, x, y, z. So that $xy-x-y$, and $yz-y-z$, and $yx-y-x$ ſhall be Squares.

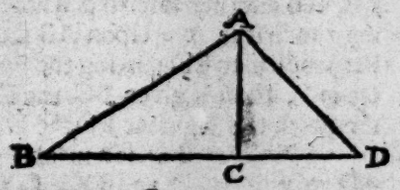
Q. 13. If a Perpendicular be let fall from the oppoſite Side of a Right-angled Triangle, thereby dividing the whole into two other, ſimilar and like the whole; 'tis demanded to give a Theorem to find innumerable Integers expreſſive of the Sides.

Q. 14. To find four more Right-angled Triangles in Integers, which ſhall have the ſame *Hypothenuſe*.

Queſtions producing Simple Equations in Geometry.

I Shall only put a few of the eaſieſt Queſtions here that I can light on, yet ſuch as ſhall be introductive to higher, and exerciſe ſome of the moſt uſeful Propoſitions in *Euclid*, that aſſiſt us in the forming Equations.

Queſt. 1. In a Triangle, ABD, all the Sides are given; the Segments of the Baſe (made by falling a Perpendicular from A) are ſought.



AB $=a$
BD $=b$
DA $=c$
BC $=y$. CD $=b-y$

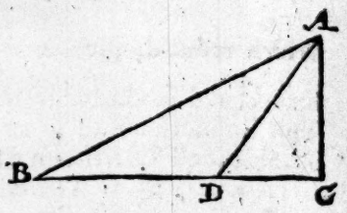
$aa-yy=$ AC Square.
$cc-bb+2by-yy=$ AC Square. } 47. *e.* 1. *Encl.*

Whereſ. $aa-yy=cc-bb+2by-yy$, or $aa=cc-bb+2by$.

Whence $aa+bb-cc=2by$. Then $y=\frac{bb+aa-cc}{2b}$

After the ſame manner putting CD $=x$, and BC $=b-x$, there ariſes $x=\frac{bb+cc-aa}{2b}$

Queſt. 2. In any Obtuſe-angled Triangle ABD, the three Sides being given, and either of the leſſer BD, produc'd till it meet a Perpend AC, the extream Segment DC is required?



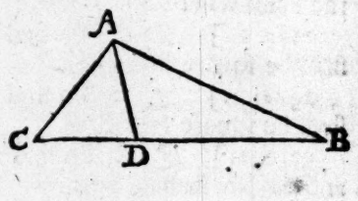
$AB = a$
$BD = b$
$DA = c$
$DC = x$

$aa - bb - 2bx - xx = ACq$
$cc - xx = ACq$ } 47. *e.* 1.

Wherefore $aa - bb - 2bx - xx = cc - xx$, or $aa - bb - 2bx = cc$.

Whence $aa - cc - bb = 2bx$, then $x = \frac{aa - cc - bb}{2b}$

Queſt. 3. In any Triangle ABC, whoſe Angle at the top is equally biſected, the Sides AB, AC, with one of the Segments DB are given, the other Segment CD is required?



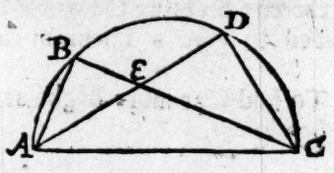
By 3. *e.* 6. AB. AC :: DB, CD, then $AB \times CD = AC \times DB$, then by Reduction.

$\frac{AC \times DB}{AB} = CD$ required.

Queſt. 4. Two Triangles ABC, ADC, being given, ſtanding upon the ſame Baſis AC, to find the Segments EC, or BE?

The Angles at B and D are both right, by 31. *e.* 3. *Encl.*
The oppoſite Angle BEA and DEC, are equal, (15. *e.* 1.)
Therefore the remaining Angles BAE, DCE are equal, (26. *e.* 1.)
Then the Triangles BEA, DEC, are ſimilar and proportional, (4. *e.* 6.)

$AB = a$
$BC = b$
$CD = c$
$DA = d$
$EC = x$
$BE = b - x$.



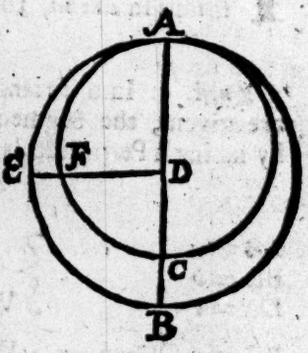
Wherefore AB. CD :: BE. DE. and DC. CE :: AB. AE.

That is $a . c :: b - x . \frac{cb - cx}{a}$. and $c . x :: a . \frac{ax}{c}$

But $DE + AE = AD$; that is $\frac{cb - cx}{a} + \frac{ax}{c} = d$; which reduced, gives

$x = \frac{acd - ccb}{aa - cc} = EC$ required.

Queſt. 5. Let there be a Circle, whoſe Diameter is AB, and another inſcrib'd whoſe Diameter is AC, touching the firſt in A: Upon AB from the Center D, erect a Perpendicular ED, cutting the Periphery of the leſſer Circle in F, there's given BC the Difference of the Diameters, with the Segment EF; the Diameters are demanded?



$AD = x$
$EF = a$
$FD = x - a$
$CB = b$

FD is a mean Proportional betwixt AD and DC, (13. *e.* 6.)
Therefore $AD \times DC = FDq$, that is
$xx - bx = xx - 2ax + aa$.

Which reduc'd, gives $x = \frac{aa}{2a - b} = AD$,

Whoſe double is the greater Diameter required; which leſſened by b, gives the Leſſer.

Of

Of Surd Quantities.

Notation of Surds.

§. 1. WHEN the Root of any Quantity can't be truly expreſs'd, (according to any known Notation) ſuch Root is call'd a *Surd*, *Irrational*, or *Imperfect Root*, whether of a *Square Cube*, &c. Thus the Square Root of 18 is Imperfect; the next to it in whole Numbers is 4. The uſual way of this Notation, is to prefix a Note of Radicality, with the reſpective Index of the Power. For Inſtance, The Square Root of 18 is $\sqrt{(2)}18$: or $\sqrt{}18$ the Biquadratic Root thereof is $\sqrt{(4)}18$: Or according to Dr. *Wallis* and Mr. *Newton*'s way of Notation, the Square Root of 18 is $\overline{18}|^{\frac{1}{2}}$; the Square Root of $aa+bb$ is $\overline{aa+bb}|^{\frac{1}{2}}$; the Cube Root of the Square Root of $aa+bb$ is $\overline{\overline{aa+bb}|^{\frac{1}{2}}}|^{\frac{1}{3}}$, or $\overline{aa+bb}|^{\frac{1}{6}}$. The Biquadratic Root of the Cube of $a+b$, is $\overline{a+b}|^{\frac{3}{4}}$; the Cube Root of $\frac{1}{27}q+\overline{\frac{1}{729}qq+\frac{1}{19683}p^3}|^{\frac{1}{2}}$, is $\overline{\frac{1}{27}q+\overline{\frac{1}{729}qq+\frac{1}{19683}p^3}|^{\frac{1}{2}}}|^{\frac{1}{3}}$: And ſo of all others, $\frac{1}{4}$ being the Index of Radicality of the fourth Power, $\frac{1}{5}$ that of the fifth Power, &c.

In Fractions the Cubic Root of $\frac{a}{b}$ is $\sqrt{(3)}\frac{a}{b}$ or $\overline{\frac{a}{b}}|^{\frac{1}{3}}$, or $\overline{\frac{a}{b}}|^{\frac{1}{3}}|^{\frac{1}{3}}$, which laſt is the beſt way, where either the Numerator or Denominator are *perfect Powers*. Thus the Cubic Root of $\frac{20}{27}$ is $\overline{\frac{20}{27}}|^{\frac{1}{3}}$, that is $\overline{\frac{20}{3}}|^{\frac{1}{3}}$, or the Cubic Root of 20 divided by 3. The Cubic Root of $\frac{a^3}{bb}$ is $\overline{\frac{a}{bb}}|^{\frac{1}{3}}$: The Square Root of $\frac{dd-e}{gf+bg+bb}$ is $\overline{\frac{dd-e}{g+b}}|^{\frac{1}{2}}$ &c.

Reduction of Surds to lower Terms when poſſible.

§. 2. All Surd Quantities may be expreſs'd as above; yet ſome are reduc'd into lower Terms by this Method. Divide the given *Surd* by the greateſt *Square*, *Cube*, *Biquadrate*, &c. contain'd therein, leaving no Remainder, (according to the Index thereof) and prefix the Root of ſuch Diviſor, whether *Square*, *Cube*, *Biquadrate*, &c. before the ſaid Surd, after the Method of Multiplication.

For Inſtance, $\sqrt{12}$, divided by the greateſt Square in it, that leaves no Remainder; *viz.* 4 gives 3 in the Quote: Hence $\sqrt{12}=2\sqrt{3}$, (in the loweſt terms) for $\sqrt{4}\times\sqrt{3}=\sqrt{12}$, but $\sqrt{4}=2$; therefore $2\sqrt{3}=\sqrt{12}$. Thus $\sqrt{75}=5\sqrt{3}$; for $5=\sqrt{25}$, and $25\times3=75$. Thus alſo $\sqrt{(3)}40$, or $\overline{40}|^{\frac{1}{3}}=2\sqrt{(3)}5$, or $2\times\overline{5}|^{\frac{1}{3}}$. In Species $\sqrt{a^2b}=a\sqrt{b}$. $\sqrt{a^2b-aabb+2aabc+abcc-ab^3+bbcc-2b^2c+b^3}=\overline{a+c-b}\sqrt{ab+bb}$. $\sqrt{\frac{48a^2b}{10}}=4a\sqrt{\frac{3ab}{10}}$. $\sqrt{\frac{aaeemm+4aamm}{ppee}}=\frac{am}{pe}\times\overline{\frac{ee+4}{1}}|^{\frac{1}{2}}$ or $\frac{am}{pe}\times\overline{ee+4}|^{\frac{1}{2}}$. This Reduction is of very great uſe, becauſe the Values of Quantities are are more eaſily diſcovered in leſſer Terms than greater, alſo in the *Approximation of Roots*. For Inſtance, If $d=\sqrt{aabb+aacc}$, then is $d=a\sqrt{bb+cc}$; and dividing each part of the Equation by a, we have $\frac{d}{a}=\sqrt{bb+cc}$, that is according to the Method of extracting Surd Quantities, $\frac{d}{a}=b+\frac{cc}{2b}$, &c. Whereas if an immediate

Immediately extraction had been made without Reduction, we ſhould have had $d = ab + \frac{acc}{2b} -$ &c. which is more perplex'd and troubleſome, eſpecially if continued to another Operation.

For the greater Eaſe in a reduc'd Notation of *Numeral Surds*, the following Table may properly be here inſerted; whoſe Uſe is briefly thus. Oppoſite to $\sqrt{45}$ in the firſt Column is $3\sqrt{5} = \sqrt{45}$ in the loweſt Terms, $\sqrt{288} = 6\sqrt{8}$ reduc'd. $\sqrt{(3)2560} = 8\sqrt{(3)5}$.

A Table of Irrational Surds, reduc'd to lower Terms.

$\sqrt{6}$	=	$2\sqrt{1\frac{1}{2}}$	$\sqrt{75}$	=	$5\sqrt{3}$	$\sqrt{200}$	=	$5\sqrt{8}$	$\sqrt{486}$	=	$9\sqrt{6}$
$\sqrt{8}$		$2\sqrt{2\frac{1}{2}}$	$\sqrt{80}$		$4\sqrt{5}$	$\sqrt{216}$		$6\sqrt{6}$	$\sqrt{490}$		$7\sqrt{10}$
$\sqrt{10}$		$2\sqrt{2}$	$\sqrt{90}$		$3\sqrt{10}$	$\sqrt{243}$		$9\sqrt{3}$	$\sqrt{500}$		$10\sqrt{5}$
$\sqrt{12}$		$2\sqrt{3}$	$\sqrt{96}$		$4\sqrt{6}$	$\sqrt{245}$		$7\sqrt{5}$	$\sqrt{512}$		$8\sqrt{8}$
$\sqrt{18}$		$3\sqrt{2}$	$\sqrt{98}$		$7\sqrt{2}$	$\sqrt{250}$		$5\sqrt{10}$	$\sqrt{539}$		$7\sqrt{11}$
$\sqrt{20}$		$2\sqrt{5}$	$\sqrt{99}$		$3\sqrt{11}$	$\sqrt{252}$		$6\sqrt{7}$	$\sqrt{567}$		$9\sqrt{7}$
$\sqrt{24}$		$2\sqrt{6}$	$\sqrt{108}$		$6\sqrt{3}$	$\sqrt{275}$		$5\sqrt{11}$	$\sqrt{600}$		$10\sqrt{6}$
$\sqrt{27}$		$3\sqrt{3}$	$\sqrt{112}$		$4\sqrt{7}$	$\sqrt{288}$		$6\sqrt{8}$	$\sqrt{640}$		$8\sqrt{10}$
$\sqrt{28}$		$2\sqrt{7}$	$\sqrt{125}$		$5\sqrt{5}$	$\sqrt{294}$		$7\sqrt{6}$	$\sqrt{648}$		$9\sqrt{8}$
$\sqrt{32}$	=	$4\sqrt{2}$	$\sqrt{128}$	=	$8\sqrt{2}$	$\sqrt{300}$	=	$10\sqrt{3}$	$\sqrt{700}$	=	$10\sqrt{7}$
$\sqrt{40}$		$2\sqrt{10}$	$\sqrt{147}$		$7\sqrt{3}$	$\sqrt{320}$		$8\sqrt{5}$	$\sqrt{704}$		$8\sqrt{11}$
$\sqrt{44}$		$2\sqrt{11}$	$\sqrt{150}$		$5\sqrt{6}$	$\sqrt{343}$		$7\sqrt{7}$	$\sqrt{808}$		$10\sqrt{8}$
$\sqrt{45}$		$3\sqrt{5}$	$\sqrt{160}$		$4\sqrt{10}$	$\sqrt{360}$		$6\sqrt{10}$	$\sqrt{810}$		$9\sqrt{10}$
$\sqrt{48}$		$4\sqrt{3}$	$\sqrt{162}$		$9\sqrt{2}$	$\sqrt{384}$		$8\sqrt{6}$	$\sqrt{891}$		$9\sqrt{11}$
$\sqrt{50}$		$5\sqrt{2}$	$\sqrt{175}$		$5\sqrt{7}$	$\sqrt{392}$		$7\sqrt{8}$	$\sqrt{1000}$		$10\sqrt{10}$
$\sqrt{54}$		$3\sqrt{6}$	$\sqrt{176}$		$4\sqrt{11}$	$\sqrt{396}$		$6\sqrt{11}$	$\sqrt{1100}$		$10\sqrt{11}$
$\sqrt{63}$		$3\sqrt{7}$	$\sqrt{180}$		$6\sqrt{5}$	$\sqrt{405}$		$9\sqrt{5}$	$\sqrt{1200}$		$10\sqrt{12}$
$\sqrt{72}$		$3\sqrt{8}$	$\sqrt{192}$		$8\sqrt{3}$	$\sqrt{448}$		$8\sqrt{7}$			
$2\sqrt{2}$	=	$\sqrt{8}$	$2\sqrt{5}$	=	$\sqrt{20}$	$2\sqrt{7}$	=	$\sqrt{28}$	$2\sqrt{10}$	=	$\sqrt{50}$
$3\sqrt{2}$		$\sqrt{18}$	$3\sqrt{5}$		$\sqrt{45}$	$3\sqrt{7}$		$\sqrt{63}$	$3\sqrt{10}$		$\sqrt{90}$
$4\sqrt{2}$		$\sqrt{32}$	$4\sqrt{5}$		$\sqrt{80}$	$4\sqrt{7}$		$\sqrt{112}$	$5\sqrt{10}$		$\sqrt{160}$
$5\sqrt{2}$		$\sqrt{50}$	$5\sqrt{5}$		$\sqrt{125}$	$5\sqrt{7}$		$\sqrt{175}$	$5\sqrt{10}$		$\sqrt{250}$
$6\sqrt{2}$		$\sqrt{72}$	$6\sqrt{5}$		$\sqrt{180}$	$6\sqrt{7}$		$\sqrt{252}$	$6\sqrt{10}$		$\sqrt{360}$
$7\sqrt{2}$		$\sqrt{98}$	$7\sqrt{5}$		$\sqrt{245}$	$7\sqrt{7}$		$\sqrt{343}$	$7\sqrt{10}$		$\sqrt{490}$
$8\sqrt{2}$		$\sqrt{128}$	$8\sqrt{5}$		$\sqrt{320}$	$8\sqrt{7}$		$\sqrt{448}$	$8\sqrt{10}$		$\sqrt{640}$
$9\sqrt{2}$		$\sqrt{162}$	$9\sqrt{5}$		$\sqrt{405}$	$9\sqrt{7}$		$\sqrt{567}$	$9\sqrt{10}$		$\sqrt{810}$
$10\sqrt{2}$		$\sqrt{200}$	$10\sqrt{5}$		$\sqrt{500}$	$10\sqrt{7}$		$\sqrt{700}$	$10\sqrt{10}$		$\sqrt{1000}$
$2\sqrt{3}$	=	$\sqrt{12}$	$2\sqrt{6}$	=	$\sqrt{24}$	$2\sqrt{8}$	=	$\sqrt{32}$	$2\sqrt{11}$	=	$\sqrt{44}$
$3\sqrt{3}$		$\sqrt{47}$	$3\sqrt{6}$		$\sqrt{54}$	$3\sqrt{8}$		$\sqrt{73}$	$3\sqrt{11}$		$\sqrt{99}$
$4\sqrt{3}$		$\sqrt{48}$	$4\sqrt{6}$		$\sqrt{96}$	$4\sqrt{8}$		$\sqrt{128}$	$4\sqrt{11}$		$\sqrt{176}$
$5\sqrt{3}$		$\sqrt{75}$	$5\sqrt{6}$		$\sqrt{150}$	$5\sqrt{8}$		$\sqrt{200}$	$5\sqrt{11}$		$\sqrt{275}$
$6\sqrt{3}$		$\sqrt{108}$	$6\sqrt{6}$		$\sqrt{216}$	$6\sqrt{8}$		$\sqrt{288}$	$6\sqrt{11}$		$\sqrt{396}$
$7\sqrt{3}$		$\sqrt{147}$	$7\sqrt{6}$		$\sqrt{294}$	$7\sqrt{8}$		$\sqrt{392}$	$7\sqrt{11}$		$\sqrt{539}$
$8\sqrt{3}$		$\sqrt{192}$	$8\sqrt{6}$		$\sqrt{384}$	$8\sqrt{8}$		$\sqrt{512}$	$8\sqrt{11}$		$\sqrt{704}$
$9\sqrt{3}$		$\sqrt{243}$	$9\sqrt{6}$		$\sqrt{486}$	$9\sqrt{8}$		$\sqrt{648}$	$9\sqrt{11}$		$\sqrt{891}$
$10\sqrt{3}$		$\sqrt{300}$	$10\sqrt{6}$		$\sqrt{600}$	$10\sqrt{8}$		$\sqrt{800}$	$10\sqrt{11}$		$\sqrt{1100}$
$2\sqrt{(3)2}$	=	$\sqrt{(3)16}$	$2\sqrt{(3)3}$	=	$\sqrt{(3)24}$	$2\sqrt{(3)4}$	=	$\sqrt{(3)32}$	$2\sqrt{(3)5}$	=	$\sqrt{(3)40}$
$3\sqrt{(3)2}$		54	$3\sqrt{(3)3}$		81	$3\sqrt{(3)4}$		108	$3\sqrt{(3)5}$		135
$4\sqrt{(3)2}$		128	$4\sqrt{(3)3}$		192	$4\sqrt{(3)4}$		256	$4\sqrt{(3)5}$		220
$5\sqrt{(3)2}$		250	$5\sqrt{(3)3}$		375	$5\sqrt{(3)4}$		500	$5\sqrt{(3)5}$		625
$6\sqrt{(3)2}$		432	$6\sqrt{(3)3}$		648	$6\sqrt{(3)4}$		864	$6\sqrt{(3)5}$		1080
$7\sqrt{(3)2}$		686	$7\sqrt{(3)3}$		1029	$7\sqrt{(3)4}$		1372	$7\sqrt{(3)5}$		1715
$8\sqrt{(3)2}$		1024	$8\sqrt{(3)3}$		1536	$8\sqrt{(3)4}$		2048	$8\sqrt{(3)5}$		2560
$9\sqrt{(3)2}$		1458	$9\sqrt{(3)3}$		2187	$9\sqrt{(3)4}$		2916	$9\sqrt{(3)5}$		3645
$10\sqrt{(3)2}$		2000	$10\sqrt{(3)3}$		3000	$10\sqrt{(3)4}$		4000	$10\sqrt{(3)5}$		5000

Multi-

Multiplication of like Surd Roots.

§. 3. The Reverſe of the preceding Method of reducing Integral Surds to lower Terms, furniſhes us with a Method of multiplying them. For Inſtance, $\sqrt{36} = 6\sqrt{1} = 6$, and $\sqrt{aa+2ab+bb} = a+b\sqrt{1} = a+b$, alſo $\sqrt{aa+ba} = a\sqrt{1+\frac{b}{a}}$, as is evident from the preceding Section; then it's alſo evident that the true Product of a into $\sqrt{1+\frac{b}{a}} = a\sqrt{1+\frac{b}{a}} = \sqrt{aa+ba}$: And ſo of all other Homogeneal Roots. *Ex. gr.*

Mult. $\sqrt{10}+\sqrt{6}$
by $\sqrt{10}+\sqrt{6}$
$\sqrt{100}+\sqrt{60}$
$\sqrt{60}+\sqrt{36}$
$\sqrt{100}+2\sqrt{60}+\sqrt{36}$

Which in ſhorter terms is $\sqrt{10}+\sqrt{6}$ / $10+2\sqrt{60}+6$ / or $16+2\sqrt{60}$

Mult. $3+\sqrt{2}-\sqrt{3}$
by $3+\sqrt{2}+\sqrt{3}$
$9+3\sqrt{2}-3\sqrt{3}$
$3\sqrt{2}+2-\sqrt{6}$
$+3\sqrt{3}+\sqrt{6}-\sqrt{3}$
Prod. $9+6\sqrt{2}+2-3$
that is $8+6\sqrt{2}$.

Mult. $\sqrt{ab+\frac{d}{gg}} : +\sqrt{\frac{d}{2}}$
by $dd-\sqrt{\frac{2}{a}}$
Product, $dd\sqrt{ab}+dd\sqrt{\frac{d}{gg}}+dd\sqrt{\frac{d}{2}}-\sqrt{2b}-\sqrt{\frac{2d}{ggg}}-\sqrt{\frac{d}{a}}$

Diviſion of like Surd Roots.

§. 4. The preceding Method of reducing Fractional Surds, (§. 2.) leads us to a Diviſion of them. For Inſtance, $\sqrt{\frac{drg+ss}{16}} = \frac{1}{4}\sqrt{\frac{drg+ss}{1}} = \frac{1}{4}\sqrt{drg+ss}$

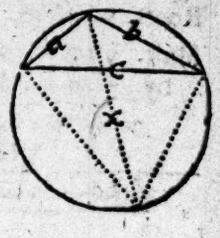
Therefore $\frac{1}{4}$ of $\sqrt{drg+ss}$, or $\sqrt{drg+ss}$ divided by 4, returns to what is was, *viz.* $\sqrt{\frac{drg+ss}{16}}$, or the Square Root of the Numerator divided by the Square Root of the Denominator.

Thus $\sqrt{4-2d}$) $\sqrt{4d+4gg+2dd-2dg}$ ($\sqrt{d+gg}$
$\sqrt{4d-2dd}$
$\sqrt{\ *.\ *}$ $4gg-2dg$
$4gg-2dg$

Alſo $\sqrt{dd+g}$) $\sqrt{2dd+dd\sqrt{dd-\sqrt{3}ddd+g\sqrt{2}+gdd-g\sqrt{3}a}}$ ($\sqrt{2+\frac{d}{d}}-\sqrt{3a}$
$\sqrt{2dd}$
$+g\sqrt{2}$
$dd\sqrt{dd}-\sqrt{3}ddd+gdd$
$dd\sqrt{dd}$ $+gdd$
$-\sqrt{3}ddd-g\sqrt{3a}$
$-\sqrt{3}ddd-g\sqrt{3a}$

$\sqrt{\frac{1}{3}d}$) $3dd\sqrt{\frac{1}{3}d}-\sqrt{\frac{1}{3}d}\frac{2}{d}-\frac{a}{b}-\sqrt{\frac{1}{3}dd}+c$ ($3dd-\sqrt{\frac{2}{d}}-\frac{a}{b}-\sqrt{dd}+\sqrt{\frac{cc}{\frac{1}{3}d}}$
$3dd\sqrt{\frac{1}{3}d}$
$-\sqrt{\frac{1}{3}d}\sqrt{\frac{2}{d}-\frac{a}{b}}$
$-\sqrt{\frac{1}{3}}\sqrt{d\frac{2}{d}-\frac{a}{b}}$
$-\sqrt{\frac{1}{3}dd}$
$-\sqrt{\frac{1}{3}dd}+c$ or $\sqrt{\frac{cc}{\frac{1}{3}d}}$ Remainder.

To ſhow the Neceſſity and great Uſe of the third and fourth preceding Sections; the former for freeing Equations from Surds, the latter for finding out the ſeveral Values of Roots in *Analytic Expreſſions*, &c. I ſhall give an Example or two, without proceeding to other Operations in Surds, my Deſign here being not to enlarge upon any Head, further than 'tis uſeful in Algebraic Practice, preſuming this ſufficient for that End; and ſuch as would know more of *Surds*, may peruſe Dr. *Wallis*'s *Treatiſe of Algebra*, cap. 25. who has writ the beſt on this Subject.



1. Suppoſe the three Sides of a Triangle a, b, c were given given me, and the Diameter of a Circle x, in which the ſaid Triangle may be inſcrib'd, is demanded?

Since in any Quadrilateral inſcrib'd in a Circle, *the two Rectangles of the oppoſite Sides are equal to the Rectangle of the Diagonals.*

I have this Equation, $b\sqrt{xx-aa}+a\sqrt{xx-bb}=xc$.

By Tranſpoſition, $b\sqrt{xx-aa}=xc-a\sqrt{xx-bb}$

$b\sqrt{xx-aa}$ $\quad$ $xc-a\sqrt{xx-bb}$

Each part ſquar'd, $bbxx-bbaa=xxcc-2xca\sqrt{xx-bb}+aaxx-aabb$

Equally expung'd, . . . $xbb=xcc-2ca\sqrt{xx-bb}+aax$

Tranſpos'd, ... $2ca\sqrt{xx-bb}=xcc+aax-bbx$

Equally divided, ... $\sqrt{xx-bb}=\frac{xcc+aax-bbx}{2ca}$

$\sqrt{xx-bb}=\frac{xcc+aax-bbx}{2ca}$

$x^2c^2+x^2a^2c^2-x^2ccbb$

$+x^2a^2c^2+a^4x^2-xxaabb$

$-x^2ccbb-xxaabb+b^4x^2$

Each part ſquar'd, ... $xx-bb=\frac{x^2c^2+2x^2a^2c^2-2x^2c^2b^2+a^4x^2-2x^2a^2b^2+b^4x^2}{4cca}$

The Fraction clear'd, $4ccaaxx-4c^2a^2b^2=x^2c^2+2x^2a^2c^2-2x^2c^2b^2+a^4x^2-2x^2a^2b^2+b^4x^2$

By Tranſpoſition to make higheſt known Power poſitive, $4ccaaxx-x^2c^2-2x^2a^2c^2+2x^2c^2b^2-a^4x^2+2x^2a^2b^2-b^4x^2=4c^2a^2b^2$

By common Reduction, . . $xx=\frac{4ccaabb}{2ccaa+2c^2b^2+2a^2b^2-c^4-a^4-b^4}$

Laſtly, If the Square Root of each part be extracted, we have the Diameter of the Circle required, *viz.*

$$x=\sqrt{\frac{4ccaabb}{2ccaa+2ccbb+2aabb-c^4-b^4-a^4}}$$

Or in lower Terms, $x=2ca\sqrt{\frac{1}{2c^2a^2+2c^2b^2+2a^2b^2-c^4-b^4-a^4}}$

2. Suppoſe in this Equation $xx+bx=d$, one of the Values of x were given me, *viz.* $x=\sqrt{d+\frac{bb}{4}}-\frac{b}{2}$, and I am to find the other, (all Quadratics having two Roots, as will be ſhewed hereafter)

Then

Then $x - \sqrt{d + \frac{bb}{4}} + \frac{b}{2}$)$xx + bx - d$ ($x + \sqrt{d + \frac{bb}{4}} + \frac{b}{2}$

$xx - x\sqrt{d + \frac{bb}{4}} + \frac{xb}{2}$

$x\sqrt{d + \frac{bb}{4}} + \frac{xb}{2} - d$

Whence also $x = -\sqrt{d + \frac{bb}{4}} - b$.

The same may also be done in Cubics.

$x\sqrt{d + \frac{bb}{4}} - d - \frac{bb}{4} + \frac{b}{2}\sqrt{d + \frac{bb}{4}}$

$\frac{xb}{2} + \frac{bb}{4} - \frac{b}{2}\sqrt{d + \frac{bb}{4}}$

$\frac{xb}{2} + \frac{bb}{4} - \frac{b}{2}\sqrt{d + \frac{bb}{4}}$

$0 \quad 0 \quad 0$

Of Quadratic Equations.

§. 1. WHatever may be resolv'd into Parts, was first compounded of Parts, as xx (supposing x an Universal Quantity) was first made by the Multiplication of x into its Self. Thus also xxx, was xx before it could arise to be xxx; and so on *ad infinitum*. Whence it follows, that in any Equation whatever, there are so many Compositions or Values of x, as is the Index of the highest Power. For Instance, $xx + bx = d$; x here has two Values. Again, in $xxx + bx = d$, there are three Values of x; and so in higher Equations.

§. 2. But from hence it does not follow that these Values are always Positive, for sometimes they are *Negative*, sometimes mix'd; sometimes only *Imaginary*, or such as include an impossibility in Nature, as will appear hereafter from their first Composition; and sometimes one or two, or more of them may be the same, yet there will be always so many and no more, as is the Index of the higher Power: But how to determine *how many there are*, which *Positive*, which *Negative*, and which *Imaginary* in any Equation whatever, is best shew'd by their Origination.

§. 3. Let x (as before) be an Universal Quantity, capable of expressing the Value of any effable or ineffable Number; and let the following Equations be made from the Alternation and *Values* of x.

1. $x = b \ldots x - b = 0$
   $x = c \ldots x - c = 0$

   $xx - bx - cx + bc = 0$

   An Equat. of the third Form. $xx - \frac{b}{c}x = -bc$

2. $x = -b \quad x + b = 0$
   $x = +c \quad x - c = 0$

   $xx + bx - cx - bc = 0$

   An Equat. of the 1st or 2d Form. $xx + \frac{b}{-c}x = bc$

3. $x = -b \quad x + b = 0$
   $x = -c \quad x + c = 0$

   $xx + bx + cx + bc = 0$

   An Imaginary Equation. $xx + \frac{b}{c}x = -bc$

1. Whence we may observe, that an Equation of the third Form (supposing $\frac{-b}{-c}$ made one Quantity, as also bc) has always two positive Roots or Values of x.

2. An Equation of the *first* or *second* Form, has always two Roots: Let $\frac{+b}{-c}$ be made one Quantity; and if $+b$ prevails, or if the Equation be of the first Form, the greater Root is Negative, and the lesser Positive. If the Equation be of the second Form, (or if $-c$ prevailes) the greater Root is Positive, and the lesser Negative; and if both the Roots be equal, the second Root will be destroyed.

The three Forms being put in their most Simple Terms, appear thus with their different Roots.

Third Form, $xx - bx = -d$

$x = \sqrt{\frac{bb}{4} - d} + \frac{b}{2}$ Greater Root Positive.

$x = \frac{b}{2} - \sqrt{\frac{bb}{4} - d}$ Lesser Root Positive.

First Form, $xx + bx = d$

$x = -\frac{b}{2} - \sqrt{d + \frac{bb}{4}}$ Greater Root Negative.

$x = \sqrt{d + \frac{bb}{4}} - \frac{b}{2}$ Lesser Root Positive.

Second Form, $xx - bx = d$

$x = \sqrt{d + \frac{bb}{4}} + \frac{b}{2}$ Greater Root Positive.

$x = \frac{1}{2}b - \sqrt{d + \frac{bb}{4}}$ Lesser Root Negative.

3. The third and last Multiplication of the two Negative Values of x, exhibit an *imaginary or impossible Equation*; for two Positive Quantities can't be equal to a Negative one: Therefore I shall pass over the Roots as expres'd in the other three preceding *Forms*, because useless.

4. The Coefficient of (or Quantity affecting) the second Term, in the first and second Form, is the Difference of the two Roots, (under a contrary Sign) according as the Negative or Positive prevails; and in the third Form 'tis the Sum of the two Roots under a contrary Sign.

5. *The Homogeneum Comparationis*, as *Vieta* calls it, or *absolute known Quantity*, that possesses one Side of the Equation, is equal to a Rectangle of the two Roots, both multiply'd under the same Sign in every Form.

6. In the third Form the Coefficient is always greater than any of the two Roots: And if a Quarter of the Square of the whole Coefficient be equal to the *absolute known Number*, the two Roots are both of the same Value, or else not.

6. If in the first and second Form, the two Roots are of the same Value, the second Term will be destroyed.

7. Lastly, Supposing the whole Equation made equal to o, so often as + follows —; or the contrary, so many Affirmative Roots there in that Equation: and so often as + follows +, and — follows —, so many are the Negative Roots, at least *real Ones*, for the Imaginary I pass over as useless.

§. 4. We are first to *order* and *prepare an Equation*, before we extract the Roots thereof, the Method may be fully and briefly compris'd under these Rules.

1. If the highest unknown Power be Negative, change all the Signs, (the Equation is not destroy'd) because 'tis impossible for a Negative Square to have any Root, since + x + gives +, and — x — gives +. For Instance, If $bx - xx = d$, then is $-bx + xx = -d$.

2. If the highest unknown Power be not first in Order, make it so, because the Extraction must begin therewith; thus $-bx + xx = -d$, becomes $xx - bx = -d$.

3. If the highest unknown Power be affected with any Coefficient, it must be clear'd either by Division or Multiplication.

For Instance, If $cxx + bx = d$, then $xx + \frac{b}{c}x = \frac{d}{c}$. If $\frac{xx}{c} + bx = d$, then $xx = bcx - cd$.

4. If there be no Coefficient to the second Term, Unity supplies the place. Ex. gr. $xx + x = d$, then is $xx + 1x = d$.

5. If there be more Coefficients to the second Term than one, they must be all reduc'd to one.

For Example, If $xx + bx + cx - \frac{d}{r}x = p$, then $xx + b + c - \frac{d}{r}x = p$.

§. 5. When

§. 5. When an Equation is prepar'd and ordered as above, the Roots of them (which I have already set down, with their respective Forms, for a greater convenience of comparing them with their Original Formations) may be extracted by several Methods; One of which may be this.

Put y (an unknown Quantity + or −, the Coefficient of the second Term, (taking always a contrary Sign) divided by the Index of the highest unknown Power, as 2 in the Square, 3 in the Cube, 4 in the Biquadrate, *&c.* And by this Artifice the second Term will be destroy'd, if a new Equation be made answerable to the respective Parts of the first. For Instance,

1. Let $xx + bx = d$, and let $y - \frac{1}{2}b = x$.
2. Then $yy - yb + \frac{1}{4}bb = \left\{\begin{matrix} xx \\ bx \end{matrix}\right\} = d$
3. Also . . . $yb - \frac{1}{2}bb = \left\{\begin{matrix} xx \\ bx \end{matrix}\right\}$
4. Where $yy - \frac{1}{4}bb = d$.
5. By Transp. $yy = d + \frac{1}{4}bb$.
6. By equal Extract. $y = \sqrt{d + \frac{1}{4}bb}$
* 7. But $x + \frac{1}{2}b\ (= y) = \sqrt{d + \frac{1}{4}bb}$
8. Then also $x = \sqrt{d + \frac{1}{4}bb} - \frac{1}{2}b$

* If $y - \frac{1}{2}b = x$
Then $y - x = \frac{1}{2}b$

Reassuming the seventh Step, where $x + \frac{1}{2}b = \sqrt{d + \frac{1}{4}bb}$. Square each part of the Equation, there arises $xx + xb + \frac{1}{4}bb = d + \frac{1}{4}bb$. Whence we have also another Method for resolving Quadratic Equations, *by adding $\frac{1}{4}$ of the Square of the whole Coefficient to both Sides of the Equation*, thereby making the first Part a true Square, and consequently capable of an exact Extraction. Thus also in an Equation of the *second Form*, $xx - \frac{bx}{c} = d$, by adding to each side $\frac{bb}{4cc}$ ($\frac{1}{4}$ of the Square of $\frac{b}{c}$) there arises $xx - \frac{b}{c}x + \frac{bb}{4cc} = d + \frac{bb}{4cc}$, and by extracting the Roots of both Sides, $x - \frac{b}{2c} = \sqrt{d + \frac{bb}{4cc}}$, and by transposing of $\frac{b}{2c}$, $x = \sqrt{d + \frac{bb}{4cc}} + \frac{b}{2c}$. And lastly, In the third Form, $xx - bx - cxx = d$; then by adding a quarter of the Square of the whole Coefficient to each Side of the Equation, we have $xx - bx - cxx + \frac{bb + 2cb + cc}{4} = \frac{bb + 2bc + cc}{4} + d$; And by equal Extraction of Roots, $x \frac{-b-c}{2} = \sqrt{\frac{bb+2cb+cc}{4} + d}$, and by Transposition $x = \sqrt{\frac{bb+2cb+cc}{4} + d} + \frac{b+c}{2}$.

§. 6. Having discovered one of the Roots of any one of the three Forms, the other may be found by Division, as was shewed before in the Division of *Surds*. Let it be proposed to find the second Root in the *third Form*, the first being given in the Equation, $xx - bx = -d$, *viz.* $\sqrt{\frac{bb}{4} - d} + \frac{b}{2}$; Then also by Transposition $x - \sqrt{\frac{bb}{4} - d} - \frac{b}{2} = 0$; Also $xx - bx + d = 0$, and *this* divided by *that*, exhibits the following Operation, the Quote of which is equal to Nothing; for if 0 divides 0, 0 will result: Then by Transposition, the second Root is found, $x = \frac{b}{2} - \sqrt{\frac{bb}{4} - d}$. Which is also Positive, and 'tis the third Form only that has two Positive Roots, and which are so ambiguous in many Algebraic Operations, that till we have tried both, 'tis difficult to know which is the most proper Solution of a Question, but in the other two Forms this Difficulty never occurs, but the enquired Root comes first.

$$x - \sqrt{\tfrac{bb}{4} - d} - \tfrac{b}{2}\ \Big)\ xx - bx + d\ \Big(\ x + \sqrt{\tfrac{bb}{4} - d} - \tfrac{b}{2} = 0$$

$$xx - x\sqrt{\tfrac{bb}{4} - d} - \tfrac{bb}{2}$$

$$x\sqrt{\tfrac{bb}{4} - d} - \tfrac{bb}{2} + d$$

$$x\sqrt{\tfrac{bb}{4} - d} - \tfrac{bb}{4} + d - \tfrac{b}{2}\sqrt{\tfrac{bb}{4} - d}$$

$$-\tfrac{bb}{2} + \tfrac{bb}{4} + \tfrac{b}{2}\sqrt{\tfrac{bb}{4} - d}$$

$$-\tfrac{bb}{2} + \tfrac{bb}{4} + \tfrac{b}{2}\sqrt{\tfrac{bb}{4} - d}$$

There's yet another way of finding the two Roots at once.

Let the Sum of the two Roots be $2p$, and their Difference $2q$, then is the greater Root $p+q$, and the leſſer $p-q$; then alſo is $x-p-q=0$, and $x-p+q=0$. Multiply for a new Equation.

$$\begin{array}{r} x-p-q \\ x-p+q \\ \hline xx-xp-xq \\ \quad -xp+pp+pq \\ \quad\quad -qx-pq-qq \\ \hline xx-2px+pp-qq=0 \\ xx+bx-dd=0 \end{array}$$

Compare the Product with any of the three Forms. Suppoſe with the firſt, xx and xx deſtroy each other, and there are yet theſe two Equations of the Parts $-2px=bx$, and $pp-qq=-d$. The firſt Equation is reduced to $-2p=b$; divide each part by 2. $-p=\frac{1}{2}b$. Wherefore alſo $pp=\frac{bb}{4}$. Which ſubſtitute in the Room of pp in the ſecond Equation, and there ariſes $\frac{bb}{4}-qq=-d$; or by Tranſpoſition $\frac{bb}{4}+d=+qq$; and by equal Extraction $\sqrt{\frac{bb}{4}+d}=+q$; therefore the two Roots will be found at once, *viz.* the greater $p+q=-\frac{1}{2}b+\sqrt{\frac{bb}{2}+d}$, the leſſer $p-q=-\frac{1}{2}b-\sqrt{\frac{bb}{4}+d}$. The ſame Method may be purſued in the other two Caſes.

§. 7. There are yet Biquadratics, as alſo higher Equations belonging to the three preceding Forms, which are of the ſame Nature as Quadratics, and may not improperly receive their Solution here.

The firſt Form —— $xxxx+bxx=d$

The Poſitive Root —— $xx=\sqrt{d+\frac{bb}{4}}-\frac{b}{2}$

A ſecond Extraction —— $x=\sqrt{\sqrt{d+\frac{bb}{4}}-\frac{b}{2}}\Big|^{\frac{1}{2}}$

The ſecond Form —— $xxxx-bxx=d$

The Poſitive Root —— $xx=\sqrt{d+\frac{bb}{4}}+\frac{q}{2}$

A ſecond Extraction —— $x=\sqrt{\sqrt{d+\frac{bb}{4}}+\frac{b}{2}}\Big|^{\frac{1}{2}}$

The third Form —— $xxxx-bxx=-d$

The greater Poſitive Root —— $xx=\sqrt{\frac{bb}{4}-d}+\frac{b}{2}$

A ſecond Extraction —— $x=\sqrt{\sqrt{\frac{bb}{2}-d}+\frac{bb}{2}}\Big|^{\frac{1}{2}}$

Theſe three Forms in Biquadratics occur pretty often; and the ſame Solution would hold for any higher Power thus qualified.

For

For Inſtance, $x^6 + bx^3 = d$, the Root of which is $xxx = \sqrt{d + \frac{bb}{4}} - \frac{b}{2}$, and by extracting the Cube Root of each, $x = \sqrt[3]{d + \frac{bb}{4}} - \frac{b}{2}\Big|^{\frac{1}{3}}$.

Thus alſo if $x^{20} + bx^{10} = d^2$, then is $x = \sqrt{d + \frac{bb}{4}} - \frac{b}{2}\Big|^{\frac{1}{10}}$. And ſo of all others thus qualified, *ad infinitum.*

Numeral Queſtions falling under Quadratic Equations.

Q. 1. CLAVIUS in his Treatiſe of Algebra, (*Cap.* 31. *Queſt.* 58.) puts this Queſtion; Two Perſons have each a certain number of Crowns; the Sum of which being ſubducted from the Sum of the Squares of the ſaid Numbers (made ſeparately) leaves 78; but their Sum being added to the Product of the two Numbers, gives 39. 'Tis demanded how many Crowns each had?

This Queſtion, ſays *Clavius*, is too difficult to be anſwer'd by Algebra, without very good knowledg in Geometry. But Algebra is now brought to that Perfection, that Geometry depends rather upon it, than it does upon Geometry, ſince we are not only able to reſolve ſuch Queſtions by pure Algebra, as no Geometrician can do by Rule and Compaſs, but alſo to find out Laws and Rules for ſeveral *Geometric Effections*, which otherwiſe would be deſperate.

But to return from this Digreſſion. Let $2y$ be the Sum of the unknown Numbers, and $2z$ their Difference, and let $78 = a$, and $39 = b$; then $y+z$ the greater Number, and $y-z$ the Leſſer. Hence,

1. The Sum of their Squares is —	$2yy + 2zz$.
2. *Idem*, leſſened by their Sum, $= a$	$2yy + 2zz - 2y = a$.
3. Clear zz —	$zz = y + \frac{1}{2}a - yy$.
4. Their Product added to their Sum $= b$ —	$yy - zz + 2y = b$.
5. Clear zz —	$zz = yy + 2y - b$.
6. From the 3*d* and 5*th* Steps —	$y + \frac{1}{2}a - yy = +yy + 2y - b$.
7. By Reduction —	$yy + \frac{1}{2}y = \frac{2b+a}{4}$
8. Add to each $\frac{1}{4}$ of the Square of the whole Coeffement —	$yy + \frac{1}{2}y + \frac{1}{16} = \frac{2b+a}{4} + \frac{1}{16}$.
9. Extract equally —	$y + \frac{1}{4} = \sqrt{\frac{2b+a}{4}} + \frac{1}{16}$.
10. Tranſpoſe $\frac{1}{4}$ —	$y = \sqrt{\frac{2b+a}{4}} + \frac{1}{16} - \frac{1}{4}$.

Which is in Numbers $\sqrt{\frac{78+78}{4}} + \frac{1}{16} - \frac{1}{4}$ or $\sqrt{39\frac{1}{16}} - \frac{1}{4} = \sqrt{\frac{625}{16}} - \frac{1}{4} = 6$. therefore 12 is the Sum of the two Numbers, and from Step 4. $36 - zz + 12 = 39$; that is $zz = 9$, then is $z = 3$; but the greater was $y+z$ by Suppoſition, therefore $6+3 = 9$ the greater Number, $6-3 = 3$ the leſſer Number ſought: Which two will anſwer the Conditions of the Queſtion.

Q. 2. The Mean of the three Geometric Proportionals being given (12), and the Sum of their Extreams (26); to find the three Numbers, let m be the Mean; $2s$ the Sum of their Extreams, and $2y$ the Difference of their Extreams, then is the greater $s+y$, and the leſſer $s-y$. Whence $s+y.\ m.\ s-y ::$, that is $ss-yy=mm$; whence $ss-mm=yy$, then is $y=\sqrt{ss-mm}$, that is $\sqrt{13\times13-12\times12}=5$; and by Poſition y or 5 is half the difference of their Extreams; but half the Sum and half the Difference of two Numbers is always the greater; ſo that 18 is the greateſt. Whence 18. 12. (12). 8 :: are the three Numbers ſought.

Q. 3. The Product of two Numbers is p, and the Sum of their Squares is q; What are the Numbers?

For their Sum put $2x$, and for their Difference $2y$; then is $x+y$ the greater Number, and $x-y$ the Leſſer; their Product is $xx-yy=p$, and the Sum of their Squares is $2xx+2yy=q$. In the firſt Equation $xx-p=yy$, in the 2d, $\frac{1}{2}q-xx=yy$; Therefore $xx-p=\frac{1}{2}q-xx$; or $2xx=\frac{1}{2}q+p$; or $xx=\frac{1}{4}q+\frac{1}{2}p$; or $x=\sqrt{\frac{1}{4}q+\frac{1}{2}p}$: Then $\sqrt{\frac{1}{4}q+\frac{1}{2}p}$, is half their Sum, and half their Difference is given, by conſequence the Numbers themſelves.

Or ſhorter thus, with one unknown Quantity; Let your Product be p, the Sum of their Squares $2q$, and the Difference of their Squares $2y$. Then is $q+y$ the greater Square, and $q-y$ the Leſſer; the Product of whoſe Roots, *viz.* $\sqrt{q+y}\times\sqrt{q-y}=p$, that is, $\sqrt{qq-yy}=p$, and to clear the Surd $qq-yy=pp$. Whence as before, $y=\sqrt{qq-pp}$, which is half the Difference of their Squares; but half the Sum and half the Difference of two Squares being given, the Squares themſelves are given, and by conſequence the Roots.

If it be demanded why I make uſe of $x+y$ for the greater unknown Number, and $x-y$ for the Leſſer, and not rather x for the Greater, and y for the Leſſer: I anſwer, That by this Method, in twenty Quadratics together, if work'd according to Mr. *Kerſey*'s Method and his Followers, I avoid for the moſt part the ſecond Term, which they cannot, and thereby ſave much Labour. Beſides ſuch Queſtions as by this Method do ariſe to an affected Quadratic, are commonly extreamly difficult, if reſolv'd according to the ordinary way: Nay even a ſimple Quadratic in this Method will ſometimes reſolve what's very troubleſom in that, as may be ſeen by ſome of the following Queſtions.

Q. 4. The Sum of the Extreams of four Numbers in Geometric Proportion, and the Sum of the Means being given to find each of the Terms.

Put $2a$ for the Sum of the Extreams, and $2x$ for their Difference, $2b$ the Sum of the Means, and $2y$ their Difference, and proceed, *&c.*

Q. 5. The Sum of the Squares of two Magnitudes, and the Ratio of their Product to the Square of their Difference being given, to find the Numbers?

Q. 6. The Product of two Numbers, with the Sum of their Cubes, to find the Numbers?

Q. 7. The Square of two Numbers, with the Sum of their Cubes being given, to find the Numbers?

Q. 8. The Difference of the Squares of two Numbers being given, with the Difference of their Cubes, to find the Squares?

For forming Equations in any Arithmetical *or* Geometrical Progreſſion.

§. 1. IN any *Arithmetical Progreſſion*, Let a be the firſt Term, ω the laſt, n the Number of Terms, s the Sum of all the Terms, and d the Common Difference.

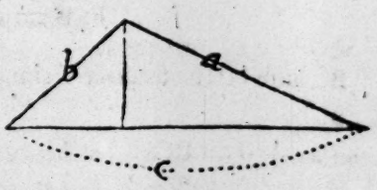
Lemma I. $\omega = a + d \times \overline{n - 1}$.
Lemma II. $s = \frac{1}{2} n \times \overline{a + \omega}$.

Any three of theſe being given, as $a + \omega + s$, or $d + n + s$, &c. the other two are alſo given: And of theſe Threes there may be ſixty Alternations; which expunging ſuch as are alike, are reduc'd to twenty Caſes by Mr. *Oughtred*; and may be all eaſily reſolv'd by the preceding *Lemma's*.

§. 2. In any *Geometrical Progreſſion*, Let a be the firſt Term (or Antecedent); b the 2d Term or Conſequent, $s - \omega$ the Sum of all the Antecedents, and $s - a$ the Sum of all the Conſequents. Then as $a . b :: s - \omega, s - a$, or $as - aa = bs - b\omega$, or $as - bs = aa - b\omega$; whence $s = \frac{aa - b\omega}{a - b}$ if the Progreſſion decreaſes; or elſe $s = \frac{b\omega - aa}{b - a}$, when it increaſes.

Of Quadratic Equations in Geometry.

Q. 1. LET the three Sides of any Triangle, a, b, c, be given; to find the length of a Perpendicular let fall upon the Baſe.

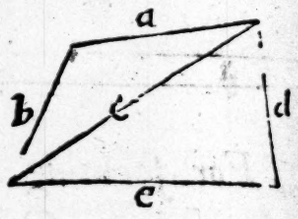


From the preceding firſt Theorem in Simple Equations; $\frac{cc + bb - aa}{2c}$ = to the leſſer Segm. of the Baſe. Whence according to the new way of Notation, $bb - \overline{\frac{cc + aa - bb}{2c}|^{2}_{1}}$, is equal to the Square of the Perpendicular; whoſe Root $\sqrt{bb - \overline{\frac{cc + bb - aa}{2c}|^{2}_{1}}}$, is the Perpendicular ſought.

From hence alſo we have this Theorem for the *Area* of Triangles (3 Sides being given) *viz.* $\frac{1}{2}c\sqrt{bb - \overline{\frac{cc + bb - aa}{2c}|^{2}_{1}}}$.

Or in a little ſhorter Terms, $\frac{1}{2}c\sqrt{bb - \overline{\frac{1}{2}c + \frac{bb - aa}{2c}|^{2}_{1}}}$.

Hence also we have a *Theorem* for the measuring of *Trapezias*, whose four Sides are given, with the Diagonal, *viz.*



$\frac{1}{2}c\sqrt{bb-\frac{1}{2}c+\frac{bc-aa}{2c}}\Big|^{\frac{1}{2}}+\frac{1}{2}c\sqrt{dd-\frac{1}{2}c+\frac{dd-ee}{2c}}\Big|^{\frac{1}{2}}$

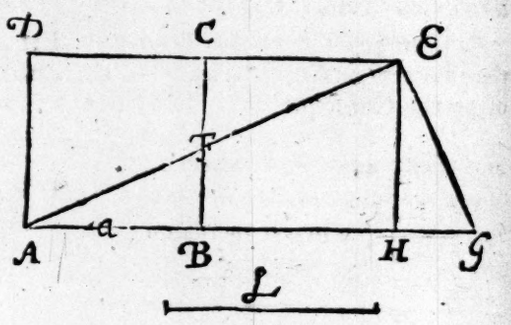
Equal to the *Area* of any *Trapezium*, a, b, c, d, e, or to avoid two Multiplications.

$\frac{1}{2}c\sqrt{bb-\frac{1}{2}c+\frac{bb-aa}{2c}}\Big|^{\frac{1}{2}}+\sqrt{dd-\frac{1}{2}c+\frac{dd-ee}{2c}}\Big|^{\frac{1}{2}}$, shall be the *Area* thereof. This is a very exact way in case of any Dispute; for the surveying any large Plains, Fields, Commons, *&c.* which are very easily reduced into *Trapezias*.

But the shortest *Theorem* that can possibly be given for the *Areas* of Triangles, is that of Mr. *Oughtred*, provided the literal Expression be not delivered at length as it has been by him, Mr. *Kersey* and others. The known Terms arising to Surfolids, I would chuse to express it thus, resuming the first Triangle, and putting s for half the Sum of the three given Sides, a, b, c, viz. $\sqrt{sxs-axs-bxs-c}$, equal to the *Area* of any Triangle, as a, b, c.

Q. 2. There's a Square given, A B C D; and it's demanded to draw a Line C E, to meet A B produc'd. So that the External Part F E, shall be equal to a given Line L.

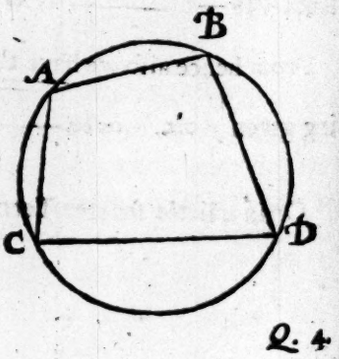
Produce A B indeterminately, as also D C, and draw E G perpendicular to A E, and E H perpendicular to A G.



Let $\left.\begin{array}{l} A\,B = a \\ F\,E = l \\ C\,F = x \\ F\,B = a - x \end{array}\right\}$

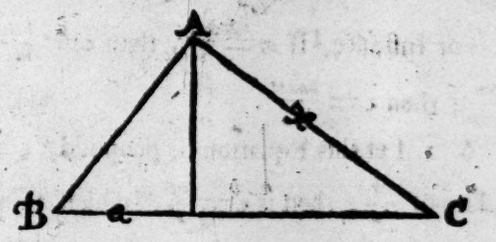
Because of the Similar Triangles we find $\frac{la-lx}{x} = AF = EG$: Also this Equation $aa+ll = BGq$. Whence again $a+\sqrt{aa+ll}, \frac{la-lx}{x} :: l.\ x$, or $xa+x\sqrt{aa+ll} = \frac{lla-llx}{x}$, or $xxa+xx\sqrt{aa+ll} = lla-llx$, or $\sqrt{aa+ll}\}xx+llx, = lla$, or $xx+\frac{llx}{\sqrt{aa+ll}+a} = lla$, which arises only to a Quadratic, and may be solv'd after the usual Method.

Q. 3. There is given the Sides of a *Trapezium* ABCD, inscrib'd in a Circle; the *Area* thereof is demanded? Which will arise to a much easier *Theorem* than that of a *Trapezium*, which can't be inscrib'd in a Circle.

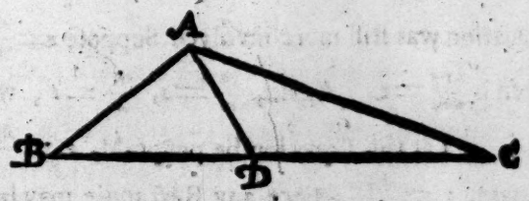


Q. 4

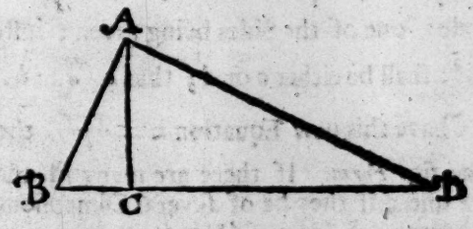
Q. 4. In a Right-angled Triangle A B C, *a* the leſſer Segment of the Baſe is only given; yet with this limitation, that the unknown Perpendicular AB, ſhall be equal to the other Segment of the Baſe; the ſide AC is demanded?



Q. 5. In any Triangle ABC, the Angle BAD, with its oppoſite ſide BD; and the Angle DAC, with its oppoſite ſide DC are given, to find all the Sides?

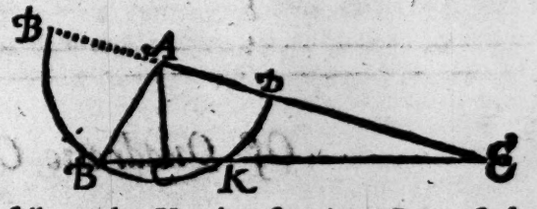


Q. 6. In any Right-angled Triangle ABD, if a Perpendicular AC be let fall from the right Angle upon the oppoſite Side, thereby dividing the whole into two Triangles ſimilar, and like the whole; at the ſame time, conſtituting five Parts, any two of them are given, to find the other three?



Q. 7. In the ſame Triangle AB+BD=*p*, and AC+DC=*q* are given, the Parts are demanded?

Q. 8. In any Right-angled Triangle ABE, the Difference of the Segments of the *Hypothenuſe* KE, with either of the Sides BA or AE, are given, to find all the other Parts?



In the ſame Triangle the Difference of the Legs BA and AE, *viz.* DE is given, and the Perpendicular let fall on the *Hypothenuſe*, *viz.* AC; to find the *Hypothenuſe*, and all the other Parts?

To give *Analitic Theorems* for the inſcribing a three, four, five, ſix, ſeven, eight, nine, ten, eleven, and twelve-ſided Figure in a Circle.

For Equations in Cubics and higher Powers.

To give *Analitic Theorems* for the *Triſection*, *Quadriſection*, *Quinquiſection*, *Sextiſection*, &c. of an Arch or Angle.

Geometric Effections or Conſtructions in ſimple Equations.

I Suppoſe all Simple Equations are reducible to this Form, $x=\frac{ab}{c}$. Then for the Conſtruction $xc=ab$, which reſolv'd into Proportionals, gives $c.a::b.x$: So that to find the Value of x, is only to find a fourth Proportional.

§. 1. The Number of Dimenſions in the Numerator, always exceeds that of the Denominator by one, becauſe of Diviſion, (for a Line is Heterogeneous to a Square Cube, &c.); And if it be not ſo at firſt, let *a* (or any other Letter be put for *Unity*); and let that (or the Powers thereof, if need be) ſupply what's wanting on either Side, without altering the Value thereof.

For Inſtance, If $z = \frac{bct}{e}$, then $z = \frac{bcl}{ed}$; if $z = \frac{mnros}{gs}$, then $z = \frac{mnros}{gsdd}$; if $z = \frac{lm}{ggr}$, then $z = \frac{lmddd}{ggrr}$.

§. 2. Let this Equation be propos'd, $z = \frac{abc}{de}$, take any Quantity, ſuppoſe m in the place of $\frac{ab}{d}$, then is $z = \frac{mc}{e}$, which is the *firſt Form*. Again, let $z = \frac{abcd}{efg}$, take $m = \frac{ab}{e}$ and $n = \frac{cd}{f}$, then is $z = \frac{mn}{g}$ of the *firſt Form*, 'tis the ſame if a Simple Equation was ſtill more involved. Suppoſe $z = \frac{abcdsfg}{hklmno}$, take $\frac{ab}{b} = p$, $\frac{cd}{k} = q$, $\frac{ef}{l} = r$, then is $\frac{pqrg}{mno} = z$. Again, $\frac{pq}{m} = s$, $\frac{rg}{n} = t$; whence $\frac{st}{o} = z$.

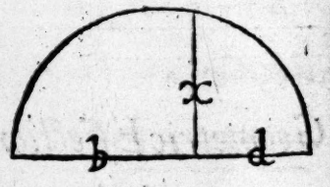
§. 3. Let this Equation be propos'd, $z = \frac{ab+ac}{b}$, take $m = b+c$, then is $z = \frac{am}{b}$. Again, $z = \frac{ab+cd}{e}$, ſince any Rectangle may be chang'd into another of the ſame Value, one of the Sides being given; inſtead of ab take a Rectangle, whoſe one Side ſhall be either c or d; thus $c . a :: b . y$, then $ab = cy$, and $y = \frac{ab}{c}$, whence we have this new Equation $z = \frac{cy+cd}{e}$; then again taking $y + d = m$, $z = \frac{cm}{e}$ of the *firſt Form*. If there are many Members of the Numerator, reduce them one by one; if they be of ſeveral Dimenſions, make them of the ſame by the Multiplication of *Unity (d)* or its Powers into them: If they have different Denominators, reduce them to one, and then work as before; in all which Reductions x, the Number ſought, will be a fourth Proportional here.

Of Quadratic Conſtructions.

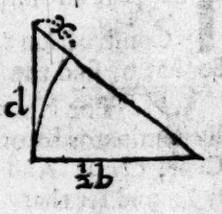
ALL Quadraticks will fall under ſome one of theſe four Forms, or what is very readily deducible from them. (1.) $xx = dd$. (2.) $xx + bx = dd$. (3.) $xx - bx = dd$. (4.) $xx - bx = -dd$.

And when reduc'd, (1.) $x = \sqrt{dd} = d$. (2.) $x = \sqrt{dd + \frac{bb}{4}} - \frac{b}{2}$. (3.) $x = \sqrt{dd + \frac{bb}{4}} + \frac{b}{2}$. (4.) $x = \sqrt{\frac{bb}{4} - dd} + \frac{b}{2}$.

§. 1. If the firſt, there's no need of any conſtruction for $x = d$; if $xx = bd$, here it's evident that x is a mean Proportional betwixt b and d; for $b . x :: x . d$: Whence $bd = xx$ as before. Deſcribe a Circle, whoſe Diameter let be $b + d$ as one Line, and at their point of meeting erect a Perpendicular x ſought, which is a mean proportional betwixt b and d, (by 13. *e* 6.)

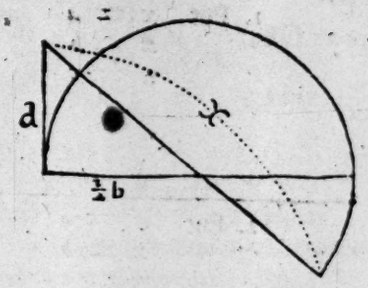


§. 2. Let the ſecond Caſe be propos'd, $x = \sqrt{dd + \frac{bb}{4}} - \frac{b}{2}$ at right Angles erect d, and $\frac{1}{2}b$; the Squares of which (by 47. *e* 1.) are equal to that of the *Hypothenuſe* $= dd + \frac{bb}{4}$, the Root of which leſſen'd by $\frac{1}{2}b$, gives x ſought.

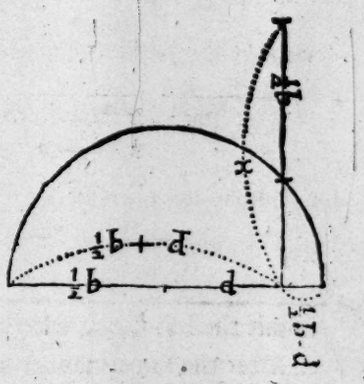


The

The third Case is $x = \sqrt{dd + \frac{bb}{4}} + \frac{b}{2}$, which differs not from the last, save only that $\frac{1}{2}b$ is not taken out of the *Hypothenuse*, but added thereto to make up the Value of x sought.



The fourth and last Case is, $x = \sqrt{\frac{bb}{4} - dd} + \frac{b}{2}$, $\frac{bb}{4} - dd$, is a Rectangle made by the Multiplication of $\overline{\frac{1}{2}b + d} \times \overline{\frac{1}{2}b - d}$, describe a Circle whose Radius shall be $\frac{1}{2}b$; to which add d in the same Line, and from $\frac{1}{2}b$ subduct d as in the Figure, then is the Perpendicular which stands on the meeting of $\frac{1}{2}b + d$, and $\frac{1}{2}b - d$, a mean Proportional betwixt them and the Square thereof, is equal to their Rectangle; to the Root of which (that is the Perpendicular it self) if $\frac{1}{2}b$ be added in one Line, it gives the Value of x sought.



All adfected Quadratics are reducible to one of the three last Forms, I shall give one perplex'd Instance for all; $xx + x = dd - aa - \frac{ddcc}{bb-aa}$, take $dm = aa$; then instead of $dd - aa$, we have $dd - dm$; take k for $d - m$, then is $dk = dd - aa$. Again, let $aa = br$, then $bb - aa = bb - br$; take $p = b - r$, and we have $bp = bb - aa$, then also is $dd - aa - \frac{ddcc}{bb-aa} = dk - \frac{ddcc}{bp}$.

Yet again, take $n = \frac{dd}{b}$ and $o = \frac{cc}{p}$, then is $dk - no = dk - \frac{ddcc}{bp}$.

Again, take $no = dg$, then is $dk - no = dk - dg$; and if h be assum'd for $k - g$, $dk - dg = dh$.

Lastly, Take a mean proportional betwixt d and h: Suppose l, then $dh = ll$, then will $dd - aa - \frac{ddcc}{bb-aa} = ll$, then also is $xx + x = ll$, which was to be done.

Those that would see more of the Practice of Geometrical Constructions, may consult *Oughtred*'s *Clavis*, and *De Cartes* Geometry.

As for the Construction of Cubic, Biquadratic, and higher Equations, 'tis necessary first to understand *Conic Sections*, therefore I omit that as foreign to my Purpose at present.

Of Cubic, Biquadratic, and higher Equations.

THE Origination of a Cubic Equation, is the same with that of an adfected Quadratic. *Ex. Gr.*

1. Put $x = a$, $x = b$, $x = c$

Then is $x - a = 0$, $x - b = 0$, $x - c = 0$

$$xxx - axx + abx - bxx + acx - cxx + bcx - bcd = 0.$$

2. Put $x = a$, $x = b$, $x = -c$

Then is $x - a = 0$, $x - b = 0$, $x + c = 0$

$$xxx - axx + abx - bxx - acx + cxx - bcx + bcd = 0.$$

3. Put $x = -a$, $x = -b$, $x = +c$

Then is $x + a = 0$, $x + b = 0$, $x - c = 0$

$$xxx + axx + abx + bxx - acx - cxx - bcx - bcd = 0.$$

I omit the laſt Caſe, where, a, b, c, are all Negative, which is impracticable.

1. After the ſame manner any Biquadratic, Cubiquadratic, or higher Equation may be deriv'd, where there will be always ſo many Values of the unknown Quantity, as is the *Index* of the *higheſt Power*, ſometives Poſitive, ſometimes Negative, ſometimes Imaginary, and ſometimes Mix'd.

2. The *Coefficient* of the ſecond Term, is the Sum of all the Roots under contrary Signs.

3. The *Coefficient* of the third Term, is the Rectangles of all the Roots ſo many ways as they can be taken, *viz.* three in a Cube, four in Biquadrate, *&c.* The Coefficient of the fourth Term, if we proceed to Biquadrates, Cubiquadrates, *&c.* is the Sum of all the Solids of the Roots, ſo many ways as they can be taken. The Coefficient of the fifth Term will be the Sum of all the Surfolids of the Roots, ſo many ways as they can be taken; and ſo on *ad infinitum.* Laſtly, The Abſolute known Number in the Square, is the Rectangle of the two Roots; in the Cube, the Solid of the three Roots; in the Biquadrate, the Surfolid of the four Roots, *&c.*

4. When the Coefficients are mix'd, if the Sum of the Negative Roots taken together, is equal to the Sum of the Poſitive, the ſecond Term is deſtroy'd in that Equation: Alſo if the Rectangles of the Negative Roots are equal to the Rectangles of the Poſitive, the third Term will be deſtroy'd; and ſo on.

As for the Determination of the Quality of Roots in a Cubic Equation, and higher if need be, *viz.* how many *Poſitive*, and how many *Negative*, &c. I think we need not follow Mr. *Harriot*'s Method, which is very much perplex'd in the deriving variety of Caſes, as by his *Reciprocals*, his *parting a Simple Quantity* into the *Form* of a Binomial, *&c.* All the ſixteen Caſes in Cubics (for ſo many I find) are very eaſily and naturally derived, only from the different Combinations of the Values of the Coefficients in the three preceding Originations.

I ſhall give one Example of my Method, by which all the reſt will be evident: Let the ſecond Caſe be reaſſum'd where the *Geneſis* ariſes, from two Poſitive Roots and one Negative.

From the different Combinations of the Coefficients in this Example, there will ariſe three of the ſixteen Caſes in Cubics before the ſecond or third Term be taken away.

Let $a + b$ taken together be either greater or leſſer than c,

$x = a$, $x = b$, $x = -c$

$x - a = 0$, $x - b = 0$, $x + c = 0$

$$xxx - axx + abx - bxx - acx + cxx - bcx - abc = 0.$$

(for

(for ſhould they be equal, the 2*d* Term would be deſtroy'd) then by expreſſing the Coefficient of the ſecond Term in one Letter: Suppoſe *p*, there can be only two Alternations according as the two Poſitive Roots, or the one Negative prevails; *viz.* $-p$, or $+p$.

For the Coefficients of the third Term: Suppoſe *c* either greater, equal, or leſſer than $a+b$. Firſt, If *c* be greater than $a+b$, *ac* and *bc* together will be greater than *ab*, becauſe *bc* by it ſelf is greater than *ab*. (2.) If *c* be equal to $a+b$, $ac+bc$ will be greater than *ab*, becauſe *ac* it ſelf is greater than *ab*, which two are but one Caſe. Again, ſuppoſing *c* leſſer than $a+b$, the Defect will be either great or little. If *c* be moderately leſs than $a+b$, then $ca+cb$ may be taken greater than *ab*, which is a ſecond Alternation. Laſtly, If *c* be taken very little, $ca+cb$ may be leſſer than *ab*; which is the third Alternation, and all that can be made in the Coefficients of the third Term.

Add theſe three Alternations to the two firſt made by the Coefficients of the ſecond Term, and let *q* ſtand for the Total of the laſt Coefficients, and they will ariſe thus.

$xxx - pxx$ } For the firſt Alter-
$xxx + pxx$ } nation.

For the Second, $xxx - pxx \left\{ \begin{matrix} -qx \\ +qx \end{matrix} \right. - abc = 0.$

For the Third, $xxx + pxx \; \{ -qx - abc = 0.$

From ſuch different Alternations, and Values of all the Coefficients of every Term in any Equation, there's a Neceſſity that ſo often as $+$ follows $+$, or $-$ follows $-$, (all the Terms being put over to one Side) ſo many are the Negative Roots; and ſo often as $+$ follows $-$, or the contrary, ſo many are the Poſitive Roots in any Equations, (I mean of real Ones).

Upon theſe Conſiderations I determine the Qualities of the Roots of all the Caſes in the three preceding *Gemeſes*, as follows.

1. Caſe,	1. $xxx-pxx+qx=+r$	All the Roots are Poſitive.
2. Caſe,	2. $xxx-pxx-qx=-r$	The two Poſitives prevail, and the Negative is moderately big.
	3. $xxx-pxx+q=-r$	The two Poſitives prevail, and the Negative is very ſmall.
	4. $xxx+pxx-qx=-r$	The Negative is greater than the two Poſitives.
3. Caſe,	5. $xxx+pxx-qx=+r$	The two Negatives prevail, and the Poſitive is moderate.
	6. $xxx+pxx+qx=+r$	The two Negatives prevail, and the Poſitive is very ſmall.
	7. $xxx-pxx-qx=+r$	The Poſitive is greater than the two Negatives.
2. Caſe,	8. $xxx-pxx*=-r$	Two Poſitives greater than one Negative.
	9. $xxx+pxx*=-r$	One Negative Root and two Imaginary.
3. Caſe,	10. $xxx+pxx*=+r$	Two Negatives greater than one Poſitive.
	11. $xxx-pxx*=+r$	One Poſitive Root and two Imaginary.
2. Caſe,	12. $xxx*+qx=-r$	One Negative Root and two Imaginary.
	13. $xxx*-qx=-r$	Two Poſitives equal to one Negative.
3. Caſe,	14. $xxx*-qx=+r$	Two Negatives equal to one Poſitive.
	15. $xxx*+qx=+r$	One Poſitive Root and two Imaginary.
1, 2, or 3*d* Caſe,	16. $xxx**=+r$	One Poſitive Root and two Imaginary.

☞ *Note*, That if in the 13*th* or 14*th* Forms, if $\frac{1}{27}p^3$ be greater than $\frac{1}{4}qq$, the two Positive Roots in the first, and the two Negative Roots in the last will be Imaginary.

From the three preceding Cases only, may also be deriv'd the 46 different Combinations in Biquadratics, and also of yet higher Equations, but this would be a Business of greater Labour than Use, and therefore I omit it.

I shall also pass over the Method of resolving these Cubics as deliver'd by *Cardan*, because his Rules reach not all Cases as where the Root is under Unity: Or upon supposition they did, what a tedious Operation (after that of taking away the second Term) would it amount to, especially if the Root sought were irrational? We must extract two Squares and two Cubes, before we can arrive to what we want. But for such as are in love with this Method, I shall barely set down the *Theorems* for them as follows, reserving a full Resolution of them, (and higher Equations,) till we come to the easier Methods of *infinite Approximations*.

$x^3 * + qx = +r$. Root. $x = \sqrt{(3)\frac{1}{2}r+\sqrt{\frac{1}{4}rr+\frac{1}{27}q^3}} - \sqrt{(3)-\frac{1}{2}r+\sqrt{\frac{1}{4}rr+\frac{1}{27}q^3}}$

$x^3 * - qx = +r$. Root. $x = \sqrt{(3)\frac{1}{2}r+\sqrt{\frac{1}{4}rr-\frac{1}{27}q^3}} + \sqrt{(3)+\frac{1}{2}r-\sqrt{\frac{1}{4}rr-\frac{1}{27}q^3}}$

$x^3 * - qx = -r$. Root. $x = -\sqrt{(3)\frac{1}{2}r+\sqrt{\frac{1}{4}rr-\frac{1}{27}q^3}} - \sqrt{(3)+\frac{1}{2}r-\sqrt{\frac{1}{4}rr-\frac{1}{27}q^3}}$

$x^3 * + qx = -r$. Root. $x = -\sqrt{(3)\frac{1}{2}r+\sqrt{\frac{1}{4}rr+\frac{1}{27}q^3}} + \sqrt{(3)-\frac{1}{2}r+\sqrt{\frac{1}{4}rr+\frac{1}{27}q^3}}$

Of dissolving Compound Equations.

FROM the Origination of Compound Equations, we have a Method of dissolving them again, (if by any means we can discover any of the Roots) *viz.* by depressing a Biquadratic to a Cubic, a Cubic to a Quadratic, a Quadratic to a Lateral. For Instance, Let the following Quadratic $xx-5x=6$, be generated from the Roots 2 and 3.

$x=2$ $\quad x-2=0$
$x=3$ $\quad x-3=0$
$\quad xx-2x$
$\quad\quad -3x-6$
$x-2=0$) $xx-5x-6=0$ ($x-3=0$
$\quad xx-2x$
$\quad\quad -3x-6$
$\quad\quad -3x-6$

Suppose I had by some means discovered that one of the Roots or Values of x was 2, then the other would be found by Division, as in the Operation. Imagine the same in Equations more Compound.

To find all the Roots of a Cubic Equation.

SUppose in a Cubic Equation, whose second Term is taken away, I have occasion for all the Roots; put $2x$ for the Sum of the two, whether Positive or Negative, $2y$ their Difference; $2x$ will also be the third, whether Negative or Positive, because it must be equal to the other two, or the second Term could not be destroy'd; then is $x+y$ the greater Root of the two, $x-y$ the Less. Let the three Roots be multiply'd with their own Signs, so many ways as they can be taken to form a Coefficient equal to that of the third Term, and there arises $-3xx-yy$.

Let

Let any Equation be propos'd $\left\{\begin{matrix} zzz-pz=-q \\ zzz-pz=+q \end{matrix}\right\}$ Then from the Combinations of the Coefficients of the third Term, we have this Equation, $-3xx-yy=-p$, or $3xx+yy=p$, or $p-3xx=yy$, or $y=\sqrt{p-3xx}=\frac{1}{2}$ the Difference of the two Roots. So that having the greatest, whether Negative or Positive, given or found first; and by consequence the Sum, or half the Sum of the other two, all the three are given at once by the preceding Theorem; only we are to remember, that $2x$ is equal to z, and by equivalent Substitution $y=\sqrt{p-\frac{3}{4}zz}$. *Ex.* $zzz-pz=-6$, or $zzz-7z=6$. Let the greatest Negative z (=to the other two Positive) be given or found, *viz.* Suppose 3; then is $\frac{1}{2}z+\sqrt{p-\frac{3}{4}zz}$, the greater $\left\{\begin{matrix} \text{Positive,} \\ \text{Negative,} \end{matrix}\right.$ and $\frac{1}{2}z-\sqrt{p-\frac{3}{4}zz}$, the lesser $\left\{\begin{matrix} \text{Positive,} \\ \text{Negative,} \end{matrix}\right.$ *viz.* $\frac{1}{2}3+\sqrt{7-\frac{3}{4}9}=1.5+.5=2$ the greater $\left\{\begin{matrix} \text{Positive,} \\ \text{Negative.} \end{matrix}\right.$

and $1.5-.5=1$ the lesser $\left\{\begin{matrix} \text{Positive,} \\ \text{Negative.} \end{matrix}\right\}$ And hence also we have another Determination of Imaginary Roots, *viz.* when these Theorems, or one of them, infer an Impossibility.

But if no Term be wanting, as in this Equation, $zzz-pzz-qz=-r$, where there are two Positive Roots, and one Negative moderately big; let $2x$ be the Sum of the Positive, $2y$ their Difference; then is $x+y$ the greater Positive, $x-y$ the Lesser, and $2x-p$ the third Negative; as is evident from the Nature of the Coefficients in the second Term; whence $2x-p=z$, or $\frac{z+p}{2}=x$.

Let the Roots, with their proper Signs, be multiply'd, so many ways as they can be taken, and their Difference equated to the Coefficient of the third Term, *viz.* $-q$, and then there will arise $-3xx-yy+2px=-q$, or $2px+q-3xx=yy$, or $y=\sqrt{2px+q-3xx}$; but x was found before equal to $\frac{z+p}{2}$, z being the Negative Root; which substituted eqivalently, gives $y=\frac{1}{4}\sqrt{pp+4q-3zz-2pz}$: Whence as before, $\frac{\frac{1}{4}\sqrt{pp+4q-3zz-2pz}+z+p}{2}$ equal to the greater Positive, and $\frac{\frac{1}{4}\sqrt{pp+4q-3zz-2pz}-z-p}{2}$, equal to the lesser Positive: But this supposes z the Negative or single Root first found. The like will serve for all the rest of the Cases, relation being had to the Signs of the Roots in forming the Theorem, the single Root being first found out, and the Sum and Difference of the other two, whether Positive or Negative, being call'd $2x$, $2y$; but if all the Roots be Positive, let $2x$ be the Sum of the lesser, $2y$ their Difference, and let the greatest be $2x+p$. Hence also in Cubic Equations having all the Terms, we have a Method for determining imaginary Roots, *viz.* when the preceding Theorems, or one of them infer an Impossibility.

Of Increasing and Decreasing the Value of an unknown Equation without destroying it.

SUppose I would double this Quadratic $xx+bx=d$; Multiply each Term by a Series of Proportionals, beginning with *Unity*, whose second Term is 2. Thus $1\times xx+2\times bx=4\times d$, which destroys the Equation; restore it again by a backward Multiplication of the same Proportionals $4\times 1\times xx+2\times 2\times bx=1\times 4\times d$; Instead of x take another unknown Quantity, y, supposed equal to $2x$, then is $xx=\frac{1}{4}yy$: And there results another Equation, which compar'd with the first, satisfies the Demand, *viz.* $\left\{\begin{matrix} 4\times 1\times xx+2\times 2\times bx=1\times 4\times d \\ yy+2by=4d \end{matrix}\right\}$

If

If I would have an Equation which shall be $\frac{1}{3}$ of this Cubic $xxx+bxx+cx=d$, multiply by a Series of Proportionals, beginning with 1, whose second Term is $\frac{1}{3}$, and put y for $\frac{1}{3}x$, then by the preceding Method we shall have this new Equation, $yyy+\frac{1}{3}byy+\frac{1}{9}cy=\frac{1}{27}d$. Or more generally thus, put a for any Number, Integer, or Fraction, and reassume the preceding Equation, $\left\{\begin{matrix} xxx+bxx+cx=d, \\ yyy+abyy+aacy=aaad, \end{matrix}\right.$ then is where the Value of y to x, is as a to 1.

The Uses of this Artifice are very great; some of which are these.

Use 1. *To free an Equation from Fractions.* Suppose $xx+\frac{1}{3}bx=d$, putting $y=3x$, we have $yy+by=9d$. Again, suppose this Equation was propos'd, $xxx+\frac{b}{a}xx=d$; that is, $xxx+\frac{b}{a}xx+ox=d$, putting $y=ax$, we have $yyy+byy*=aaad$.

Use 2. *To take away the second Term in any Equation.* Suppose $xx+bx=d$, put $y-\frac{1}{2}b=x$.

Then is $yy-by+\frac{1}{4}bb=xx$ and $\ldots+by-\frac{1}{2}bb=bx$ $\Big\}=d$.

$yy-\frac{1}{4}bb=d$.

By the same Method the second Term in a Cubic may be destroy'd; only remember that $y\pm\frac{1}{3}$ the Coefficient in the Square, $y\pm\frac{1}{3}$ the Coefficient in the Cube $y\pm\frac{1}{4}$, the Coefficient in the Biquadrate, *&c.* must be taken with the contrary Sign to the second Term; for if the same Sign be taken, the second Term will doubled not taken away.

Use 3. *To take away all the Coefficients in a Cubic Equation.* Which I think has not yet been done by any one; and perhaps others may make better use of it than I can at present. Suppose the second Term in a Cubic Equation taken away, or else that this is given $xxx+bx=d$, assume $y=x\sqrt{\frac{1}{b}}$, ($\sqrt{\frac{1}{b}}$, being a mean Proportional betwixt $\frac{1}{b}$ and Unity); Then multiply by a term of continual Proportionals in that Series as before; and instead of $xxx+bx=d$, we have $1xxxx+\sqrt{\frac{1}{b}}\circ+\frac{b}{b}x=\frac{d}{b}\sqrt{\frac{1}{b}}$ that is by equivalent Substitution $yyy+y=\frac{d}{b}\sqrt{\frac{1}{b}}$.

In all the preceding Cases, as soon as the Value of y is found, the Value of x is also discover'd, y being always taken in some certain Proportion to x.

Of the Punctation of Adfected Equations.

LET a Biquadrate be made from the Binomial $x+a=p$, according to the former *Genesis of Powers*, and it will arise to this,

$xxxx+4bxxx+6bbxx+4bbbx+bbbb=pppp$. Again, let a Biquadratic be found from several different Values of x, thus;

$\left\{\begin{matrix} x-b=0 \\ x-c=0 \\ x-d=0 \\ x-f=0 \end{matrix}\right\}$ And we have this *Genesis*. $\left\{\begin{matrix} xxxx-bxxx+bxxx \\ -cxxx-bdxx \\ -dxxx+cdxx-bcdx \\ -fxxx-bfxx-bcfx \\ +cfxx-bdfx \\ +dfxx-cdfx+bcdf=0. \end{matrix}\right.$

Now confounding the *Homogenal Coefficients*, and putting them all into one, we may represent their Totals thus ——— $x^4-mx+nnx-pppx=-qqqq$.

In

In this and the preceding *Geneſis*, (from one Value of x, omitting the diſtinction of the *Uncia*) 'tis evident that the Coefficients encreaſe in their Power, as the higheſt unknown Term decreaſes, and that the *laſt abſolute known Quantity* is of the ſame Power as the Firſt. Imagine the ſame under different Signs; alſo if one or more of the Terms be intercepted or loſt, the remaining will yet keep their Places. Hence in *Numeral Equations*, ſuppoſe $xxxx + 3x^3 + 16xx + 125x = 1000$, the Coefficients are firſt Lateral, then Quadratic, then Cubic, *&c.* and 'tis no matter whether they are figurate Numbers or not, *viz.* exact Squares, Cubes, *&c.* they are always ſuppoſed of that Nature. Hence we have theſe two Conſectaries.

1. *If the Root of the Coefficient (in any Place) be continually multiply'd to its degree of Adfection, and then by it ſelf again, the laſt Product ſhall be of the ſame Power with the higheſt unknown Term in the Equation.*

For Inſtance, $x^4 - mx^3 + n^2x^2 - p^3x = -qqqq$, the Coefficient m is a Lateral, (therefore it ſelf a Root); and if Cub'd, (for ſo is the higheſt Power which it is join'd to, or its degree of Adfection) it becomes mmm; which multiplied again by it ſelf, becomes m^4 of the 4*th* Power, as is $xxxx$. Again, in nnx^2, the Root of nn is n, its degree of Affection is a Square nn, which multiply'd into it ſelf, *viz.* nn, becomes $nnnn$ a Biquadrate. Laſtly, p^3x the Root of p Cube, is p, its degree of Affection, is only Lateral: Therefore p (the Side) $\times$ ppp (the Coefficient it ſelf) produces p^4.

2. *If the abſolute known Power be divided by any one of the Coefficients, the Quote will be of the ſame degree of Adfection with the place of ſuch Coefficient*, viz. *Lateral, if the Adfection be Lateral; Quadratic, if the Adfection be Quadratic*, &c.

For Inſtance, $x^4 + 3xxx + 16xx + 125x = 1000$: if 1000 be divided by 3, the Quote is Cubic; if by 16, the Quote is Quadratic; if by 125, 'tis Lateral. From hence then, and the preceding *Geneſis*, we have a Method of Pointing any adfected Equation in order to its Extraction.

It's evident from the *Formation of Powers*, that a Square muſt be pointed every ſecond Figure, (beginning from the Right) a Cube every third, a Biquadrate every fourth; and ſo on. For ſuppoſe 57209 was given to be ſquar'd, 5 has four places after it; therefore its Square 25 will have eight Ciphers; or if Cub'd, 'tis 125 with twelve Ciphers annex'd; and ſo on, every Cipher or Place following a Figure in the Root, making 2 in the Square, 3 in the Cube, *&c.*

Let any Equation be propos'd, (in order to its pointing). Suppoſe without any Coefficient $xxx - xx = 186494880$. From the preceding *Geneſis* 186494880 is Cubic, becauſe ſo is the higheſt unknown Term, therefore let it be pointed at every third Figure thus, 186494880. But ſince it alſo contains in it the Negation of xx, therefore imagine it alſo pointed as a Square, thus, 186494880, but with the ſame Number of Points, becauſe xxx is no oftner contain'd in 186494880, than xx is ſuppoſed to be taken out of it.

There are three Caſes that can only happen in pointing.

1. Where the number of Figures in the Coefficients are regular, and will admit of an equal Number of Points according to their reſpective Powers. For Inſtance, $xxxx + 13xxx + 237xx + 5927x = 100000$, where each place has two Points, determining the Value of x to conſiſt of two Integers; ſo that we have for the firſt pointing this Equation $x^4 + 1x^3 + 2xx + 5x = 10$ near; whereby the Value of x in its firſt Figure is readily diſcover'd. 'Tis the ſame if interrupted, *viz.* $xxxx + 9532x = 123579$, then $x^4 + 9532x = 12$; or if the natural Order of Places be inverted, *viz.* $x^4 + 3537xx\ 35xxx = 2598372$, where $x^4 + 35xx - 3x^3 = 259$.

2. The ſecond Caſe is call'd a *Devolution*, where the Number of Figures is too ſhort in ſome of the Coefficients: And here it's eaſier than before to find the Value of x as to the firſt Figure, for we have no more to do than prefix ſo many Ciphers in each, as are the number of Figures that are wanting.

For Inſtance, $xxx + 24x = 587914372$; that is, $xxx + 00024x = 587914372$. Whence $xxx + 0x = 587$, or $xxx = 587$.

3. The third, which is called a *Caſe of Anticipation*, where ſome of the Coefficients run beyond their juſt Number of Punctations, and this is much more troubleſome than either of the former.

For Inſtance, $xxx + 4xx + 376958x = 753922$; which pointed as before, would be $xxx + 04xx + 376958x = 753922$; or $x^3 + 3769x = 753$, where $x = .1$, whereas 'tis equal to 2 preciſely. The beſt Expedient I can meet with in this, (if the Figures have but one Punctation) is that of Dr. *Wallis*, who ſuppoſes the firſt Figure ariſing to be 1, 10, 100, 1000, *&c.* Or downwards, .1. .01 .001, *&c.* And then having found by trial whether it lies betwixt 1 and 10, or 10 and 100, *&c.* He takes the Medium 5, 50, *&c.* till he meets with it. But this Expedient will alſo prove fruitleſs for the ſecond Figure, if the *Anticipation* be large: we have therefore but one way left us that I know of, and that is the incomparable Method of a *Converging Series*, lately diſcover'd by the ingenious Mr. *Raphſon*; which though it might be help'd by Punctation, as in the two preceding Caſes, yet it has no abſolute Neceſſity of it, for by aſſuming any Number at pleaſure, no matter how far off the Truth, (as to the poſſibility of its accompliſhing the End) it will converge to it infinitely quicker than by any Eſſays or Trials whatever in the moſt perplex'd *Anticipations*, therefore extreamly ready and eaſy in the other Caſes. The further Uſes of it are beſt explain'd by Examples; however I thought it neceſſary to ſay ſomething of Punctations here, having occaſion to uſe them hereafter, becauſe Mr. *Raphſon* has not done it in his *excellent Series*, but only refer'd to *Vieta*, *Oughtred*, &c. which perhaps every one has not at hand, or can't read them in Latin. But Mr. *Raphſon*'s Buſineſs was not Repetition, or a troubling himſelf with what was requiſite to be known before he could be underſtood, preſuming reaſonably enough, that his Reader was not ignorant of what Advancements had been made before him in the *Exegiſis Numeroſa* of the aforementioned Authors.

Of Infinite Approximations, *or a Numeral* Converging Series *for all Adfected Equations whatever.*

Example 1.

LET any Equation be propos'd, ſuppoſe $xx = a$.

1. Put p (a known Quantity) $+ y$ (an unknown Quantity) equal to x. Then is $p + y|^2$, or $pp + 2py + yy = (xx =) a$.

2. By Tranſpoſition $2py = a - pp - yy$, or $y = \frac{a - pp - yy}{2p}$; or which is the ſame thing, $y = \frac{a - pp}{2p} - \frac{yy}{2p}$. But $x = p + y$ by Poſition. *Ergo*, $x = p + \frac{a - pp}{2p} - \frac{yy}{2p}$, in which Equation $p + \frac{a - pp}{2p}$ is wholly a known Quantity, and $- \frac{yy}{2p}$ an unknown.

3. Put $q = p + \frac{a - pp}{2p}$, and $- z = - \frac{yy}{2p}$, then is $x = q - z$. Then alſo is $qq - 2qz + zz = (xx =) a$; and by Tranſpoſition and Diviſion as before $z = \frac{qq - a}{2q} + \frac{zz}{2q}$. But $x = q - z$ by Poſition; *Ergo*, $x = q + \frac{a - qq}{2q} - \frac{zz}{2q}$, where the known $q + \frac{a - qq}{2q}$, is evidently leſs than the former known Quantity q, becauſe $qq < a$.

4. Put

4. Put $r = q + \frac{c-qq}{2q}$, and $-v = -\frac{xx}{2q}$; and by the ſame Method of procedure we ſhall have $x = r + \frac{c-rr}{2r} - \frac{vv}{2r}$ the known Quantities which are too great being ſtill decreas'd.

5. Put $s = r + \frac{c-rr}{2r}$ and $-\omega = -\frac{xx}{2r}$, and by a like repeated Proceſs there will ariſe $x = s + \frac{c-ss}{2s} - \frac{\omega\omega}{2s}$; which laſt $-\frac{\omega\omega}{2s}$ will be ſo very ſmall, that we need make no ſubduction for it, if the Value of a be not above 32 Figures, which I think is ſufficient for Practice.

Hence then by collecting the whole together we have this Series;
$p + \frac{c-pp}{2p}\ (=q)\ q + \frac{c-qq}{2q}\ (=r)\ r + \frac{c-rr}{2r}\ (=s)\ s + \frac{c-ss}{2s} = x$ *proximè*; or according to Mr. *Raphſon*'s Method, $p + \frac{c-pp}{2p}$ only, repeated at pleaſure is equal to x, where the Value of p always changes at every Operation; and the Value of a is firſt 1 Punctation, then 2, then 4, then 8, then 16, and ſo on as far as any one pleaſes to proſecute it, from the ſame Conſiderations which I mention'd in the preceding Series for Extraction of Square Roots.

Examp. 2. Let this adfected Quadratic be propos'd, $xx + bx = c$.

1. Put $x = p + y$; then is $\left.\begin{matrix} pp + 2py + yy \\ bp + by \ldots \end{matrix}\right\} \begin{matrix} = xx \\ = bx \end{matrix} \Big\} = c$. That is,

$y = \frac{c-pp-bp-yy}{2p+b}$; or which is the ſame thing, $y = \frac{c-pp-bp}{2p+b} - \frac{yy}{2p+b}$, but $x = p + y$; therefore $x = p + \frac{c-pp-bp}{2p+b} - \frac{yy}{2p+b}$. Now putting $q = p + \frac{c-pp-bp}{2p+b}$, and $-z = -\frac{yy}{2p+b}$. By the ſame Procedure repeated at pleaſure, we have this Series,

$p + \frac{c-pp-bp}{2p+b}\ (=q)\ q + \frac{c-qq-bq}{2q+b}\ (=r)\ r + \frac{c-rr-br}{2r+b}\ (=s)\ s + \frac{c-ss-bs}{2s+b} = x$.

Or according to Mr. *Raphſon*'s Method, $p + \frac{c-pp-bp}{2p+b}$ only repeated as far as there is occaſion, is equal to x, p acquiring a new Value every Operation; alſo b and c changing their Value, reaching firſt to one Punctation, then to two, then to four, then to eight, and ſo on, or at leaſt as far as there are Punctations; and when there's no more their Value is fix'd, tho p will always converge and vary *ad infinitum*.

Examp. 3. Let this adfected Cubic Equation be propos'd, $x^3 + bxx + cx = d$.

Putting $x = p + y$, we have this new Equation reſulting. $\left.\begin{matrix} ppp + 3ppy + 3pyy + yyy \\ bpp + 2bpy + byy \ldots \\ cp + cy \ldots\ldots \end{matrix}\right\} \begin{matrix} = x^3 \\ = bxx \\ = cx \end{matrix} \Big\} = d$.

Then is $\frac{d-ppp-bpp-cp}{3pp+2bp+c} - \frac{3ppy+3pyy+yyy+bpy+byy+cy}{3pp+2bp+c} = y$; But $p + y = x$. Therefore $x = p + \frac{d-p^3-bp^2-cp}{3p^2+2bp+c} - \frac{3ppy+3pyy+yyy+bpy+byy+cy}{3pp+2bp+c}$. Now putting q for all that's known, as before, and $-z$ for all that's unknown, and ſo repeating the ſame Proceſs arbitrarily, we ſhall have this Series;

$p + \frac{d-p^3-bp^2-cp}{3p^2+2bp+c}\ (=q)\ q + \frac{d-q^3-bq^2-cq}{3q^2+2bq+c}\ (=r)\ r + \frac{d-r^3-br^2-cr}{3r^2+2br+c}$, &c. $= x$.

And ſo on in all higher Equations, repreſenting always all thoſe Quantities which are affected with the *Powers* of the unknown *aſſum'd Quantity* by another unknown, thereby driving forward and decreaſing that unknown Value, till it becomes ſo

ſmall that it may ſafely be rejected, and what is known taken for Truth it ſelf, as being indeterminately near it: remembring alſo, that all the Values of the Coefficients, as alſo the laſt *abſolute known Number* change, ſo long as the Punctations laſt; but no longer, though the Value of the aſſum'd Quantity will ever *converge* and alter.

Before I proceed to Examples, I ſhall give the *Series* for the four Caſes in Quadratics, as alſo the fourteen uſeful Caſes in Cubics, leaving others to calculate higher as they have occaſion.

The four Quadratic Series.

1.	$xx+bx=c$	$x=\frac{c-pp-b\times p}{2p+b}+p$
2.	$xx-bx=c$	$x=\frac{c+bp-pp\times p}{2p-b}+p$
3.	$-xx+bx=c$	$x=\frac{c+pp-bp\times p}{b-2p}+p$
4.	$xx=c$	$x=\frac{c-pp}{2p}+p.$

The fourteen Cubic Series.

1.	$xxx=c$	$x=\frac{c-p^3}{3p^2}+p$
2.	$xxx+cx=d$	$x=\frac{d-pp-c\times p}{3pp+c}+p$
3.	$xxx-bx=d$	$x=\frac{d+cp-pp\times p}{3pp-c}+p$
4.	$-xxx+cx=d$	$x=\frac{d+pp-c\times p}{c-3pp}+p$
5.	$xxx+bxx=c$	$x=\frac{c-pp-bp\times p}{3p+2b\times p}+p$
6.	$xxx-bxx=c$	$x=\frac{c+bp-pp\times p}{3p-2b\times p}+p$
7.	$-xxx+bxx=c$	$x=\frac{c+pp-bp\times p}{2b-3p\times p}+p$
8.	$x^3+bx^2+cx=d$	$x=\frac{d-pp-bp-c\times p}{c+3p+2b\times p}+p$
9.	$x^3-bxx+cx=d$	$x=\frac{d+bp-pp-c\times p}{c+3p-2b\times p}+p$
10.	$x^3+bxx-cx=d$	$x=\frac{d+c-pp-bp\times p}{3p+2b\times p-c}+p$
11.	$x^3-bxx-cx=d$	$x=\frac{d+bp-pp-c\times p}{3p-2b\times p-c}+p$
12.	$-x^3+bxx+cx=d$	$x=\frac{d+pp-bp-c\times p}{c+2b-3p\times p}+p$
13.	$-x^3+bxx+cx=d$	$x=\frac{d+pp+bp-c\times p}{c-3p-2b\times p}+p$
14.	$-x^3+bxx-cx=d$	$x=\frac{d+pp+c-bp\times p}{2b-3p\times p-c}+p$

No one that underſtands how to convert a Literal Theorem into a Numeral Operation, can be ignorant how to apply the preceding *Theoretic Series*. Yet notwithſtanding the Method is ſo very natural and eaſy, I ſhall give ſome Examples.

I have formerly taken notice, that in adfected Quadratics, if the *Homogeneum Comparationis* ariſes high, this *Series* is preferable to the common Way; but in all higher Equations whatever, it infinitely exceeds any *Exegeſis Numeroſa* that has yet been found out.

Let this Adfected Quadratic be propos'd, $xx+587x=987459$, and let the Value of x be found to five or ſix Places in Decimals; which is the ſame thing as to find the Value of x (if Rational) where the *abſolute known Quantity* ariſes to twelve Places.

Let

Let $b = 587$, and $d = 987459$. $xx + 587x = 987459$
After Punctation we have first $xx + 5x = 98$ $xx + bx = d$
2dly, $xx + 58x = 9874$
3dly, $xx + 587x = 987459$

Also $b = 5$, next $b = 58$, lastly, $b = 587$.
Again, $d = 98$, next $d = 9874$, then $d = 98745$. And if further prosecuted, add Ciphers at pleasure according to the preceding Rules.

$x = \frac{d - pp - bp}{2p + b} + p$, Theorem 1st:

Or $x = \frac{d - p - bxp}{2p + b} + p$, Theor. 2d.

An Operation according to the first Theorem.

$8 = p^{1st}$ $98 = d$ $*\overline{.746}|\frac{3}{7} = 556516$
-64 437902
-40 -994418
-104 987459
$21)-6.0(-.2$ $1492)-6459(-3.34$
$+8$ $+746.66$
$78 = p^{2d}$ $742.66 = p^{3d}$
$78|\frac{3}{7} = 6084 = pp$ $742.66|\frac{3}{7} = 551543.8756$
$4524 = bp$ 435941.42
-10608 -987485.2256
9174 487459
$156)-734(-3.4$ $207282)-26.2456(-.012689$
$+78$ $+742.66$
$746 = p^{3d} *$ $742.647311 = x$ sought.

II. EXAMPLE.

PUT $xxx - 2x = 5$, or $xxx - bx = c$.

Here's no need of any Punctations in this Example.

$2 = p^{1st}$ $5 = c$ $*5 = c$
4 4.1892
9 $9.1892 = bp + c$
$-8 = p^3$ $-9.18741550536 = p^3$
$18)1.0(.1$ $11.16205)-.00054155036(-.000048517$
$+2$ $+2.0946$
$2.1 = p^{2d}$ $2.094551483 = x$, sought.
$5 = c$
4.2
4.2
$-9.261 = p^3$
$11.23)-.061(-.0054$
$+2.1$
$2.0946 = p^{3d} *$

Theorem $x = \frac{c + bp - p^3}{3pp - b} + p$

III. EXAMPLE.

$xxx+74xx+8729x=560783$
$xxx+bxx+cx=d$

(1) $x^3+7x^2+87x=560.$ (2) $x^3+74x^2+8729x=560783.$

(1) $b=7.$ (2) $b=74.$

(1) $c=87.$ (2) $c=8729.$

(1) $d=570.$ (2) $d=560783.$

1. Theor. $x=\frac{d-p^3-bp^2-cp}{c+3p^2+2bp}+p.$

2. Theor. $x=\frac{d-p^3-bpxc}{c+3p+2bxp}+p.$

An Operation according to the second Theorem.

$4=p^{1\beta}$

$116=pp$
$128=pp$
$187=cp$
-131 Total
$4=p$
-524
$560=d$

$191)-36(-.1$
$+4=p^{2d}$
$41=p^{*}$

$*41|^2_1=11681=pp$ ☞ $41.48|^2_1=11720.5904=pp$
$13034=bp$ $13069.52..=bp$
$18729=c$ $18729....=c$
-13444 Total. -13519.1104
$41=p$ Mult.by $41.48=p$
13444 Produces -560772.699392
53776 $560783=d$
-551204 A 16.300608
$560783=d$

$19140)+9579(*.48$
$41.=p^{2d}$
$41.48=p^{3d}.$ ☞

A $20030)10.300608(.000514$
$41.48=p^{3d}$
$41.480514=x$ sought.

IV. EXAMPLE.

$-xxxxx+7xxxx-20xxx+155xx=10000$
Or, $xxxxx+bxxxx-cxxx+dxx=f.$

$-5=p^{1\beta}$

$-3125=p^5$
$-2500=cp^3$
$-4375=bp^4$
$-3875=dp^2$
-13857
$10000=f$

$-9765)-3875(.4$
$-5=p^{1\beta}$
$-4.6=p^{2d}*$
$-3500=4bp^3$
$-1550=2dp$
$-3125=5p^5$
$-1500=3cpp$
-9575 Divisor.

Theor. $x=\frac{f-p^5-cp^3-bp^4-dp^2}{-4bp^3-2dp-5p^5-3cpp}+p.$

$*-2059.62976=p^5$
$-1946.72=cp^3$
$-3134.2192=bp^4$
$-3279.8=dp^2$
-10420.36896
$+10000$

$-7659)-420.36896($ $+.055$
$-4.6=p^{2d}$
$-2725.408=4bp^3$ $-4.545-=x$ *propè*.
$-1426.000=2dp$
$-2238.728=5p^5$
$-1269.600=3cpp$
-7639.736 Divisor.

V. EX-

V. EXAMPLE.

LET this Equation be propos'd for the Septisection of an Angle.

$-x^7+7x^5-14x^3+7x=1.5$ Theor. $x=\frac{q+p^7+dp^3-cp^5-bp}{5cp^4+b-7p^6-3dpp}+p.$

Or, $-x^7+7x^5+14x^3+7x=1.5$

$.2=p^{1\beta}$

$+7.056000=5cp^4+b$ $+1.6120128=q+p^7+dp^3$

$-1.680448=-7p^6-3dp^2$ $-1.4032400=-cp^5-bp$

$5.375552\ldots\ldots)$ $.2097728(.03$

$+2$

$.23=p^{2\delta}$

$.23=p^{1\delta}$

$+7.09794435=5cp^4+b$ $+1.67037204825447=q+p^7+dp^3$

$-2.22283625=-7p^6-3dp^2$ $-1.6145054401\ldots=-cp^5-bp$

$4.8751081\ldots\ldots)$ $+.055866608154 47(+.0115$

$+.23$

$.2415=p^{3\delta}$

$.2415=p^{1\delta}$

7.11905196957719 $+1.69723543656325544455$

-2.45092317574650 -1.69625021013057815625

$4.66812879383069)$ $+.00098522643267728825($ $+.00021105$

$+.2415$

x sought $=$ $.24171105=p^{1b}$

When the Powers arise very high, as in this Example, they are most conveniently manag'd by Logarithms.

To shew the Universality of this Method, and that there's no absolute necessity of Punctations, or taking a Quantity near the Truth, let there be taken a Quantity never so distant from Truth, it will at a few Operations converge to it; which is a great Convenience above all other Methods, in which if you be once out, you must begin again; but in this an Error is but a new Trial, and you are never out of the way, for if you prosecute that Error you will find the Truth, and there is no need of considering the Occasion of it, further than as a Caution for the next Operation. One Example will abundantly evince this.

Let there be propos'd $xx+bx=d$ } Begin as far of Truth as you please.

Or, $xx+5x=646$ } Suppose $p=1$.

$1=p^{1\beta}$

Theor. $x=\frac{d-pp}{2p+b}+p$

or by Red. $x=\frac{d+pp}{2p+b}$

646

+1

$7)647(92=p^{2d}$

63

17

92

184

828

+8464

646

$189)9110(48=p^{3d}$

756

1550

48

384

192

2304

646

$101)2950(29=p^{4b}$

202

930

$29=p^{4b}$

29

261

58

841

646

$63)1487(23=p^{5b}$

126

227

23

69

46

529

646

$51)1175(23.03=p^{4b}=x$ sought.

102

155

153

200

These

These Examples I have borrowed out of Mr. *Raphson*, where such as please may see great Variety: But his Method is so natural, easy, and yet general, that half of these Examples would have given a very dull Reader sufficient Instruction in all Cases that can happen, as well as a particular Satisfaction in partaking the Benefits of such an happy Discovery, which no doubt but future Ages will very gratefully acknowledg.

FINIS.

ERRATA.

PAge 1. §. 7. line 2. in *Compounds*, expunge *s*.
Pag. 2. Parag. I. line 4. for 3*da*, read 13*da*.
Idem. Parag. IV. l. 7. for *s*, read *ss*.
Pag. 3. Parag. 4. l. *ult.* for $-a^3$, read $+3a^3$.
Pag. 4. Parag. 3. l. 11. for $-c^3bd$, read $+c^3bd$.
Idem. l. 12. for $+cb$, read *bb*, and for $-c^3bd$, read $+c^3bd$.
Pag. 6. L. 1. l. 3. expunge *Coroll. id.* and prefix it to *Lemma* II.
Idem. §. III. l. 2. betwixt *therefore* and *divide*, insert *actually*.
Idem. §. V. l. *penult.* for $d+e$, read $dd+e$.
Pag. 7. §. VII. l. 6. betwixt *ac* and *bb*, insert a Point.
Idem. §. VIII. l. 6. for *b* twice, read *b* twice.
Pag. 8. Parag. *ult.* l. 9. for *aatn*, read *aaatn*.
Idem. §. II. l. *ult.* for *bcddr*, read 6*cddr*.
Pag. 10. In the Line expressing the Root, for *a* & *b*, read *b* & *a*.
Idem. In the Figure of the Cube, for *aa* mend *aab*, and in the next Cell underneath write *bba*.
Idem. In the Figure of the Biquadrate, mend *Biquadr*.
Idem. §. IV. *ult.* for *quadratim*, read *gradatim*.
Pag. 11. l. 1. for $2612\frac{3}{7}$, read $6212\frac{3}{7}$.
Idem. §. V. l. 6. betwixt *have* and *Ciphers*, insert *five*.
Idem. §. VI. l. 6. instead of $2ab$, read $2a)2ab$, &c.
Idem. Parag. 1. l. 1. expunge *the*.
Pag. 12. l. 26. for 3357, read 3375.
Idem. §. VII. l. 7. expunge *Squared* after *which*, and insert it after *Quote*.
Pag. 14. §. IX. l. 18, 19, 20, expunge wholy.
Pag. 15. Parag. 2. l. 3. set 8 one place nearer to the right hand.
Pag. 17. l. *penult.* for $qq-10$, read $qq+10$.
Idem. l. *ult.* for $qq-100$, read $qq-10$.
Pag. 18. In Division, the second Line, for -10, read $+10$.
Pag. 19. l. 3. for $+bx$, read $-bx$.
Pag. 22. *Quest.* 8. l. *ult.* instead of $aa-xx$, read $aa-aa-xx$, or, &c.
Pag. 23. *Quest.* 13. the Quest. is Universal for all Triangles whatever.
Idem. *Quest.* 14. after *four*, read *or*.
Pag. 26. l. 1. expunge *ly* in *immediately*.
Pag. 27. §. 4. l. 16 & 21. instead of $\sqrt{\frac{cc}{\frac{1}{4}d}}$, read $\sqrt{\frac{cc}{\frac{1}{4}d}}$.
Pag. 29. Parag. 3. l. *ult.* for *imaginary Equation*, read *impracticable Equation*.
Pag. 30. Parag. 3. l. 2. for *imaginary or impossible*, read *impracticable Equation*.
Pag. 33. Q. 1. l. 28. for $\sqrt{\frac{b25}{16}}$ read $\sqrt{\frac{625}{16}}$.
Pag. 35. Q. 1. l. 8. for $cc+aa-bb$, read $cc+bb-aa$.
Pag. 36. Q. 2. l. 3. for CE and AB, read AE and DE.
Idem. l. 16. instead of $\sqrt{aa+\frac{+ll}{+a}}$ read $\sqrt{aa+\frac{+ll}{+a}}$.
Pag. 39. in the Title expunge *Quadratic*, &c.
Idem. in the last line but two, expunge *Biquadratic and higher*.
Pag. 40. in the Title, expunge *Biquadratic*, &c.
Pag. 41. l. 5 and 9. for lesser, read less.

Numerical RECREATIONS.

CHAP. I.

Of *Digit*, *Article*, *Mixt*, *Square* and *Cube Numbers*; and ſome *Obſervations* upon them.

TO paſs by the Common *Species* and *Rules* of *Arithmetick*, both *Vulgar* and *Decimal*, in *Whole Numbers* and *Fractions*; as alſo the *Extraction* of the *Square* and *Cube Roots*; ſuppoſing my Reader to be already acquainted with them; I ſhall proceed to treat of ſome other *Numerical Practices*, wherein the *Properties* and *Privileges* which ſome particular *Numbers* have over others; as alſo in the reſolving of ſeveral *Enigmatical Queſtions*, to recreate the Spirits of the Ingenious Student in the *Art* of *Numbers*: And I ſhall begin with the *Nine Digits*, viz. 1, 2, 3, 4, 5, 6, 7, 8, 9.

I. *Of the Digit* 1.

The Digit 1 hath a Property which no other Number hath beſides it ſelf; for it neither Multiplieth nor Divideth, but leaveth the Number to be ſo Multiplied or Divided ſtill the ſame: As if it were required to Multiply 365 by 1, it will ſtill be but 365: Alſo if you were to Divide 365 by 1, the Quotient will be the ſame; for 1 may be had 365 times in 365; and 365 Unites that is multiplied by 1, maketh it no more than 365: And from hence it alſo followeth, that if any Number be divided by 1, there will nothing remain, whereas Numbers divided by any other Digit are liable to.

II. *Of the Digit* 2.

If you would multiply readily any number by the Digit 2, it is but doubling of that number; ſo 365 being doubled makes 730, which is equal to the Product of 365 multiplied by 2. ——Alſo, if you would divide any number by 2, it is but taking the half of that number; ſo if 730 were to be divided by 2, the Quotient will be 365, for the half of 730 is 365. ——But this Digit number 2 hath another Property or Privilege above any other Digit, Article, or Mixt Number; for there is no other whole Number to be found, which being added to it ſelf, or

multiplied in it ſelf, that ſhall produce the ſame number; but 2 added to 2, maketh 4 for the Sum; and 2 multiplied by 2, produceth 4 for the Product, which is alſo the Rectangle or Square of 2. —— And it is moreover worth the taking notice of, That no Square Number, how large ſoever, can terminate or end with the Digit 2.

III. *Of the Digit* 3.

If you would multiply any number by 3, to the given number add the double thereof, and the Sum ſhall be equal to the Product; ſo 365 multiplied by 3, will be 1095; for the double of 365 is 730, to which 365 being added, the Sum is 1095; and ſo much would 365 multiplied by 3 produce. —— Alſo, to divide any number by 3, take one third part thereof; ſo one third part of 1095 is 365, which is equal to the Quotient. And with this Digit 3, no Square Number can terminate.

IV. *Of the Digit* 4.

If you would multiply any number by 4, you muſt double the duplication thereof, and the Sum is the Product; ſo the double of 365 is 730, which doubled is 1460, and that is the Product of 365 multiplied by 4. —— And if you would divide any number by 4, one fourth part thereof is the Quotient; ſo one fourth part of 1460 is 365, equal to the Quotient.

V. *Of the Digit* 5.

If you would multiply any number by 5, add a Cypher to the given number, then the half of that number will be equal to the Product; ſo 365 multiplied by 5, will produce 1825; for if to 365 you add a Cypher, it makes it 3650, the half whereof is 1825, equal to the Product. —— On the contrary, if you would divide any number by 5, double the number, and cut off the laſt figure towards the right hand, (which will always be either a 5 or a Cypher) and the remainder ſhall be the Quotient. —— So if you would divide 1825 by 5, the double of 1825 is 3650, from which cut off the 0 towards the right hand, and it leaves 365, equal to the Quotient.

VI. *Of the Digit* 6.

If you would multiply any number by 6, add a Cypher to the given number, and take the half thereof, to which add the given number, the Sum ſhall be equal to the Product of that number multiplied by 6: So if you would multiply 365 by 6, a Cypher added to the given number makes it 3650, the half whereof is 1825, to which 365 added, it makes 2190, equal to the Product of 365 multiplied by 6. —— On the contrary, To divide any number by 6, take half the number, one third part of that half ſhall be the Quotient: So if you would divide 2190 by 6, the half of 2190 is 1095, one third part whereof is 365, equal to the Quotient.

I. *Observation.*

¶ Between these two last mentioned *Digits* 5 and 6, there is a secret Property; for if you multiply either of them in themselves, the numbers produced by such multiplications shall terminate in themselves; so 5 multiplied in 5, produceth 25; and 6 multiplied in 6 produceth 36, terminating in themselves 5 and 6. —— Also, if any greater numbers be multiplied by 5 or 6, they will continually terminate or end in 5 and 6; so 365 multiplied by 5, produceth 1825, terminating in 5; and 186 multiplied by 6, produceth 1116, terminating in 6.

II. *Observation.*

¶ The number 6 hath another eminent Property, for all its aliquot parts are equal to himself; as his half (which is 3), his third (which is 2), and his sixth (which is 1), being added together, do make 6. And of Numbers that have this Property, there are but 10 to be found between *One*, and *One Million of Millions*, which are these exhibited in this following Scheme; and in these Numbers you may observe an Order; for every of the Odd Places, as the I, III, V, VII, and IX, do terminate in 6, whose half is 3, an Odd Number; and the Even Places, namely, the II, IV, VI, and VIII, do terminate in 8, the half whereof is 4, an even number. And if you will farther proceed to find more of these numbers, the 20*th.* number having this qualification, will be this, 15111577451553768931328.

I	6
II	28
III	496
IV	8128
V	120816
VI	2096128
VII	33550336
VIII	536854528
IX	8589869056
X	137438691328

VII. *Of the Digit* 7.

If you would multiply any number by 7, add a Cypher to the given number, and take the half thereof, to which half add the double of the number given, the Sum of them shall be the Product of the given number multiplied by 7. —— So if you would multiply 365 by 7, add a Cypher to it, and it makes 3650, the half whereof is 1825, to which add 730, the double of 365 the given number, and the Sum will be 2555, which is the Product of 365 multiplied by 7. — On the contrary, if you would divide any number by 7. ——— Double the number given, and cut off the last figure to the right hand; then take the seventh part of that number and double it, and substract it from the former number, the remainder shall be the Quotient. —— So if you would divide 2555 by 7, that number doubled is 5110, from which cut off the Cypher to the right hand, and it is 511, one seventh part whereof is 73, the double whereof is 146, which substracted from 511, the remainder is 365, and that will be the Quotient of 2555 being divided by 7. ——And with this Digit 7, no Square Number can terminate.

VIII. *Of the Digit* 8.

To multiply any number by 8, double the number given, and ſubſtract that double from the given number, (a Cypher being added to the given number), that Remainder ſhall be the Product of the given number, multiplied by 8. ——So if you would multiply 365 by 8, the double of 365 is 730, a Cypher added to 365 is 3650, from which ſubſtract the double of 365, namely, 730, and the remainder will be 2920, the Product of 365 multiplied by 8.——In like manner, if you would divide any number by 8, take half the number given ſucceſſively three times, the third half ſhall be the Quotient of the given number divided by 8. ——So if 2920 were a number given to be divided by 8, the half of it is 1460, the half of that is 730, and the half of that is 365, which is the Quotient of 2920 divided by 8. ——And this Digit 8 hath another Property in it, for no Square Number can terminate with it.

IX. *Of the Digit* 9.

To multiply any number by 9, add a Cypher to the given number, and from that number ſubſtract the given number, the Remainder ſhall be the Product. ——So if you would multiply 365 by 9, add a Cypher to it, it makes 3650, from which ſubſtract 365, and the remainder will be 3285, equal to the Product of 365 being multiplied by 9. ——On the contrary, if you would divide any number by 9, take the third part of the given number twice ſucceſſively, the ſecond remainder ſhall be the Quotient: ——So if you would divide 3285 by 9, one third of 3285 is 1095, and one third of 1095, is 365, which is the Quotient of 3285 divided by 9.

III. *Obſervation.*

¶ This Digit 9 hath a Privilege above all the other Digits; for if you take any number, the Nines taken out of the groſs ſum of that number, or of all the parts thereof ſeverally, the remaining Digit will be ſtill the ſame: *Example*; In the number 36 there are 4 Nines contained therein; and ſo if you multiply 9 by 4, it will produce 36; and if you take the Nines out of 36, they are four alſo. ——In like manner, if you take the Nines out of this number 248, it is all one as if you ſhould take the Nines out of the ſimple figures 2, 4, and 8, which do make 14, from which 9 being taken, there will remain the Digit 5; and alſo if you divide 248 by 9, the Quotient will be 27, which ſhews that there are 27 times 9 in 248, and the Digit 5 remaining.

X. *How the Nine* Digits *may be diſpoſed in ſuch* Order, *that the* Number 15 *ſhall be accounted in* Rank, File, *and* Diagonally, 32 *times.*

In the Nine Digits there are Four Even, and Five Odd; now if you ſet the four Even Digits, 2, 4, 6, and 8, at the four corners of a Square, and 5 (which is the middlemoſt of the five odd Digits) in the Center

or

or middle of the Square, as is done in the Margent then between every two even Digits put ſuch an odd Digit as ſhall make the number in that line both in Rank and File 15; as between 2 and 4, put 9, which together make 15, and ſo the reſt; the which 9 Digits thus diſpoſed, 15 may be accounted in *Rank*, *File*, and *Diagonally*, 32 times: Example,

	Rank		
	2	9	4
File	7	5	3
	6	1	8
	Rank		File

In Rank,

Rank 1.	2 9 4 4 9 2 9 2 4 9 4 2	equal to 15.
Rank 2.	7 5 3 3 5 7 5 7 3 3 7 5	equal to 15.
Rank 3.	6 1 8 8 1 6 1 6 8 8 6 1	equal to 15.

In File,

File 1.	2 7 6 6 7 2 7 2 6 6 2 7	equal to 15.
File 2.	9 5 1 1 5 9 5 1 9 5 9 1	equal to 15.
File 3.	4 3 8 8 3 4 3 4 8 3 8 4	equal to 15.

Diagonally,

Diagonal Dexter,	2 5 8 8 5 2 5 2 8 5 8 2	equal to 15.
Diagonal Sinister,	4 5 6 6 5 4 5 4 6 4 6 5	equal to 15.

IV. *Obſervation.*

¶ There is a notable harmony between ſome Numbers, as particularly in theſe two, 220 and 284: For the addition of the Aliquot Parts of the one, do make up the Sum of the other: So

The Aliquot Parts of 220 are	110	
	55	
	44	
	22	
	20	
	11	
	10	
	5	
	4	
	2	
	1	
Whoſe Sum is	284	

The Aliquot Parts of 284 are,	142	
	71	
	4	
	2	
	1	
Whoſe Sum is	220	

XI. *Of*

XI. *Of the* Article Numbers, 10, 100, 1000.

If you would multiply any number by 10, 100, or 1000, *&c.* it is done by adding of ſo many Cyphers towards the right hand of the given number, as there are Cyphers in the Article number by which you are to multiply: So

If you are to multiply { 365, 7642, 96421 } by { 10, 100, 1000 } the Product will be { 3650, 764200, 96421000 }

On the contrary, any number may be divided by an *Article Number*, by cutting off from the given number ſo many figures towards the right hand, as there are *Cyphers* in the *Article Number* by which you are to divide: So

If you would divide { 7628, 34086, 932073 } by { 10, 100, 1000 } the Quotient will be { 762. 8/10, 340. 86/100, 932. 073/1000 }

And the like of any other.

XI. *Of the Numbers* 11, 12, 13, 14, 15, 16, 17, 18, *and* 19.

If you would multiply any number by any of theſe numbers that hath a Unite in the Tenth place, obſerve the method following, where you may ſet down the Product in one ſingle line; as in the Margent;

786
12
——
9432

Let 786 be a number given to be multiplied by 12; ſet them down as in the Margent, and ſay, Two times 6 is 12; ſet down 2, and keep 1 in mind: ——Then, 2 times 8 is 16, and 1 in mind is 17, and 8 over the 1, is 23; ſet down 3, and bear 2 in mind: Then, 2 times 7 is 14, and 2 in mind is 16, and 8 over 1 is 24: ſet down 4, and bear 2 in mind; which 2 add to the laſt figure 7, it makes 9; ſo that 786 multiplied by 12, produces 9432. And ſo of others; as in Examples,

4264	60304
16	19
68224	1145776

XII. *Of*

XII. *Of* Square *and* Cube *Numbers.*

It is obſerved before, that no Square Number, can terminate in any of theſe four Digits 2, 3, 7, or 8: Notwithſtanding, there are many ſubtle Properties in Square Numbers: As,

1. If from an Unite you do ſucceſſively add the next odd digit number, the ſum of thoſe two Numbers ſhall produce Square Numbers: As, if to 1 you add 3, the next odd number, the Sum is 4, a Square Number; to which if you add 5, the next odd Digit, it makes 9, a Square Number; to which if you add 7, the next odd Digit, it makes 16, a Square Number; and ſo on, as in this little Table.

For	1	3	makes	4
	4	5		9
	9	7		16
	16	9		25
	25	11		36
	36	13		49
	49	15		64
	64	17		81
	81	19		100

A Square Number.

2. If a Series of Eleven Square Numbers be orderly ſet one under another, the terminating figure of the firſt, and the terminating figure of the laſt ſhall be the ſame. And ſo if the terminating figure of the firſt number be a Cypher, the ſecond ſhall be 1, the third 4, the fourth 9, the fifth 6, the ſixth (or middlemoſt) 5, the ſeventh 6, the eighth 9, the ninth 4, the tenth 1, the eleventh 0, as in this Table. So likewiſe if Cube Numbers be ſucceſſively added from Unity, thoſe Numbers ſhall alſo be Square Numbers.

Roots	Squares
80	6400
81	6561
82	6724
83	6889
84	7056
85	7225
86	7396
87	7569
88	7744
89	7921
90	8100

XIII.

A Table of the Nine Digits, each of them being Multiplied into all the Nine Digits, and the several Products of those Multiplications Added.

I.				II.				III.			
1	1	1	1	1	2	2	2	1	3	3	3
2	1	2	2	2	2	4	4	2	3	6	6
3	1	3	3	3	2	6	6	3	3	9	9
4	1	4	4	4	2	8	8	4	3	12	3
5	1	5	5	5	2	10	1	5	3	15	6
6	1	6	6	6	2	12	3	6	3	18	9
7	1	7	7	7	2	14	5	7	3	21	3
8	1	8	8	8	2	16	7	8	3	24	6
9	1	9	9	9	2	18	9	9	3	27	9
		45	45			90	45			45	54

IV.				V.				VI.			
1	4	4	4	1	5	5	5	1	6	6	6
2	4	8	8	2	5	10	1	2	6	12	3
3	4	12	3	3	5	15	6	3	6	18	9
4	4	16	7	4	5	20	2	4	6	24	6
5	4	20	2	5	5	25	7	5	6	30	3
6	4	24	6	6	5	30	3	6	6	36	9
7	4	28	1	7	5	35	8	7	6	42	6
8	4	32	5	8	5	40	4	8	6	48	3
9	4	36	9	9	5	45	9	9	6	54	9
		180	45			225	45			270	54

VII.				VIII.				IX.			
1	7	7	7	1	8	8	8	1	9	9	9
2	7	14	5	2	8	16	7	2	9	18	9
3	7	21	3	3	8	24	6	3	9	27	9
4	7	28	1	4	8	32	5	4	9	36	9
5	7	35	8	5	8	40	4	5	9	45	9
6	7	42	6	6	8	48	2	6	9	54	9
7	7	49	3	7	8	56	1	7	9	63	9
8	7	56	1	8	8	64	0	8	9	72	9
9	7	63	9	9	8	72	9	9	9	81	9
		315	45			360	42			405	81

CHAP.

XIV. *Some Remarks upon the Nine Digits, as they are disposed in the Table.*

I. *Of the Digit* 1.

LIttle can be said concerning this *Digit*, for that it neither Multiplieth nor Divideth, but leaveth the Number which is Multiplied or Divided still the same: But this you may take notice of, That any two Terms, taken equidistant from the Middle Term, do make 10, and the Middle Term (which is 5) doubled does make 10 also.

So the { 1, 2, 3, 4 } and { 9, 8, 6, 5 } are every of them equal to 10.

Again; If you Multiply the last term (9) by the middlemost term (5), that Product shall be equal to the Sum of all the Products added together; for 1, 2, 3, 4, 5, 6, 7, 8, and 9, added together, do make 45.

Also, If you leave out the last term (9), and take the two extreams successively, and add them together, every of their Sums shall be equal to 9.

As { 8, 7, 6, 5 } and { 1, 2, 3, 4 } are all of them equal to 9.

II. *Of the Digit* 2.

In the Addition of the Products of the Multiplication of this *Digit*, you have one of every of the Nine Digits, tho not orderly disposed, yet they stand so, that if you add the two extreme terms equidistant from the middle term together, the Sum of each of them shall be equal to (11).

So { 9, 8, 7, 6 } and { 2, 3, 4, 5 } are each of them equal to 11.

And if you leave out the last term 9, then every two extream terms taken together, shall be equal to (9) the last term.

As { 2, 4, 6, 8 } and { 7, 5, 3, 1 } each equal to 9.

And again, If you add the numbers in the fourth row under this Digit together, you will find the Sum of them to be 45, which multiplied

by the Digit (2) makes 90, equal to the Sum of the Products of all the Multiplications of this Digit, which ſtand in the third *Row* under Digit II.

III. *Of the Digit* 3.

In this *Digit* you may obſerve (as in all the reſt) that the Products of the ſeveral Multiplications do proceed in an *Arithmetical Progreſſion*, the Common Difference being that of the *Digit*: So in this of the *Digit* (3). Wherefore if you add any two terms equidiſtant from the middle term, the Sum of thoſe two terms ſhall be equal, and the double of the middle term equal thereunto alſo: So in this

3		27	
6		24	
9	and	21	are all equal to 30.
12		18	
15 doubled			

In this Digit you may obſerve, that all the Additions of the Products are one of theſe three *Digits*, viz. 3, 6, or 9; and now if you multiply every one of theſe ſeverally by (3) the common difference, the Products of each of them will be 9, 18, 27; which added together do make 54, which is equal to the Sum of the fourth Row under this Digit (3). And in this fourth Row of Additions, if you add the two extream terms, as 3 and 9 together, their Sum will be 12, to which the middle term (6) doubled, is alſo equal.

IV. *Of the Digit* 4.

In the fourth Row under this Digit (4) you have all the 9 Digits produced; and if you omit the laſt term, every two of the reſt added together ſhall be equal to 9; and the Sum of all that Row added together makes 45; which if you multiply by the digit (4), the Product will be (180) equal to the third Row of all the Products added together.

V. *Of the Digit* 5.

This Digit hath all the Nine Digits in the fourth Row of Additions (as the Firſt, Second, and Fourth had), and therefore hath the ſame qualifications; for omitting the laſt term, every two figures taken equidiſtant from the extreams, ſhall be equal to (9), and the Sum of all that Row ſhall be (45); which multiplied by the common difference (5) the Product ſhall be (225) equal to the Sum of all the Products of the Multiplications in the third Row. In which Row alſo, if you add the two extream terms together (5 and 45), the Sum of them is (50), and is equal to the double of the middle term (25) And ſo alſo in the fourth Row, if you add the two extream Digits (9 and 5) together, their Sum (14) ſhall be equal to the double of the middle term (7).

VI. *Of the Digit 6.*

This Digit hath the ſame qualifications as had the Digit (3), for all the fourth Row of Additions are one of theſe Digits 6, 3, or 9: And omitting the laſt term, every two terms taken equidiſtant from the extreams, ſhall be equal to 9, and the ſum of all of them equal to 54, equal to the laſt number of the third row: And in the third row, if you add any two numbers equidiſtant from the two extreams, their ſum will be 60, equal to the double of the middle term (30). And farther; the ſum of the fourth row being (54), that multiplied by (5), a number leſs by one than (6) the common difference, (becauſe it exceeds 5), the Product will be 270, equal to the ſum of all the Products in the third row.

VII. *Of the Digit 7.*

In this Digit (as in all the reſt) the third row, which proceeds by multiplying of the Digit 7 into all the other Digits, deſcends in Arithmetical Progreſſion, the common difference being (7); and ſo if you add the two extream numbers together, as (7) and (63), the ſum of them will be (70), which will be equal to the double of the middle number (35). And the like will be by the addition of any two of them equidiſtant from the two extreams, or from the middle number,

As { 14 and 56 / 21 and 49 / 28 and 42 } are all of them equal to 70.

And ſo is the double of (35 the middle term alſo) equal to 70.

Again; the laſt number in the third being (63) multiply it by (5), a number leſs by 2 than the common difference, and the Product will be (315) equal to the ſum of all the Products in that third row added together.

VIII. *Of the Digit 8.*

In this the third Row increaſes by an Arithmetical Progreſſion, the common difference being (8); ſo that the two extream terms (8 and 72) being added together, do make (80), equal to the double of the middle term (40), and ſo any two that are equidiſtant from the middle term: And for the ſum of them, multiply the laſt term (72) by (5) and the Product will be (360) equal to the ſum of all the Products in the third row: And for the fourth row, they all deſcend till they come to (0) the laſt being (9) as it is in all the other Unites.

IX. *Of the Digit 9.*

The third row under this Digit proceeds in an Arithmetical Progreſſion, ſo that the two extream terms (8 and 81) make (90), equal to the double of (45) the middle term; and ſo do any two of the reſt that are taken equidiſtant from both the extreams, or from the middle

term (45): And for the sum of the numbers in this third row, the last number (81) multiplied by (5) gives (405) for the sum of all that row. As for the numbers (or Sums) in the fourth row, they are all equal one to the other, namely (9), and they being all added together, their sum is equal to the last term in the third row.

CHAP. II.

Of Comparative Arithmetick; *or of the Relation of Numbers in* Quantity.

Comparative Arithmetick is performed by Numbers, as they are considered to have relation one to another in *Quantity* or *Quality*.

1. *Relation in Quantity*] is the reference or respect that the Numbers themselves have one to another: As when the comparison is made between 6 and 2, or 2 and 6; —— 5 and 3, or 3 and 5. And here the numbers propounded are always two, whereof the first is called the *Antecedent*, and the second the *Consequent*; so in the first *Example* 6 is the *Antecedent*, and 2 the *Consequent*; and in the second, 2 is the *Antecedent*, and 6 the *Consequent*.

2. *Relation in Quality*], consists either in the *Difference*, or else in the *Rate* or *Reason* that is found between the Terms propounded.

3. *Difference.*] The *Difference* of two numbers is the remainder which is left after Subtraction of the lesser out of the greater; so 6 and 2 being the Terms propounded, 4 is the *Difference* between them, for 2 subtracted from 6, the remainder is 4.

4. *Rate* or *Reason.*] The *Rate* or *Reason* between two numbers, is the *Quotient* of the Antecedent divided by the Consequent: So if it be demanded what *Rate* or *Reason* 6 hath to 2; the Answer is, *Triple Reason*; for if you divide the Antecedent 6, by the Consequent 2, the Quotient will be 3, 2 being contained in 6 just three times. ——In like manner is there sub-triple Reason between 2 and 6; for if you divide 2 by 6, the Quotient is $\frac{2}{6}$, or $\frac{1}{3}$, because 6 being not found once in 2, there remains 2 for Numerator, 6 the Divisor being the Denominator.

This *Rate* or *Reason* of Numbers is either *Equal*, or *Unequal*.

5. *Equal Reason*] Is the relation that equal numbers have one to another, as 5 to 5, ——6 to 6, ——7 to 7, *&c.* For here, one being divided by the other, the Quotient is always an Unite; for if it be demanded, how often 5 may be had in 5, the answer will be, 1, or Unity.

6. *Unequal Reason*] is the relation that unequal numbers have one unto another; and this is either of the *Greater* to the *Lesser*, or of the *Less* to the *Greater*. ——*Unequal Reason of the Greater to the Lesser*], is, when the greater term is Antecedent, as of 6 to 2, or 9 to 7, and the like; for here the Quotient of the Antecedent divided by the Consequent, is always greater than Unity; so 6 divided by 2, the Quotient is 3, and 9 divided by 7, is $1\frac{2}{7}$. ——But *Unequal Reason of the Lesser to the Greater*], is, when the lesser term is Antecedent, as of 2 to 6, or 7 to 9, *&c.* —— And here the Quotient of the Antecedent divided by the

the Conſequent, is always leſs than Unity; ſo 2 divided by 6, the Quotient is $\frac{2}{6}$, or $\frac{1}{3}$, and 7 divided by 9, is $\frac{7}{9}$

Each of theſe kinds of *Unequal Reaſon* is again ſubdivided into Five other kinds or varieties; whereof the three firſt are *Simple*, and the other two are *Mixt*. ——The Simple kinds of *Unequal Reaſon* are, (1.) *Manifold*. (2.) *Superparticular*. (3.) *Superpartient*.

7. *Manifold Reaſon of the Greater to the Leſs*], is when the Conſequent is contained in the Antecedent divers times, without any part remaining; as 4 to 2, 8 to 4, 16 to 8; which is called Double *Reaſon*, becauſe the Leſs is contained twice in the Greater; ſo 6 to 2 is *Triple Reaſon*, and 8 to 2 *Fourfold Reaſon*, *&c.* And here, the Quotient of the Antecedent divided by the Conſequent, is always a whole number; as 8 divided by 2, the Quotient is 4, a whole number. ——The oppoſite of this kind, *viz.* Of the *Leſs* to the *Greater*, is called *Sub-manifold*: Examples hereof are, 2 to 4, 4 to 8, 8 to 16, *&c.* Alſo 2 to 6, 2 to 8, 2 to 10, *&c.*

8. *Superparticular Reaſon of the Greater to the Leſſer*], is, when the Antecedent contains the Conſequent once; and beſides, an Aliquot part of the Conſequent; that is, one half, one third, one fourth, or one fifth, *&c.* of the Conſequent; as 3 to 2, 4 to 3, 5 to 4, 6 to 5, and the like: So here, 3 divided by 2, the Quotient is $1\frac{1}{2}$; and 4 divided by 3, the Quotient is $1\frac{1}{3}$; 5 by 4, the Quotient is $1\frac{1}{4}$; wherefore I ſay, 2 and half 2 (which is 1) do conſtitute 3; ſo 3, and $\frac{1}{3}$ of 3 do conſtitute 4; and ſo of the reſt. —— For here, the Quotient of the Antecedent divided by the Conſequent, is a Mixt Number, whoſe whole part, and the Numerator of the Fraction, is always an Unite. ——The oppoſite Reaſon of this kind is *Subſuperparticular*, as 2 to 3, 3 to 4, 4 to 5, 5 to 6, *&c.*

9. *Superpartient of the Greater to the Leſs*], is, when the Antecedent contains the Conſequent once, and divers parts of the Conſequent beſides; as 5 to 3, 7 to 5, 7 to 4, 8 to 5, 9 to 5, 11 to 7, *&c.* Here 5 divided by 3, the Quotient is $1\frac{2}{3}$, and therefore 5 contains 3 once, and $\frac{2}{3}$ of 3, which is 2, and they two together do conſtitute 5. ——Here the Quotient of the Antecedent divided by the Conſequent, is a *Mixt Number*, whoſe whole part being a Unite, hath always for the Numerator of the Fraction a Number compoſed of more Unites than one; ſo the conference being made between 5 and 3, and 5 the Antecedent being divided by 3 the Conſequent, the Quotient is $1\frac{2}{3}$. ——The oppoſite of this Reaſon is *Subſuperpartient*; Examples hereof are, 3 to 5, 5 to 7, 4 to 7, 5 to 8, 5 to 9, 7 to 11, *&c.*

The Mixt kinds of *Unequal Reaſon* are two, *viz.* (1.) *Manifold Superparticular*. And (2.) *Manifold Superpartient*.

10. *Manifold Superparticular Reaſon*], is, when the Antecedent contains the Conſequent divers times, and an *Aliquot part* of the Conſequent beſides; as 5 to 2, 10 to 3, 17 to 4, 21 to 5, and the like. ——Here the Quotient of the Antecedent divided by the Conſequent, is a Mixt Number, whoſe whole part conſiſting of more Unites than one, hath always an Unite for the Numerator of the Fraction annexed unto it; ſo 5 divided by 2, hath for the Quotient $2\frac{1}{2}$, and 21 divided by 5, the Quotient

ent is $4\frac{1}{3}$. —And the oppoſite of this Reaſon is *Submanifold Superparticular*; as 2 to 5, 2 to 7, 3 to 7, 4 to 9, *&c.*

11. *Manifold Superpartient Reaſon*], is, when the Antecedent contains the Conſequent divers times; and divers parts of the Conſequent beſides; as 8 to 3, 17 to 5, 19 to 4, 28 to 5, *&c.* —Here the Quotient of the Antecedent divided by the Conſequent, is a Mixt Number, whoſe whole part, as alſo the Numerator of the Fraction annexed unto it, is always a number compoſed of more Unites than one: —So 8 divided by 3, the quotient is $2\frac{2}{3}$, and 28 by 5, the quotient is $5\frac{3}{5}$, *&c.* —And the oppoſite hereof is *Submanifold Superpartition*; as 3 to 8, 5 to 17, 4 to 19, 5 to 28, and the like.

Theſe are the ſeveral *Varieties* of the *Rates* or *Reaſons* that are found among *Numbers*; ſo that no two *Numbers* can be named, but the *Rate* or *Reaſon* between them is comprehended under one of theſe laſt Five Heads.

CHAP. III.

Of Arithmetical, Geometrical, *and* Muſical *Proportions.*

I. *Of Arithmetical Proportion.*

Arithmetical Proportion (or Habitude) is an equality of *Differences*: That is to ſay, when ſeveral Numbers have one and the ſame *Difference*: And this Habitude is twofold, *viz.* (1.) *Continued.* (2.) *Diſcontinued.*

1. *Arithmetical Proportion Continued*], is, when of ſeveral Numbers, the Second exceedeth the Firſt, by the ſame number of Unites, as the Third exceeds the Second, and as the Fourth doth the Third; and ſo *in infinitum.*

As thus,

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, *&c.* do differ from each other by (1), or One Unite. And 1, 3, 5, 7, 9, *&c.* differ by (2) or Two Unites. Alſo 1, 6, 11, 16, 21, 26, *&c.* differ by (5) or Five Unites. And this Orderly proceeding of Numbers from the leſſer to the greater, is that which we properly call *Arithmetical Progreſſion.*

2. *Arithmetical Proportion Diſcontinued*], is, when the Second Number exceeds the Firſt, by the ſame number of Unites as the Fourth doth the Third; but not as the Third doth the Second.

As for Example;

1, 3, 7, 9, are four diſtinct *Arithmetical Proportionals Diſcontinued.* —For 3 exceeds 1 by the ſame number of Unites (*viz.* 2.) as 9 exceeds 7, but not as 7 doth 3.—So again, 2, 7, 10, 15, are four *Arithmetical Proportionals Diſcontinued*; for 7 exceedeth 2 by (5), and ſo doth 15 exceed 10; but 10 doth not exceed 7 by (5), but by (3). And this is *Arithmetical Proportion Diſcontinued.*

II. *Of*

II. *Of Geometrical Proportion.*

Geometrical Proportion, or Habitude,] is the equality of *Ratio*'s; or it is that which shews what part or parts one Number is of another; —— Or more plainly thus; It is an Increase by a *Common Multiplication.*

So these Numbers, 1, 2, 4, 8, 16, 32, 64, *&c.* are Numbers in *Geometrical Proportion.* where the *Common Multiplier* is [2], that is, the first multiplied by 2, produceth the second; the second multiplied by 2, produceth the third, and so on *in infinitum.* ——— Or in these Numbers, 3, 9, 27, 81, 243, *&c.* where the *Common Multiplier* is [3.] And this is called *Geometrical Progression.*

Of both these kinds of Progression I have inserted Tables.

Arithmetical Progression, the Common Difference being

One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Ten
1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	96	108	120
13	26	39	52	65	78	91	104	117	130
14	28	42	56	70	84	98	112	126	140
15	30	45	60	75	90	105	120	135	150
16	32	48	64	80	96	112	128	144	160
17	34	51	68	85	102	119	136	153	170
18	36	54	72	90	108	126	144	162	180
19	38	57	76	95	114	133	152	171	190
20	40	60	80	100	120	140	160	180	200
21	42	63	84	105	126	147	168	189	210
22	44	66	88	110	132	154	176	198	220
23	46	69	92	115	138	161	184	207	230
24	48	72	96	120	144	168	192	216	240
25	50	75	100	125	150	175	200	225	250

Geo-

Geometrical Progression.

1	1	1
2	3	4
4	9	16
8	27	64
16	81	256
32	243	1,024
64	729	4,096
128	2,187	16,384
256	6,561	65,536
512	19,683	262,144

1	1	1
5	6	7
25	36	49
125	216	343
625	1,296	2,401
3,125	7,756	16,807
15,625	46,536	117,649
78,125	279,216	823,543
390,625	1,675,296	5,764,801
1,953,125	10,651,776	40,353,607

1	1	1
8	9	10
64	81	100
512	729	1,000
4,096	6,561	10,000
32,768	59,049	100,000
262,144	531,441	1,000,000
2,097,152	4,782,969	10,000,000
16,777,216	43,046,721	100,000,000
134,217,728	387,420,489	1,000,000,000

Arithmetical Progression.	Geometrical Progression.
1	10
2	100
3	1,000
4	10,000
5	100,000
6	1,000,000
7	10,000,000
8	100,000,000
9	1,000,000,000
10	10,000,000,000
11	100,000,000,000
12	1,000,000,000,000
13	10,000,000,000,000
14	100,000,000,000,000
15	1,000,000,000,000,000
16	10,000,000,000,000,000
17	100,000,000,000,000,000
18	1,000,000,000,000,000,000
19	10,000,000,000,000,000,000
20	100,000,000,000,000,000,000
21	1,000,000,000,000,000,000,000
22	10,000,000,000,000,000,000,000
23	100,000,000,000,000,000,000,000
24	1,000,000,000,000,000,000,000,000
25	10,000,000,000,000,000,000,000,000

In

In this *Progreſſion* it is viſible, how *Addition* and *Subtraction* in *Arithmetical Progreſſion*, anſwers to *Multiplication* and *Diviſion* in *Geometrical Progreſſion*. —— For, As in *Geometrical Progreſſion*, 1000 multiplied by 100,000, produces 100,000,000: So in *Arithmetical Progreſſion*, the numbers anſwering to 1000, and 100,000, are (3) and (5,) which being added together make (8,) which anſwers to this Product 100,000,000. ——And again, As in *Geometrical Progreſſion* (100,000,000) being *divided* by (100,000) the *Quotient* is (1000.) So in *Arithmetical Progreſſion*, If from the correſpondent number, belonging to the former *Product* 100,000,000, (*viz.* 8,) you ſubtract any of the other correſpondent numbers, *viz.* (5,) the remainder will be the other correſpondent number (*viz.* 3,) which anſwers to 1000.

III. *Of Muſical Proportion.*

Muſical Proportion or Habitude,] is, when the firſt number hath the ſame proportion to the third, which the difference between the firſt and ſecond hath to the difference between the ſecond and the third.

As in theſe numbers [3, 4, 6,] whereas 3 is the half of 6: So 1, which is the difference between 3 and 4 the half of 2.

CHAP. IV.

Numerical Theorems.

I. *If* Numbers *(how many ſoever) exceed one another by an equal* Interval; *the* Interval *between the* Greateſt *and the* Leaſt, *is* Multiplex *of that equal* Interval: *according to the multitude of the Numbers propounded, leſs by* One.

LET the numbers propoſed be theſe four 1, 3, 5, 7, whoſe common *Interval* is 2. Then (by the *Hypotheſis*) 6 is the *Interval* between the *Greateſt* 7, and the *Leaſt* 1. ——And likewiſe the three numbers 2, 2, 2, are every of them equal to the *common difference*, and equal one to the other. And the multitude of them (*viz.* three) equal to the multitude of the number given, (*viz.* four) leſs by *One*. —And laſtly, The *Aggregate* of theſe three numbers (*viz.* ſix) is equal to the Interval between 7 the *Greateſt*, and 1 the *Leaſt*, viz. 6.

II. *If* Numbers (*how many ſoever*) *contain the one the other by an equal* Ratio, *then the* Greateſt *of thoſe* Numbers, *is* Multiplex *of the* Powers *of the Denomination of that equal* Ratio *multiplied by the* Leaſt; *according to the Multitude of the Numbers* given, leſs by One.

LET the *Numbers* given be theſe four, *viz.* 2, 6, 18, 54. And let the Denomination of the *Ratio* be 3. Then (by the *Hypotheſis*) the firſt multiplied by 3 (the equal *Ratio*) is equal to the ſecond (*viz.* 6,) and the ſecond multiplied by (3) is equal to the third (*viz.* 18) and the third multiplied by (3) is equal to the fourth (*viz.* 54.) *Et ſic*, &c.

The	First	2	is equal to	2.
	Second	6		2 into 3. (The first Power of the Ratio, or the single Ratio.
	Third	18		2 into 3, into 3. (The second Power of the Ratio, or the Ratio squared.
	Fourth	54		2 into 3, into 3, into 3 (The third Power of the Ratio, or the Ratio Cubed.

Or, let the four Numbers be 1, 3, 5, 7, and the common difference 2. Then,

The	First	1	is equal to	1
	Second	3		1 more by 2.
	Third	5		1 more by 2, more by 2.
	Fourth	7		1 more by 2, more by 2, more by 2.

Here the Greateſt Term (7) is equal to the Leaſt (1,) and as many *differences* as there are more *Terms* beſides the *Leaſt* (*viz.* ſix.) And therefore the *Greateſt* Term 7, leſs by the Leaſt 1, (*viz.* 6) is *Multiplex* of the Difference, according to the number of Terms leſs by One.

III. *If there be three* Numbers *in* Arithmetical Proportion; *the Sum of the two* Extreams *is equal to the double of the* Means.

LET the three *Numbers* be 2, 4, 6, whoſe common difference is 2. Then,

The firſt Term 2, is only 2.
The ſecond Term is 2, more by the difference once (*viz.* 4.)
The third Term is 2, more by the difference twice (*viz* 6.)

And ſo it is evident, that twice (2) the firſt Term, more by twice the difference (4) is equal both to the Sum of the firſt and third Terms; and alſo to the double of the mean term (4,) for 2 and 6 is 8; and ſo alſo is the double of 4.

IV. *If three* Numbers *be in* Proportion, *the Number contained under the* Extreams, *is equal to the* Square *made of the* Mean: *And if the* Number *contained under the* Extreams, *be equal to the* Square *of the* Mean, *thoſe three* Numbers *ſhall be in proportion.*

LET the three Numbers be (2, 6, 18.)
And let the common Ratio be (3.)
The firſt Term is (2.)
The ſecond Term is 2 into 3 (*viz.* 6.)
The third Term is 2 into the Square of 3 (*viz.* 9) equal to 18.

And from hence it is evident, That the firſt Term drawn into the third (*viz.* 2 into 18,) is equal to 2 into 2 (*viz.* 4,) and that into the Square of 3 the common Ratio (*viz.* 9) For 2 into 18 is equal to (36.) And ſo is 4 the ſquare of 2, into 9 the ſquare of 3 equal to 36. Therefore, the Product of the firſt multiplied by the third, is equal to the ſquare of the Mean.

V. *If*

V. *If there be four* Numbers *in* Arithmetical Proportion; *the Sum of the first and fourth, is equal to the Sum of the second and third.*

LET the four Numbers be in continual Proportion, as these 4, 12, 20, 28, whose common difference is (8.)

The First Term is 4.
The Second Term is 4, and once the Difference 8, (*viz.* 12.)
The Third Term is 4, and twice the difference 16, (*viz.* 20.)
The Fourth Term is 4, and thrice the Difference 24, (*viz.* 28.)

Again, Let the four Numbers be four discontinued Proportionals; as these, 4, 12, 30, 38. And their common difference (8.) Then

The First Term is 4.
The Second Term is 4, and once the Difference 8, (*viz.* 12.)
The Third Term is 30.
The Fourth Term is 30, and once the Difference 8, (*viz.* 38.)

In the four *Continual Proportions* 4, 12, 20, 28 it is evident,
That twice the first term 4 (*viz.* 8.) more by thrice the difference 8 (*viz.* 24,) is equal to (32,) which is both the *Sum* of the *Extreams*, (4 and 28,) and of the *Means* (12 and 20.) Also,
In the four *discontinued Proportionals* 4, 12, 30, 38, it is also as evident,
That the first term 4, more by the third term 30, more by once the *difference* 8, is equal to 42, which is equal to the *Sum* of the *Extreams* (4 and 38,) and also of the *Means* (12 and 32.)

VI. *If there be four* Numbers *in* Proportion, *the Number produced of the* first *and* fourth, *shall be equal to the* Number *produced of the* second *and* third. *And if the* Number *produced of the* first *and* fourth, *be equal to that produced of the* second *and* third; *These* Four Numbers *shall be* Proportional.

LET the four *Terms* be four *Continual Proportionals*, as these (2, 6, 18, 54.) And let the common *Ratio* be (3.) Then,

The First Term is 2.
The Second Term is 2 into 3, (*viz.* 6.)
The Third Term is 2 into the square of 3 (*viz.* 9.)
The Fourth Term is 2 into the Cube of 3 (*viz.* 27.)

Again, Let the four Numbers be four *Discontinued Proportionals*, as these, 2, 6, 54, 162, and the equal Ratio (3.)

The First Term is 2.
The Second Term is 2 into the common Ratio 3 (*viz.* 6.)
The Third Term is 54.
The Fourth Term is 54 into the common Ratio 3 (*viz.* 162.)

In the Continual Proportionals it is evident by the Multiplication of Powers:

For the ſquare of the firſt term 2 (*viz.* 4,) multiplied by the Cube of (3) the common *Ratio* (*viz.* 27,) the Product will be 108, which is equal to 2 the firſt term, multiplied by 54 the fourth term, and alſo of 18 by 6, the two mean terms multiplied into each other.

In the four *Diſcontinued Proportionals* 2, 6, 54, 162, it is alſo as evident.

For, 2 (the firſt term) multiplied into 3 (the common *Ratio*) produceth (6,) and that into the third term 54, produceth 324; which is equal, both to the Product of 2 into 162 (the two *Extreams*,) and alſo of 6 into 54, the two *Means*.

VII. *In all* Continual Arithmetical Progreſſions, *the Sum of the* Extreams *is equal to the Sum of any two of the other* Means, *taken equidiſtant from the* Extreams, *and to the* double *of the* middle Term; *when the Number of Places be* Odd.

LET there be theſe ſeven Numbers in *Continual Arithmetical Progreſſion*, *viz.* 3, 6, 9, 12, 15, 18, 21, whoſe common difference is 3.

Firſt, Theſe four *Numbers* 3, 6, 18, 21, are four *Proportionals*: Therefore (by the Vth hereof) *The Sum of their* Means, *and the Sum of their* Extreams *are* equal.

For the Sum of { 3 and 21, the two *Extreams* / 6 and 18, the two *Means*, } are equal to 24.

Secondly, Theſe four Numbers 6, 9, 15, 18, are four *Proportionals*; and therefore, *The Sum of their* Extreams *and* Means *are equal.*

For the Sum of { 6 and 18, the two *Extreams*, / 9 and 15, the two *Means*, } is equal to 24.

Thirdly, Theſe three Numbers 9, 12, 15, are three *Proportionals*; and therefore (by the ſame Vth hereof) *The Sum of the* Extreams *is equal to the* double *of the* Mean.

For the Sum of { 9 and 15, the two Extreams, / 12 and 12, the Mean doubled, } are equal to 24.

And ſo { 3 and 21 / 6 and 18 / 9 and 15 / 12 doubled } are all equal one to another, *viz.* 24.

VIII. *In*

VIII. *In* Geometrical Proportionals Continual (*how many ſoever the* Places *be*) *the* Product *of the* Extreams *is equal to the* Product *of any two of the* Mean Terms, *equidiſtant from the* Extreams, *and of the* Square *of the* Middle Term; *when the Number of Places be* Odd.

LET the Number of Terms be ſeven, *viz.* 2, 6, 18, 54, 162, 486, 1458, and their common *Ratio* 3.

Firſt, Theſe four Numbers 2, 6, 486, 1458, are four *Proportionals*; and therefore the *Product* of the two *Extreams* ſhall be equal to the *Product* of the two *Means*: So

The Product of { 2 into 1458, the *Extreams*, / 6 into 486, the *Means*, } is equal to 2916.

Secondly, Theſe four Numbers 6, 18, 162, 486, are *Proportionals*: And

The Product of { 6 into 486, the *Extreams*, / 18 into 162, the *Means*, } is equal to 2916.

Thirdly, Theſe three Numbers 18, 54, 162, are three *Proportionals*; and therefore the *Product* of the two *Extreams*, is equal to the *Square* of the *Mean*. For

The Product of { 18 into 162, the two *Extreams*, / 54 into 54, the *Mean* into it ſelf, } is equal to 2916.

And ſo { 2, 6, 18, 54 } into Square { 1458, 486, 162 } are all equal one to the other, and each of them equal to 2916.

CHAP. V.

Concerning the Rule of Three, *or* Golden Rule, *both* Single *and* Compound.

THIS Rule may (and moſt properly) be called the *Rule* of *Proportion*; for that it teacheth (in any caſe) by having *Three Numbers* given, to find a *Fourth*, which ſhall be in Proportion to them: And for the Reaſon of the manner of Working this *Rule*, the *Theorems* delivered in the foregoing *Chapter* will be Subſervient.

Now, this Rule is either *Single* or *Compound*.

I. *Of*

I. *Of the Single Rule of Three.*

The *Single Rule of Three*, is, When *Three Numbers* are *Given*, and a *Fourth* Proportional *Number required*. And this Rule is either *Direct*, or *Inverse*, or *Reciprocal*.

I. *Of Direct.*

The *Single Rule of Three Direct*, is, When *Three* Numbers are given, and a *Fourth* is demanded, which bears the same Proportion to the *Third*, as the *Second* bears to the *First*. —As in this following *Example*:

If 4 *Acres of Ground cost* 80 *l. What will* 8 *Acres of the like Ground cost?*

For the better understanding of this Operation, look back to the VI^th^ *Section* of the foregoing *Chapter*, where you shall find it demonstrated: That, *If there be Four Proportionals, the Product of the First and Fourth, is equal to the Product of the Second and Third.*

Wherefore, In this Example,

If the Product of 8 in 80 (the second and third Terms) produce 640; If that 640 be divided by 4, the first term, the Quotient will be 160; and is the fourth Proportional sought.

And for Proof of this,

Let the fourth term found (*viz* 160,) be multiplied by the first term (4,) the Product will be the same with the Product of the second term (80,) multiplied by the third term (8,) namely (640.)

And from hence we may argue thus,

If the Product of (80 by 8) be equal to the Product of (4 by the unknown number)

Then,

The Quotient will be the very same, whether I divide the Product of (80 by 8,) or whether I divide the Product of (4 by the unknown number) by 4. For either of the Products being 640, the Quotient must needs be 160.

And now it is manifest,

That if I multiply 4 by a 160, and divide that Product back again by 4, it will give 160 for the Quotient: Because, Whatsoever *Multiplication* doth, is undone again by *Division*. And this is the true and genuine *Reason* of the *Operation* of the *Rule of Three*.

And that is to say,

As (4) is found in (80) just 20 times; so (8) is found in (160) just 20 times.

Or thus,

As 4 multiplied by 20 makes (80,) so (8) multiplied by (20) makes (160.)

Another Example.

If 80 *l. will buy* 4 *Acres of Land, How many Acres will* 160 *l. buy?*

If you divide the Product of 4 into 160 (*viz.* 640) by (80,) the Quotient will be (8.) For, as (80) contains (4) 20 times, so (160) being divided by (20,) the Quotient will be (8) also.

2. *Inverse,*

2. *Inverse, (or Reciprocal.)*

The Single Rule of Three Inverse, is, When there are *Three Numbers* given, and a *Fourth* required, which shall bear the same *Proportion* to the *Second*, as the *Third* doth to the *First*.

As in this Example.

If a certain quantity of Pecks of Oats will keep 8 Horses for 12 days; How many days will the same quantity of Pecks keep 16 Horses?

Caution.

The most Authors that have writ of Arithmetick, *have made two distinct Rules of this Golden Rule, calling the one* Direct, *and the other* Inverse; *whereas (in truth) they are but One: Only care must be taken how to place the Terms given.*

So in this *Example*, Look what Proportion 16 Horses bear to 8 Horses; the like Proportion do 12 days bear to a fourth number of days: And the Terms ought to be thus placed.

As 16 Horses, to 8 Horses; so 12 days to 6 days. And then the Operation is the very same as before: For the Product of 8 by 12 is (96,) which being divided by 16, the Quotient is (6) days.

II. *Of the Double, or Compound Rule of Three.*

THE *Double*, or *Compound Rule of Three*, is, When more Numbers than *Three* are given: And it is either *Direct* or *Inverse*.

I. *Direct.*

Example 1. *If 4 Men spend 19 Pound in 3 Months; How many Pounds will 8 Men spend in 9 Months?*

The Solution is thus performed.

(1.) If 4 Men spend 19 Pound, what will 8 Men spend?

Set the Numbers thus.

Men	Pounds	Men	Pounds
4	19	8	38

For 19 in 8 (the two *Mean Terms*) produce 152, which divided by 4 (the given *Extream*) the *Quotient* is 38 (the other *Extream*.)

(2.) *If 38 Pound be spent by any number of Men in 3 Months; How many Pounds will be spent by the same number of Men in 9 Months?*

Set the Numbers thus:

Months	Pounds	Months	Pounds
3	38	9	114

For,

For, 38 in 9 (the two *Mean Terms*) produce 342; which divided by 3 (the given *Extream*) the *Quotient* will be 114 (for the other *Extream.*) And ſo many *Pounds* will 8 *Men* ſpend in 9 *Months.*

II. *Inverſe.*

Example 2. *If 9 Buſhels of Oats will ſerve 8 Horſes for 12 days; How many days will 24 Buſhels ſerve 16 Horſes?*

The Solution is thus performed.

1. As 9 Buſhels is to 12 days, ſo is 24 Buſhels to 32 days.

Set the Number thus.

Buſhels	Days	Buſhels	Days
9	12	24	32

For, 12 in 24 (the two *Means,*) produce 288, which divided by 9 (the given *Extream,*) the *Quotient* is 32 (for the other *Extream.*)

2. As 16 *Horſes* is to 8 *Horſes*; ſo is 32 *Days* to 16 *Days.*

Set the Numbers thus.

Horſes	Horſes	Days	Days.
16	8	32	16

For 8 in 32 (the two *Means*) produce 256, which divided by 16 (the given *Extream,*) the *Quotient* will be 16 for the other *Extream.*

And let this ſuffice for the *Reaſon* of the *Operation* of the *Golden Rule.*

CHAP. VI.

Of the Increaſe of Swine, Corn, Sheep, *&c.*

I. *Of Swine.*

Unto what Sum of Money may all the Pigs that One Sow, with all the Pigs of her Race, and the Increaſe iſſuing of them, in Twelve Years time, amount unto?

SUppoſe the Sow brings forth but ſix Pigs at a Litter, of which we will allow two to be Barrow (which is as little as can be ſuppoſed.) And then imagine that every of thoſe four, bring as many every Year; and the increaſe of them the like, during the Term of twelve Years.

This

This is performed by *Geometrical Progression*, where the common *Ratio* is 4, and the Sum of all the Numbers is 22369620, i. e. *Twenty two Millions, 369 thousand, Six hundred and twenty Pigs.*

Now for the maintenance of these *Pigs:* Suppose *Five shillings* be allowed for to maintain one of these *Pigs* for a *Year*, (which is little enough, for it is not half a Farthing a day,) yet there must be 22369620 *Crowns* to maintain them all for one Year, which number being divided by 4, the Quotient will be 5592405, which is *Five Millions, Five hundred ninety two Thousand, Four hundred and five Pounds* Sterling for one Year.

The Increase.	Year.
4	I
16	II
64	III
256	IV
1024	V
4096	VI
16384	VII
65536	VIII
262144	IX
1048576	X
4194304	XI
16777216	XII
22369620	

II. *Of Corn.*

What quantity of Ground *will be capable of receiving one Grain of Corn and the increase of it for eight Years*; and to what number of Grains will the increase of that one Grain arise unto.

FOR the first Year, let there be allowed *one quarter of an Inch of Ground* to sew the *one Grain* in (which is, indeed, too little) and this *Corn* we will suppose had in the ear, at the Years end, 40 *Corns* (for that is but reasonable) then the second Year we must allow 40 square quarters of Inches of *Ground* to sew those 40 *Corns* in, that is, 10 square inches of Ground. And the third Year, supposing those 40 Corns to produce 40 *Ears*, and in each *Ear* 40 *Corns*, as before, they will be in the third Year increased to 1600 *Corns*; so that there must be 1600 *square quarters of Inches of Ground* to sow that Increase in, which is 40 *square Inches*, and thus continuing till the eight Years be expired, the *Increase* would be at the eight Years end 163840000000 Corns, as in the *Table*, in the Margin (where the common *Ratio* is 40). And the Sum of all the *Corns* in the *Table* do amount unto 168041025641 *Corns*, and consequently so many *square quarters of Inches of Land* must be allowed for the *sowing* of those *Corns* in; one fourth part whereof is 42010256410 *square Inches* of Land to sow this increase in, that is, *Forty two Millions of Millions, ten Millions, Two hundred*

Increase in Corns, and Ground.	Year.
1	I
40	II
1600	III
64000	IV
2560000	V
102400000	VI
4096000000	VII
163840000000	VIII
Corns 168041025641	
Inches of Land 42010256410	

dred sixty four Thousand, Four hundred and ten Inches. Now note, That in one Acre of Land, there is contained 6272640 square Inches; wherefore divide the 420102564 10 Inches, by 6272640 (the *Inches* in one *Acre*) and the *Quotient* will be 6697 *Acres*; and so much Land would be required to contain the sowing of the increase of one *Corn* in *eight Years*; which if Land were rated but at *Five Shillings* the *Acre* for a *Year*, it would amount to 1674 Pounds Five Shillings a year *Rent* for the Ground to Sow this one Corns Increase in, in eight years time.

Upon such a Condition, a *Country Fellow* agrees with a *Rich Farmer* to serve him for eight Years, he requiring no other *Wages*, than *One Corn*, and *One quarter of an Inch of Land* to Sow it in, for the first Year, and so on for the whole eight Years: Now I do question at the eight Years end which was the *Richest Man*, the *Farmer* or his *Servant*.

III. *Of Barley.*

Unto what Quantity of Barley *will one Corn arise unto, that one Corn being doubled* (*according to* Geometrical Progression) 72 *times: Also the* Worth *of so much* Barley *in Money: The* Weight *of so much: How it should be removed: And where stowed.*

I. *For the Quantity of it.*

IF we allow *Ten thousand* Corns to a *Pint* (which is more than enough) then 5120000, that is, *Five millions, one hundred and twenty thousand* Corns will make a *Quarter*; notwithstanding (for the ease of such as will make trial) we will allow 10000000, *Ten millions of Corns* to make a *Quarter*; by which number, if you divide the whole number of Corns, that will arise in the 72 place, being doubled (because we make a Quarter to contain 10000000 *Corns*; whereas it does contain little more than half so many (*viz.* 5120000 *Corns*) and that may be done by cutting off the seven last figures towards the right hand: Then the Quotient of that division will be found to be 472236648286964: And so many whole *Quarters* of *Barley* would the number of Corns in the 72 place have amounted to, besides some odd *Bushels* which we omit.

II. *For the* Worth *of so much Barley.*

Suppose it were rated at 15 Pence the Bushel (which is a very low rate) that is 10 Shillings the *Quarter*, and then it would amount unto 236118324143483 Pounds Sterling; which Sum rendred in words is, *Two hundred thirty six millions of millions; One hundred and eighteen, three hundred twenty four millions; One hundred forty three thousand; Four hundred eighty three.*——And thus, If you reckon *Land* for ever worth twenty Years Purchase: If you divide this Sum of Pounds by 20, the Quotient is 11805916207174, that is, *Eleven millions of millions, eight hundred and five thousand, nine hundred and sixteen millions, two hundred and seven thousand, one hundred seventy four Pounds* a year. And again, If you divide this number of Pounds by 365 (the *days* in one *year*) the Quotient

Quotient will be 3234497591o, that is, *Thirty two thouſand three hundred forty four Millions, nine hundred ſeventy four Thouſand, nine hundred and ten Pounds a day* for ever.

III. *For the* Weight *of this Barley.*

If we allow eight *Buſhels* (or one *Quarter*) to weigh two hundred Weight (but doubtleſs it weighs more) then the whole number of *Quarters*, viz. 4722366482869̇64, multiplied by 2, gives the *Weight* of all the *Barley* to be 9444732965739̇28 *hundred Weight*. And if you divide this number by 20 (the number of *Hundreds* in one *Tun*) the Quotient will be 472236648286̇96 Tuns, that is, *Forty ſeven Millions of Millions, two hundred twenty three thouſand ſix hundred ſixty four Millions, eight hundred twenty eight Thouſand, ſix hundred ninety ſix Tuns.*

IV. *For the Stowage of this Barley.*

If we ſuppoſe one Ship to contain a *Thouſand Tun*; then muſt there be 4722366482̇8, that is, *Forty ſeven thouſand two hundred twenty three Millions, ſix hundred ſixty four thouſand, eight hundred twenty eight*, ſuch Ships to carry it: And, if there were *four Millions* of *Nations* in the *World*, and every one of thoſe *Nations* had *Ten thouſand Sail* of Ships of a *Thouſand Tun* a-piece, yet all thoſe *Ships* would not contain it. By what is here delivered, you may ſee how prodigiouſly *Numbers* do increaſe by being multiplied according to *Geometrical Progreſſion.*

IV. *Of other Grain.*

Muſtard-ſeed is a *Grain* of very great Production; for would one imagine that *one Grain* of *Muſtard-ſeed*, and its Increaſe for 20 Years, cannot be contained within the *Viſible World*; nay, if it were 100 times greater than it is, it holding nothing beſides, from the Center of the *Earth* to the *Firmament*: For proof of which by Art, Let us ſuppoſe, that one Grain of Muſtard-ſeed ſown, to bring forth a *Tree* or *Branch*, in each extendure of which might be a Thouſand Grains; but we will ſuppoſe only One thouſand in the whole Tree: And then proceeding to a *Progreſſion* of 20 Years, where the common *Ratio* muſt be 1000, in leſs than 17 Years you ſhall have ſo many *Grains* which will ſurpaſs the *Sands* which are able to fill the whole *Firmament*: For, according to the Suppoſition of *Archimedes*, and of *Tycho-Brahe*, of the Greatneſs of the *Firmament*, the number of Grains of Sand to fill that capacity, will be ſufficiently expreſſed by an Unite and 49 Cyphers: But the number of *Grains* of *Muſtard-ſeed*, at the end of 17 years, will be an Unite and 52 Cyphers; and beſides, Grains of *Muſtard-ſeed* (though very ſmall) are far greater than grains of *Sand*: It is therefore evident, That at the 17th years end, that which ſhall ſucceſſively ſpring from one *Grain* only, cannot be contained within the limits of the whole *Firmament*; what then would it be, if multiplied again by 1000 for the 18th Year, and that again by 1000 for every years increaſe till you come to the 20th year? It is very evident, that ſuch a heap of Muſtard-ſeed would be One hundred thouſand times greater than the Earth.

V. *Of Sheep.*

Unto what Sum of Money will 100 *Sheep and the Increaſe of them, being preſerved for the ſpace of* 16 *Years, amount unto?*

SUppoſe a Man at the beginning of the Year were poſſeſſed of 100 *Sheep*, and that every one of them ſhould produce but one *Lamb* every year, unto what will theſe 100 with their increaſe amount to at 16 years end?

Years.	Increaſe.
I	200
II	400
III	800
IV	1600
V	3200
VI	6400
VII	12800
VIII	25600
IX	51200
X	102400
XI	204800
XII	409600
XIII	819200
XIV	1638400
XV	3276800
XVI	6553600

This is eaſily reſolved by a *Geometrical Progreſſion*, as in the Margin; wherein you may ſee, that at the expiration of the 16th year the *Sheep* will be increaſed to 6553600, which is, *Six Millions, five hundred fifty three Thouſand, ſix hundred Sheep:* Which if reckoned but at *five Shillings* a-piece, it will amount unto 1638400, that is, *One Million, ſix hundred thirty eight Thouſand, ſix hundred* Pound.

VI. *Of Men.*

IT may ſeem hard to conceive, that from eight Perſons which were ſaved after the *Deluge*, or *Noah's Flood*, ſhould ſpring ſuch a World of People to begin a *Monarchy* under *Nimrod*, being but 200 years after the *Flood*; and that amongſt them ſhould be raiſed an *Army* conſiſting of *Two hundred thouſand* Fighting Men: But it is eaſily proved: For, if we take but one of the Children of *Noah*, and ſuppoſe that a New Generation of People begin at every 30 years, and that it be continued to the ſeventh Generation, which is 210 years; for then, of one only Family there would be produced 111305, that is, *One hundred and eleven thouſand, three hundred and five* Souls to begin the World; although in that time Men lived longer, and were more capable of Generation, Multiplication and Increaſe. Now ſuch a Number ariſing only from a ſimple Production of only *One* yearly, would be far Greater, if one Man ſhould have many Wives (as in ancient times they had.) And from hence it is alſo, that the Children of *Iſrael*, who came into *Egypt* only *Seventy* Souls, yet after 210 years Captivity, they came forth with their *Hoſts*, ſo that there was told *Six hundred thouſand Fighting Men*, beſides *Old People, Women* and *Children*: And he that ſhall ſeparate but one of the Families of *Joſeph*, it would be ſufficient to make up that Number: And how much more would it be then, if many *Families* were joyned together?

CHAP.

CHAP. VII.

Of Changes in Bells; in Musical Instruments, &c.

I. *Of Changes in Bells.*

IT is often disputed among common *Ringers*, what Number of *Changes* may be made in 5, 6, 7, 8, or any other Number of *Bells*. This and such like *Questions* may be easily resolved; for it is but multiplying every number from the *Unite* successively into each others *Product*, unto the number of *Unites* assigned; according to which the following *Table* is made, as is easy to perceive: And so the Number standing against VI in the *Table*, is 720, and so many *Changes* may be made upon *Six* Bells; 5040 upon Seven, *&c.*

II. *Of Voices.*

If it were required to know how many *Consorts* Ten *Voices* will make, each man keeping his own *Note*, but altering his Place; against the Number X in the *Table* you find this number 3628800; which is *Three millions, six hundred twenty eight thousand, and eight hundred*; and so many Consorts may be made of Ten *Voices*.

It it also the same in *Stringed Instruments*; and the *Gamauth* may be varied.

A Table of CHANGES, *&c.*

I	a	1
II	b	2
III	c	6
IV	d	24
V	e	120
VI	f	720
VII	g	504,0
VIII	h	403,20
IX	i	362,880
X	k	362,880,0
XI	l	399,168,00
XII	m	479,001,600
XIII	n	622,702,080,0
XIV	o	871,782,912,00
XV	p	130,767,436,800,0
XVI	q	209,227,898,880,00
XVII	r	355,687,428,096,000
XVIII	s	640,237,370,572,800,0
XIX	t	121,645,100,408,832,000
XX	v	243,290,100,817,664,000,0
XXI	w	510,909,421,717,094,400,00
XXII	x	112,400,072,777,760,768,000,0
XXIII	y	258,521,167,388,849,766,400,00
XXIV	z	620,448,401,733,239,439,360,000

accord-

according to this, answerable to the *Number* standing against XXII, namely, 1124^0072777760768o000 *Notes.*

III. *Of Changes of Places.*

Into how many several *Positions* may the Twelve Figures about the *Dial-Plate* of a *Watch* or *Clock-Dial* be placed? The work is easily done, for the Number standing against XII in the *Table* will resolve you, *viz.* 479001600, which is, *Four hundred seventy nine millions, one thousand and six hundred times.* And according to this way there was a pretty Bargain made between a *Young Scholar* and a *Countrey Gentleman*; which is this:

A Young Scholar being come to a Countrey Town where he intended to reside some time, lit into a Gentleman's House, where there were in Family the Master, Mistress, and Four Children, which with himself made Seven. At dinner they discoursed concerning the Scholar's Board there for a Year, for which the Gentleman demanded a certain Sum of Money; which the Scholar thinking too much, made this Overture; That he would give him so much for a Year as he did demand, provided, That for that same Money he should have his Board so long time as he could daily place those Seven Persons that were then at the Table, in a several and distinct Order, so that they should never all of them sit in the same Places as then they did: The Gentleman condescends: The Question is, How many days may the Scholar sojourn with the Gentleman, before all these Changes of Places come about?

Look in the Table for the *Number* standing against VII, and you shall find it to be 5040, so that the *Scholar* (according to this Agreement) must sojourn with him *Five thousand and forty* days, which is Fourteen years wanting 70 days; and that at Three-Pence a day will amount unto *Threescore and three Pounds* Sterling.

IV. *Of the Letters of the Alphabet.*

From hence it is no marvel, that from the mutability of Transmutations, out of the 24 *Letters* of the *Alphabet*, there ariseth and is made such Variety of *Languages* as are in the world, and such infinite number of *Words* in each *Language*, seeing the diversity of *Syllables* produceth that effect; and also by the interchanging and placing of *Consonants* among the *Vowels*, and amongst themselves; since this Alphabet of XXIV Letters may be varied so many times as the Number against XXIV amounteth to; *viz.*

620448401733239439360000.

Which is,

Six hundred and twenty thousand, four hundred forty eight myriads, four hundred and one thousand seven hundred thirty three thousand two hundred thirty nine millions of millions, four hundred thirty nine thousand millions, three hundred and sixty thousand.

Now if from hence we should allow that a man may read or speak *One hundred thousand Words* in an *hour*, which are as many as are contained in all the *Gospels* of the *Four Evangelists*, and in the *Acts* of the

Apostles

Apoſtles alſo, (a Task too great for a man to do in ſo ſhort a time) and if there were *Four thouſand ſix hundred and fifty thouſand millions of men*, they could not ſpeak all the *Words* that the 24 Letters are capable to make (according to the hourly proportion aforeſaid) in 70000 *Threeſcore and ten thouſand years*; which number of *Words* if they ſhould be written in *Books*, each *Book* being 15 inches long, 12 broad, and 6 thick, the *Books* made of the aforeſaid Tranſmutation of the 24 Letters, would be 387780370899287 88. And if a *Library* of a *Mile Square* every way, of 50 foot high, and in which were 250 *Galleries* of 20 Foot broad apiece, it would contain but *Four hundred millions* of the ſaid *Books*: So that there muſt be to contain the reſt no leſs than 96945092, *Ninety ſix millions, nine hundred forty five thouſand, ninety two*, ſuch *Libraries*: And if the *Books* were extended over the ſuperficial ſurface of the whole *Earth*, it would be a Decupal Covering to the ſame.——And it is from hence that *Tacquet* in the 8*th. Chapter* of his 5*th. Book* of *Arithmetick* affirms, That the *Permutation* of 24 *Letters* are ſo numerous, that a *Thouſand millions* of able *Clerks*, in a *Thouſand millions* of *years* (not ſparing *Dominicals* nor *Feſtivals*) were not able to tranſcribe them. And *Galdinus* aſſerts, That the *Books* which might be compiled of the variety of 23 *Letters* only (accounting 1000 *Pages* to each *Volume*, and 100 *Lines* to each *Page*, and 60 *Letters* to each *Line*, and not any two *Words* in any of thoſe *Volumes* the ſame) would do more than twice cover the whole Superficies of the *Earth* and *Sea*: Nay farther, he ſeems to be of opinion, that the Paper of thoſe *Volumes* laid ſingly *Sheet* by *Sheet*, would cover the very *Firmament*.

And as in the Tranſpoſition of *Letters*, ſo alſo in the Tranſpoſition of the *Nine Digit Figures*, which with the *Nul.* or *Cypher* makes *Ten*, are capable of 3628800, that is, of *Three millions, ſix hundred twenty eight thouſand, and eight hundred* Changes. And in Mr. *Henry Briggs* his *Logarithms* to 100000, conſiſting of *Sixty nine* Sheets, in each *Sheet* Four *Pages*, and in each *Page Four hundred and fifty numbers*, in all 124200; i. e. *One hundred twenty four thouſand and two hundred Numbers*. And in the *Canon* of *Sines Tangents* and *Secants* of the ſame Book, conſiſting of 22 *Sheets* and a half, in which are *Forty Pages*, and in each *Page Three hundred and ſixty numbers*; in all 16200; *i. e. Sixteen thouſand and two hundred* Numbers, which with thoſe of the *Logarithms*, make together 140400, i. e. *One hundred and forty thouſand and four hundred numbers*, and not two *Numbers* exactly the ſame in both the *Books*: Yet notwithſtanding this vaſt *Number*, it is leſs than the *Number* againſt X in the *Table* by 3488400: Nay, it is very little above the *Twenty ſixth* part thereof: So that if *Twenty ſix* ſuch *Books*, all of *Numbers*, and each *Book* to contain *One hundred and forty thouſand four hundred numbers*, and not any two *numbers* the ſame in all the *Books*, they would but contain the number of the Number that the Table exhibits againſt X in the Table.

CHAP.

CHAP. VIII.

Of Arithmetical Verſifying.

Shewing an Artificial way, How from any Six *or* Five *of the* Nine Digit Numbers (*promiſcuouſly taken, or ſet down at all adventures*), *to make both* Hexameter *and* Pentameter Latin Verſes, *which ſhall be* Good Latin, *and* Good Senſe; *altho the Party which writes the* Verſe *from the* Six *or* Five Figures *ſo ſet down, doth not underſtand any thing of the* Latin Tongue.

The Fundamental SCHEME.

	1ſt. *Fig.*	2d. *Fig.*	3d. *Fig.*	4th. *Fig.*	5th *Fig.*	6th. *Fig.*
1	Perfida	Verba	Vides	Promittunt	Fœdera	Certa
2	Horrida	Jura	Viro	Monſtrabunt	Somnia	Multa
3	Turbida	Fata	Tibi	Prædicunt	Pocula	Semper
4	Sordida	Dicta	Scio	Producunt	Dogmata	Sola
5	Candida	Dona	Reor	Caſaubunt	Crimina	Tantum
6	Turpia	Vota	Malis	Portabunt	Pignora	Quædam
7	Aſpera	Facta	Mihi	Confirmant	Jurgia	Prava
8	Triſtia	Bella	Inquam	Procurant	Sidera	Sæpe
9	Impia	Damna	Aliis	Concedunt	Tempora	Plana
	3	4	5	2	1	

How to make Hexameter *Verſes by the Scheme.*

Let Six Figures be written by any perſon, as theſe 8, 6, 4, 3, 9, 5, or any other.

Find 8 *the Firſt*, 6 *the Second*, 4 *the Third*, 3 *the Fourth*, 9 *the Fifth*, 5 *the Sixth* — *Figure in the Firſt Column of the Scheme towards the Left Hand, and under the Title* — *Firſt Figure*, *Second Figure*, *Third Figure*, *Fourth Figure*, *Fifth Figure*, *Sixth Figure* — *You ſhall find this Word* — Triſtia, Vota, Scio, Prædicunt, Tempora, Tantum.

Which Words make this *Latin Hexameter* Verſe,

8 6 4 3 9 5
Triſtia Vota Scio, Prædicunt Tempora Tantum.

And from any Five Figures collected by the bottom of the Scheme (as theſe 1, 9, 7, 2, 6,) may be made this or any other *Pentameter* Verſe. *Example*, 1, 9, 7, 2, 6.

1 9 7 2 6
Fœdera Concedunt Aſpera Jura Malis.

CHAP.

CHAP. IX.

Of Numerical Enigmatical Problems.

PROBL. I.

If 136 *Pound be to be divided between two perſons, ſo that one muſt have* 18 *Pound more than the other; How much muſt each perſon have?*

FRom the given Sum (136) ſubſtract (18) the overplus that one perſon muſt have above the other, and the Remainder will be (118) the half whereof is (59); and ſo much muſt the one perſon have: And if you add (18) to (59) the Sum will be (77); and ſo much muſt the other perſon have; for theſe two numbers (77 and 59) added together, do make up the whole Sum of (136 Pound).

PROBL. II.

If 136 *Pound be to be divided between two perſons, in ſuch ſort, that the Leſſer Share ſhall have ſuch Proportion to the Greater Share, as* 2 *hath to* 5; *What muſt each perſon have?*

FOR the Working of this or the like Queſtion, this is the RULE, or Analogy.

As the Sum of the two Proportional terms 2 and 5, (*viz.* 7.)
Is to the Sum given to be divided (*viz.* 136.)
So is (2) the Leſſer of the Proportional Terms given,
To ($38\frac{6}{7}$) the Leſſer Share: And ſo is (5) the Greater Proportional Number given, To ($97\frac{1}{7}$) the Greater Share.

So if you multiply the given Number (136) by (2) the Leſſer Proportional, the Product will be (272) which being divided by the Sum of the Proportionals (7), the Quotient will be ($38\frac{6}{7}$), which is the Leſſer Share. —— Or if you multiply the given number (136) by (5) the Greater Proportional, the Product will be (680), which divided by (7), the Quotient will be ($97\frac{1}{7}$), for the Greater Share; and theſe two ($38\frac{6}{7}$ and $97\frac{1}{7}$) added together, do make (136), equal to the Number given to be divided.

PROBL. III.

There is a certain number of Pounds to be diſtributed among Six perſons, in ſuch ſort that each perſon from the firſt ſhall have 7 *Pounds more than the other, and at the end the laſt perſon had juſt as many Pounds more as the firſt: What was the Sum of Money, and how much had each perſon?*

LET the Six perſons be repreſented by theſe Six Letters, A B C D E F; then ſuppoſe that the firſt perſon A had 3 Pounds, then B muſt

A	3	35
B	10	42
C	17	49
D	24	56
E	31	63
F	38	70
	35	315

must have 10, C 17, D 24, E 31, and F 38, as in the Margent: But (by the tenure of the Problem) the ſhare of F ſhould be but juſt double to that of A, but it is much more; wherefore ſubſtract A's ſhare 3, from F's ſhare, 38, and the remainder will be 35; which is the number of Pounds that A muſt have; and then B muſt have 7 more (*viz.* 42.) and C 49, D 56, E 63, and F 70, which is juſt double to what A had; which anſwers the Queſtion as to that part: Then, theſe ſhares added together, do make 315 Pound, and that was the number of Pounds to be divided.

PROBL. IV.

There were Five Bags of Money, and in the biggeſt Bag there was as many more Pounds as there was in the leaſt, and in the middle Bag there was half as much as there was in the biggeſt and leaſt together; and each Bag from the leaſt had in it 30 Pound more than the other: What number of Pounds was in each Bag?

LET the Five Bags be repreſented by theſe Five Letters, G H K L and M, and ſuppoſe there was in the leaſt Bag G 4 Pound, then muſt there be in the Bag H 34, and in K 64, and in L 94, and in M 124. But the Bag M holds much more than double that of G, wherefore ſubſtract G 4, from M 124, and there will remain 120, and ſo much Money muſt be in the leaſt Bag G, as in the Margent: Then (becauſe every Bag exceeds other by 30 Pound), there muſt be in H 150 Pound, in K 180 Pound, in L 210 Pound, and in M 240: And then M is double to G, for 120 doubled is 240, equal to M, and G and M together is equal to 360, the half whereof is 180, and ſo much is there in the middle Bag K; and the Money in all the Bags is equal to 900 Pounds: And ſo is the Queſtion anſwered in all particulars.

G	4	120
H	34	150
K	64	180
L	94	210
M	124	240
	120	900

PROBL. V.

If 136 Pound be to be divided into two parts between A and B, ſo that B ſhall have 25 Pound more than A, what is each ſhare?

FRom 136 (the Sum) ſubſtract 25 (the exceſs that B muſt have more than A), and the remainder will be 111, the half whereof is 55 *l.* 10 *s.* for A; and that added to 25 the exceſs, it makes 80 *l.* 10 *s.* for B; for 55 *l.* 10 *s.* and 80 *l.* 10 *s.* is equal to the Sum 136 *l.*

PROBL. VI.

If 560 *Pound be to be divided between* A *and* B, *in such proportion as* 2 *Pound hath to* 7 *Pound, what must each have?*

THE Sum of the two Proportional Terms 2 and 5 make 7; wherefore by the Golden Rule,

As 7 is to 560, so is 2 the lesser Proportional to 160 the lesser Share; and so is 5 the greater Proportional to 400 the greater Share; which together make 560 Pound, the Sum to be divided.

PROBL. VII.

There was a May-Pole which consisted of three pieces of Timber, of which the first (or lowermost) was 13 *foot long, the third (or uppermost piece) was as long as the lowermost piece, and half the middle piece; and the middle piece was as long as the uppermost and lowermost together: How high was this May-Pole, and how long each Piece?*

MUltiply the length of the first piece 13, by 3 and by 4, so shall the two Products be 39 and 52, for 13 by 3, is 39 for the length of the uppermost piece; and 13 by 4, is 52 for the length of the middle piece, which 52 is equal to 13 the lower piece, and 39 the upper piece added together; and the height of the May-Pole was therefore 104 Foot.

PROBL. VIII.

A Grandfather *living in the house with his* Son, *and his* Son's Wife, *and their* Three Sons, *and discoursing concerning their* Ages, *the* Father *said, I am as old as my* Wife *and* Second Son, *and four years over.* ——*The* Wife *said that she was as old as all her three* Sons, *and three years over.* ——*The* Eldest Son *said that he was as old as his two younger* Brothers, *and* 3 *years over.* ——*And the* Second Son *said he was four times the age of his younger brother, and one year over. And when all their Ages were added together, they proved to be the just Age of the* Grandfather, *who was then* 125 *years old. What was the age of the* Father, Mother, *and all the three* Sons?

FOR the resolving of this and the like *Questions*, suppose the youngest Son to be *One* year old, then the second must be 4 years, and 1 over, and the eldest as old as both the other, that is, 5 years, and 3 over. Also the *Mother* must be as old as all the three *Sons*, which is 10 years, and 3 over: And consequently by the Tenor of the *Question*, the *Father* being the Age of his *Wife* and *Second Son*, must be 14 years old, and 4 over.

Which set down in this manner,

The Youngest Son was old	1 Year.
The Second Son was then	4 Years, and 1 over.
The Eldest Son was then	5 Years, and 3 over.
The Mother then must be	10 Years, and 3 over.
The Father must then be	14 Years, and 4 over.
All which added together make	34 Years, and 11 over.

But according to the tenor of the *Queſtion*, all their Ages together ſhould be equal to the Age of the *Grandfather*, which was 125 years: Wherefore, to find the Age of the *youngeſt Son*, you muſt work by the Rule of Proportion thus: Saying,

As 34 years
Is to 1 year (the ſuppoſed Age of the *youngeſt Son*)
So is 125 years (the Age of the *Grandfather*.)
To $3\frac{12}{34}$, or in Decimals thus, 3. 352 years,
For the Age of the youngeſt Son.

And then

The Youngeſt Son	3. 352			3. 352
The Second Son	13. 408	and 1 over,	that is,	14. 408
The Eldeſt Son	16. 760	and 3 over,		19. 760
The Mother	33. 520	and 3 over,		36. 520
The Father	46. 928	and 4 over,		50. 928
	113. 968	and 11 over	(that is)	124. 968

The Sum here is 113. 968 years, to which the 11 years over being added in their due places, as in the ſecond part of the Example they are, the Sum will be 124. 968 years, equal to the *Age* of the *Grandfather*; and the *Queſtion* reſolved in every particular.

PROBL. IX.

If 222 Crowns *be to be divided among three* Perſons *A B and C, in ſuch proportion that C ſhall have 9 times as many as B, and 6 times as many as A wanting Ten.—— And B ſhall have 7 times as many as A, and 25 over: How many Crowns muſt A have, and alſo B and C?*

Suppoſe A to have only one *Crown*, then by the *Queſtion*, B muſt have 7 Crowns, and 25 over; and C muſt have 9 times as many as B, that is 63, and 6 times as many as A, that is 6, in all 69 wanting 10; which ſet in this manner.

A 1 Crown.
B 7 Crowns, and more 25.
C 69 Crowns wanting 10.

The Sum 77 Crowns, more 25, wanting 10.

Subſtract the 10 wanting from the 25 more, and then there will remain 15 over; Subſtract 15 from 222, and there will remain 207, for a dividend, and the Sum 77 muſt be the diviſor; wherefore divide 207 by 77, and the Quotient will be $2\frac{53}{77}$ (or in Decimals 2. 688) for the ſhare of A, and then A having 2. 688, B muſt have 7 times as many, that is 18. 816, and 25 over; and C muſt have nine times as many as B, which is 169. 344, and ſix times as many as A, which is 16. 128, which together

A having 2. 688.
B muſt have 18. 816 and more 25.
C muſt have 185. 472 wanting 10.
The Sum 206. 976.

together makes 185. 472, wanting 10; all which being added together as above, do make 206. 976, to which the 25 over being added, and the 10 wanting ſubſtracted in their due places, the Sum will be 221. 976, that is, 222 Crowns, and anſwers the Queſtion.

A	2. 688,		2. 688.
B	18. 816 more 25,	that is,	43. 816.
C	185. 472 wanting 10,		175. 472.
	206. 976 more 25 want 10.		221. 976.

PROBL. X.

There are four Weights *A B C D, all which together do weigh 324 ½ Pound, of which B weighs more than A by 9 Pound, C weighs three times as much as A, and four times as much as B; and D weighs 6 times A, 3 times B, and once and a half C; their reſpective weights are demanded.*

LET the weight A be ſuppoſed to weigh only one Pound, then B muſt weigh 1 Pound and 9 over; and C muſt weigh 3 times A which is 3, and 4 times as much as B which is 4, which together make 7 for C; and then D muſt weigh 6 times A, 3 times B, once and a half C, and 4 times B, which together make 19½; all which added together do make 28½ more by 9, which ſhould be equal to 324½; Subſtract the 9 which is over, from 324½, and the remainder will be 315½ for a Dividend, and the Sum 28½ is the Diviſor: Divide 315½ by 28½ (or decimally 315. 5. by 28. (5) the Quotient will be 11. 07 for the weight of A. Then

A	1.		
B	1 more 9.		
C	7.		
D	19½.		
Sum	28½ more 9.		

A weighing	11. 07,		11. 07
B muſt weigh	11. 07 + 9,	or	20. 07
C muſt weigh	77. 49,		77. 49
D muſt weigh	215. 86,		215. 86
Their Sum	315. 49 + 9,	or	324. 49

which anſwers the Queſtion.

PROBL. XI.

Three Merchants *build a* Ship *which coſt 6000 Pound, of which A paid ¼ part, B ⅓ part, and C ⅛ part; What did each pay?*

MUltiply the Denominators of the ſeveral Fractions ¼ ⅓ ⅛, and they make 120; divide this 120 by each ſeveral Denominator, as by 4, by 5 and by 6, and the Quotients will be 30, 24 and 20,

For 120 divided by	4	giveth for its Quotient	30
	5		24
	6		20
		The Sum whereof is	74

Then

Then ſay by the Golden Rule of Proportion,

If 74 come of 6000, } 30 } Anſwer { A 2432. 43
what will come of } 24 } { B 1945. 95
{ 20 } { C 1621. 62

The Sum —— 6000. 00

PROBL. XII.

There are three Weights ABC, of which, the leaſt Weight A, weighs 30 *Pound, and the other two together do weigh ſeven times as much as A, and they are in proportion one to the other as* 4 *is to* 5.

THE Weight A weighing 30 Pound, the other two Weights together weighing 7 times A which is 30, muſt weigh 210 Pounds; and the Proportional Numbers 4 and 5 being added together do make 9. Then by the Rule of Proportion:

As 9, the Sum of the Proportional terms
Is to 210 the Sum of the two unknown Weights;
So is 5, the Greater Proportional Number,
To the Weight of the Greater Weight.

Wherefore, Multiply 210 by 5, the Product will be 1050, which divided by 9, gives in the Quotient 116⅔ for the Weight of the bigger Weight; and that ſubſtracted from 210, the remainder is 93⅓, for the leſſer Weight, and ſo the three Weights are

A 30
B 93⅓ } 240 the Weight of all three.
C 116⅔

PROBL. XIII.

Two Travellers ſet out from two Towns which are 140 *Miles diſtant, upon one and the ſame day; One travels* 8 *miles a day, the other* 6: *In how many days Travel will they meet each other?*

SUppoſe one D (or Day) for the time that they ſhall meet, in which time one would have travelled 8, and the other but 6 miles of the Journey, which together make 14 miles; whereas they ſhould have been 140 miles: Wherefore, divide 140 by 14, the Quotient will be 10, and in ſo many days will they meet: For 10 times 8 is 80 Miles; and 10 times 6 is 60 Miles; which together make 140 Miles the whole Journey.

PROBL.

PROBL. XIV.

There is a Ciſtern *which hath Four* Cocks *of ſeveral Siſes, which Ciſtern holds* 8 *Barrels of Water? If the firſt or leaſt Cock be opened, the Water will be* 6 *hours in running out: If the ſecond Cock be only opened, the Water will iſſue out in* 4 *hours: If the third be only opened, it will run out in* 3 *hours; and if the fourth and biggeſt be only opened, it will run out in* 2 *hours: In what time will it run out, if all the four Cocks be ſet running together?*

IF the leaſt Cock will vent 8 Barrels (that is, all the Water) in 6 hours, the ſecond would vent 12 Barrels in that time, the third 16, and the fourth 24 Barrels, in all 60 Barrels.

Here is now a Proportion between 60 Barrels and 8 Barrels, and between 6 hours and the time that all the Water will be vented in, if all the Cocks were opened: Wherefore ſay,

If 60 *Barrels will run out in* 6 *hours; In what time will* 8 *Barrels run out?*

Multiply 6 by 8, the Product is 48, which ſhould be divided by 60, but being it is leſs, the Quotient will be $\frac{48}{60}$ parts of an hour, or 48 Minutes, and in ſuch time will all the Water run out, if all the four Cocks be ſet open.

PROBL. XV.

A Man dies, and leaves his Wife big of her firſt Child; and by Will bequeaths his Eſtate in manner following: That if the Child his Wife then went withall ſhould prove a Daughter, then his Wife ſhould have two Thirds, and the Daughter one Third of his Eſtate: If it ſhould be a Son, then the Son was to have two Thirds, and the Mother one Third of his Eſtate, which Eſtate was 2600 *Pound.——After the deceaſe of the Father, the Mother was delivered of two Sons and one Daughter. How muſt the Eſtate be divided amongſt them, according to the Will of the Father?*

BY the Teſtators Will it is evident that for one Third that the *Daughter* had, the *Mother* was to have two Thirds; and the *Son* was to have double to the *Mother*. Then

Suppoſe the		to have	*l.*	*s.*	*d.*
	Daughter		1	0	0
	Mother		2	0	0
	1 Son		4	0	0
	2 Son		4	0	0
		The Sum	11	0	0

This 11 Pound ſhould be equivalent to 2600 Pound, wherefore divide 2600 by 11, and the Quotient will be 236 *l.* 7 *s.* 3 *d.* 1 *q.* for the Daughters ſhare: And then will

The

			l.	s.	d.	q.
The	Daughters	share be	236	07	3	1
	Mothers		472	14	6	2
	1 Sons		945	09	1	0
	2 Sons		945	09	1	1
		In all	2600	00	0	0

Equal to the whole Eſtate.

PROBL. XVI.

If there be 160 *Pound* (*or* 38400 *Pence*) *to be diſtributed among an equal number of Poor Priſoners, Lame Men, Widows and Children, ſo that every Priſoner ſhall have a Shilling, every Lame Man Six Pence, every Woman Four Pence, and every Child Three Pence: How many of ſuch People will this Money relieve?*

THE Sum of Pence in 160 Pound being 38400, ſet that down for a Dividend; then add 12 *d*, 6 *d*, 4 *d*, 3 *d* together, and they make 25, by which divide 38400, and the Quotient will be 1536; and ſo many Perſons muſt there be of each kind: For

			l.	s.	d.		
1536	Shillings	is	76	16	0	for	Priſoners.
	Sixpences		38	08	0		Lame Men.
	Groats		25	12	0		Women.
	Threepences		19	04	0		Children.
		In all	160	00	0		

And now if you multiply 1536 by 4, the Product will be 6144, and ſo many will be the whole number of Perſons to be relieved.

PROBL. XVII.

What Number is that, which if you divide it by 9 *there ſhall remain* 8, *divide it by* 8 *there ſhall remain* 7, *divide it by* 7 *there ſhall remain* 6, *and ſo by* 6, 5, 4, 3 *and* 2, *and being divided by* 2 *there ſhall remain One?*

THere are ſeveral ſuch Numbers to be found, and to find the leaſt that is thus qualified, do thus,——Set down all the nine Digits orderly one under another, and againſt 1 ſet a Cypher; then under 1 ſet 2, and add 2 and 1 together, they make 3, which ſet againſt 2; then ſay 3 times 3 is 9, and 2 over 3 is 11, ſet 11 againſt 3, and ſay 4 times 11 is 44, and 3 over 4 is 47, ſet 47 againſt 4, and thus continuing you ſhall find 725759 to ſtand againſt 9, and that is the leaſt number that is capable to be divided as aboveſaid, for the remainder ſhall be always a Unite leſs than the number you divided by.

1	0
2	3
3	11
4	47
5	239
6	1439
7	10079
8	80639
9	725759

2519
5039
7559
10079
12599
15119
17639
20159

There are infinite other Numbers which have the like property, ſome of which are here inſerted. For,

If any of thoſe Numbers be divided by	There will remain
2	1
3	2
4	3
5	4
6	5
7	6
8	7
9	8

CHAP. X.

Of Numbers thought upon.

IF any Number be doubled, and that doubled be multiplied by 5, the Product of that Multiplication ſhall be the ſame with the Number firſt doubled, cutting off the Cypher which will always be on the Right hand: So 472 being doubled, makes 944, which multiplied by 5, produceth 4720, from which the Cypher being cut off there will remain the firſt Number 472.——And from hence it is uſual to bid one think any Number, then bid him double his number thought, and then multiply that double by 5, and give you the Product, which will be the very number he thought, with a Cypher before it; which omitted, you tell him what number he thought: But this way is ſo plain and obvious, that any one will diſcover it: I ſhall therefore ſhew ſome other ways not ſo liable to diſcovery.

First Way.

BID him which thinketh double his number thought, unto that double add 4, and multiply that Sum by 5, and unto that Product bid him add 12; then bid him tell you this laſt Product, unto which do you privately add a Cypher, and from the Sum ſubſtract 320, the remainder (two Cyphers being cut off) will be the number thought.

Example.

The Number thought upon	365
The double thereof	730
To which add 4, it makes	734
That multiplied by 5, makes	3670
To which add 12, it makes	3682

This laſt Product 3682 being given you, add unto it a Cypher, and it makes 36820, from which ſubſtract (always) 320, and the remainder will be 36500, from which the two Cyphers being taken, there will be left 365, the Number thought.

The Second Way.

BID the Party that thinketh, break his number into any two parts; then bid him multiply each of thoſe parts by themſelves, and alſo the parts one by the other twice; then bid him add all thoſe four Products together, and give you the Sum of them; from which Sum do you Extract the Square Root, and that Root ſhall be the number thought.

Example.

Let the Number thought upon be 365; which let be broken into theſe two parts 221 and 144, which together make 365.

Multiply 221 by 221, the Product is	48841
Multiply 144 by 144, the Product is	20736
Multiply 221 by 144, the Product is	31824
The ſame again	31824
The Sum of the four Products	133225

The Square Root whereof is 365, the Number firſt thought upon.

A Third Way.

LET any Perſon think (and ſet down) what Number he pleaſeth; then bid him multiply that Number, and their Products ſucceſſively by all the nine Digits; then bid him give you the laſt Product; which Product, if you divide it by 362880, the Quotient ſhall be the Number thought.

So if the Number thought (or ſet down) were 321, that Number multiplied by One (1) produceth only 321:

	321	2		642
	642	3		1926
	1926	4		7704
And	7704	5	Produceth	38520
	38520	6		231120
	231120	7		1617840
	1617840	8		12942720
	12942720	9		116484480

Multiplied by

Now

Now if you divide this laſt Product 116484480 (in any caſe) by 362880, the Quotient will be 321, equal to the Number thought.

A Fourth Way.

LET any Perſon think (or ſet down) any Number, then let him multiply that Number by all the nine Digits; then let him add all the Products together, and give you the Sum of all the Products ſo added together; which Sum, if you divide (in any caſe) by 45, the Quotient will be the number thought. So

Let the Number thought be 26. That Multiplied by	1	Produceth	26
	2		52
	3		78
	4		104
	5		130
	6		156
	7		182
	8		208
	9		234
The Sum	45	The Sum	1170

Which Sum 1170 being divied by 45 (the Sum of the nine Digits) the Quotient will be 26, the Number thought.

A Fifth Way.

BID a Perſon think (or ſet down) any Number, then bid him multiply the ſame Number by any other Number, what he pleaſes, (telling you what Number he multiplies by.) Then let him multiply that Product by what other Number he pleaſes (telling you what he multiplies by, as before.) And ſo continue multiplying three, four, or as many times as he pleaſes: In the mean time do you ſet down the Number by which he firſt multiplied, and multiply that Number by the ſecond Number that he multiplied by, and that Product by what he multiplied by the third time, and as many times as he multiplies; and when he hath done, bid him give you his laſt Product, which if you divide by your laſt Product, the Quotient will be the Number which he thought upon.

Example.

Suppoſe the Number thought upon be 36, which he firſt Multiplies by 7, and the Product is 252, which he again will multiply by 3, and the Product is 756; which again he will multiply by 9, and the Product will be 6804; which laſt Product when he hath given you, then privately ſet down 7 (the Number that he firſt multiplied by) and multiply it by 3 (the ſecond Number he multiplied by) and the Product will be 21, which multiply by 9 (the third and laſt Number he multiplied by) and the Product will be 189, by which, if you divide his laſt Product 6804, the Quotient will be 36, the Number thought upon.

Otherwiſe,

If you bid him add all his Products together in one Sum, and give you the Sum of them, then do you add all your Products together, and by that divide his Sum, the Quotient will give the Number thought. So

His	First, Second, Third	Product was	252, 756, 6804	Your	First, Second, Third	Product was	7, 21, 189
	The Sum		7812				217

Now if you divide 7812 (his Product) by 217 (your Product), the Quotient will be 36, as before.

VI. *If ſeveral perſons, three, four, five,* &c. *ſhould each of them think upon a ſeveral Number* (*under Ten*), *to tell what Number each perſon thought upon.*

BID the firſt perſon *double* the Number he thought, and add 5 to it, which Sum multiply by 5, and to it add 10; if from this Number you privately ſubſtract (always) 35, the firſt figure of this remainder towards the left hand, ſhall be the Number that the firſt Party thought upon.

Example,

Let one perſon think — 9
The double thereof is — 18
To which add 5, it makes — 23
This 23 multiplied by 5, makes — 115
To which add 10, it makes — 125
From which 125, ſubtract — 35
And the remainder is 90; from which the Cypher towards the right hand being omitted, the remainder is 9, the number thought.

Thus may you do if *One* perſon only think a number; but if *Two, Three,* or *Four* perſons think ſeverally, it will be much the ſame.

Example,

Suppoſe there were three perſons, A, B, and C, and each of them ſhould think theſe three numbers, A 4, B 9, and C 6.

The Number which A thought is — 4
Which doubled, makes — 8
To which 5 added, it makes — 13
Which multiplied by 5, produceth — 65
To which add 10, and it makes — 75
To this Product bid B privately add the Number he thought 9
It makes — 84
Which bid him multiply by 10, it makes — 840
To this Product let C privately add his Number thought, *viz.* 6
It makes — 846
Which multiplied by 10, produceth — 8460

This

This being done, bid them give you this laſt Product 8460, from which do you privately ſubſtract 3500, and there will remain 4960; the Cypher being omitted, there will remain theſe three Digits, 4, 9, and 6, for the Numbers that A B and C ſeverally thought upon.

Note, That if one perſon only Think, then (as before) ſubſtract from the laſt Product only 35; if two perſons, ſubſtract 350; if three, 3500, *&c.*

VII. *There lies in a heap* 132 (*or any other number which you know*) *of Counters, or other Pieces of Money, or any other things whatſoever: Then if Three perſons take each of them a certain number of Pieces out of the heap, unknown to you, to know how many Pieces each party took.*

LET the number of Counters in the heap be 132, and let the three perſons be A, B, and C: Bid one of the Parties, as A, take from the heap 4, 8, 12, 16, 32, *&c.* or any other number that may be divided by 4, and keep them in his hand: Then bid B for every four Pieces that A took, let him take 7 Pieces: And bid C for every four Pieces that A took, take 13 Pieces: Which when they have done, tell what number of Counters are left, and ſubſtract them from the Number you laid down, and the remainder will be the Sum of Counters which all of them have in their hands, which number do you privately divide by 3, the Quotient ſhall be double to the Number which A took.

Example,

Let the Number of Counters which you laid down be 132. Then ſuppoſe that A took out 16, that is, four times 4, then B muſt take out 7 for every four that A took, which is four times 7, or 28; and C muſt take out 13 for every four that A took, and that is 4 times 13, or 52. This done, tell how many Counters there are left upon the Table, which you find to be 36; this ſubſtracted from the whole heap, which was 132, there will remain 96; and ſo many Pieces have A B and C in their hands; this Number 96 divide by 3, and the Quotient will be 32, the half of which is 16, and ſo many Pieces did A take; then B muſt have taken 28, and C 52.

VIII. *If there be two Pieces of Money, as a Ninepence and a Shilling, or any other two Pieces* (*provided one be Even, and the other Odd*), *let any perſon take one of them in one hand, and the other in his other hand, to tell in which hand the Ninepence* (*or Odd Piece*) *is, and in which the Shilling* (*or Even Piece*) *is.*

BID the Party that he double the Piece that he hath in his Left hand, and triple the Piece which he hath in his Right hand; then bid him add the two Numbers together, and ask him whether it be Even or Odd; if it be Even, the Shilling is in the Right hand; but if it be Odd, the Ninepence is in the Right hand.

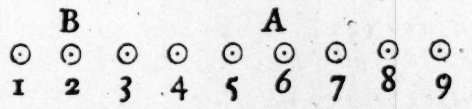
Or,

If you bid the Party double the Piece which he hath in his Right hand, and triple that which he hath in his Left, and if the Number be Even, the Odd Piece is in the Right hand, and the Even in the Left.

IX. *If*

IX. *If any Number of Counters (ſuppoſe 9) or other Pieces of Money, Stones, or the like, be laid in a Row, to tell what Number (not exceeding the number of Pieces) any one thinketh upon.*

LAY Nine Counters on a row, (as in the Figure.)



Then do you (privately) call that which lieth next your Left hand One, the next Two, the next Three, *&c.* then that towards your Right hand will be 9. Then bid any perſon think any number, not exceeding Nine, and then bid him lay his finger upon any of the Pieces; then (you knowing privately which Piece it is, whether the firſt, ſecond, third, or any other) add 9 privately to the Number of the Piece he laid his finger upon.

Thus if the Party ſhould think 2, and he ſhould lay his finger upon the Counter A, which is the ſixth, to this 6 add 9 (the Number of Pieces in all) and it makes 15; then bid the Party count from the Number which he thought upon (beginning at the Counter A) backwards, till he make his number thought on 15, which Number 15 will end at the Counter B, which is the ſecond Counter, and denotes the Number he thought on, to be 2.

The like may be done with any Number more or leſs than 9; only remember, that as now you added 9, you muſt always add the Number of the Counters or Pieces you laid down, whether 6, 10, 15, *&c.*

CHAP. XI.

A Plain and Eaſie Method of Extracting the Square Root *of any Number (how great ſoever) without the help of* Multiplication *or* Diviſion.

THIS Method of *Extracting* of the *Square Root*, I cannot but attribute to Sir *Sam. Moreland*, Baronet, altho it doth not much differ from the manner of *Extraction* by *Nepier's Bones*: For as thoſe *Rods* which my Lord *Nepier* (the Inventor of them) calls *Rabdologia*, Sir *Samuel* doth reduce the Poſition of them into ſeveral *Tables*, which he calls *Tariffa's*. The manner of making or preparing whereof I ſhall here ſhew:

How to make a Tariffa *for One or more* Figures, *that is, for any Digit, or mixt Number.*

Example: Suppoſe I would make a *Tariffa* for the Digit Number 4. As in Table (or *Tariffa*) I.

Firſt, Set down the *Nine Digits* orderly, in a Column towards the left hand, and in Figures bigger than ordinary, that they may take up the ſpace of two lines of ſmaller Figures.

Secondly, Againſt the Digit 1, more to the right hand, and at the top or upper part of the line ſet 4, the Figure for which the *Tariffa* is to be made.

Thirdly, Double 4, and it makes 8, which ſet under 4, againſt the Digit 2, and at the upper part of the Line.

Fourthly, Add 4 to 8, and they make 12, which ſet againſt the Digit 3: Alſo to 12 add 4, it makes 16, which ſet againſt the Digit 4; to which add 4, and it makes 20, which ſet againſt the Digit 5, and ſo continually adding 4, you ſhall have 36 to ſtand againſt the Digit 9; for 4 times 9 is 36.

Fifthly, In a Column yet one place forward towards the Right hand, ſet the *Squares* of the ſeveral *Digits*; as againſt 1 ſet 1, againſt 2 ſet 4, againſt 3 ſet 9, againſt 4 ſet 16, and againſt 9 you will have 81: And thus is your *Firſt Tariffa* for the ſingle Figure 4 finiſhed.

Now to make a *Tariffa* for two or more Figures, the ſame method is ſtill to be obſerved. As for

Example: Suppoſe I would make a *Tariffa* for 46.

Firſt, In a Column towards the Left hand ſet down the Nine Digits orderly, and in a bigger Figure.

Secondly, Set your Number to be *Tariffed* 46, againſt 1, as in the ſecond *Tariffa*.

Thirdly, Double 46, and it makes 92, which ſet againſt 2; then add 92 to 46, and they make 138, which ſet againſt 3; and ſo by the continual addition of 46, you ſhall find 414 to ſtand againſt the Digit 9, for 9 times 46 is 414.

Laſtly, In a Column yet more to the Right hand, ſet the *Squares* of the Nine *Digits*; as againſt 1 ſet 1; againſt 2 ſet 4; againſt 3 ſet 9; againſt 4 ſet 16, as in the firſt *Tariffa*.

And in the ſame manner may you make a *Tariffa* for any number, how great ſoever by the directions of the two foregoing *Examples*; and as is done for the number 468 in the *Third Tariffa*; and for 4684 in *Fourth Tariffa*; &c.

Digits.	First Teneriffa.	Squares of the Digits.	Digits.	Second Tariffa.	Squares of the Digits.	Digits.	Third Tariffa.	Squares of the Digits.	Digits.	Fourth Tariffa.	Squares of the Digits.
1	4	1	1	4 6	1	1	4 6 8	1	1	4 6 8 4	1
2	8	4	2	9 2	4	2	9 3 6	4	2	9 3 6 8	4
3	1 2	9	3	1 3 8	9	3	1 4 0 4	9	3	1 4 0 5 2	9
4	1 6 1	6	4	1 8 4 1	6	4	1 8 7 2 1	6	4	1 8 7 3 6 1	6
5	2 0 2	5	5	2 3 0 2	5	5	2 3 4 0 2	5	5	2 3 4 2 0 2	5
6	2 4 3	6	6	2 7 6 3	6	6	2 8 0 8 3	6	6	2 8 1 0 4 3	6
7	2 8 4	9	7	3 2 2 4	9	7	3 2 7 6 4	9	7	3 2 7 8 8 4	9
8	3 2 6	4	8	3 6 8 6	4	8	3 7 4 4 6	4	8	3 7 4 7 2 6	4
9	3 6 8	1	9	4 1 4 8	1	9	4 2 1 2 8	1	9	4 2 1 5 6 8	1

How to Extract *the* Square Root *of any* Number.

LET 54877476 be a Number given, and let the Square Root thereof be required

First ſet down the Number, and make a Prick over the firſt figure thereof towards the right hand, and ſo over every ſecond figure, as is uſual in Vulgar *Arithmetick*, and then will the Number given being ſo Pointed, ſtand thus,

5̇4̇8̇7̇7̇4̇7̇6̇

And there being five Points, it ſhews that the *Root* thereof will conſiſt of *Five Figures*, whereof to begin your *Extraction*, find the neareſt *Square Number* to 5, the figure under the laſt Point, and that is 4, the *Root* whereof is 2, put 2 into the *Quotient*, and ſet the *Square* thereof 4, under 5, and ſubſtracting 4 from 5, there will remain 1, to which 1 bring down the two figures belonging to the *Second Point*, namely 4 and 8, and then it will be 148, and the work will ſtand thus,

5̇4̇8̇7̇7̇4̇7̇6̇ (2
4

148

Then double 2, the figure in the *Quotient*, and it makes 4; for which figure 4 make a *Tariffa*, as is before taught, (as the *Firſt Tariffa* is) and in

in that *Tariffa* find the neareſt number to 148, which is leſſer, and you ſhall find that number to be 129, againſt which (in the Colume of *Digits*) ſtands 3, put 3 in the *Quotient*, and ſubſtract 129 from 148, and the Remainder will be 19; to which bring down the two figures belonging to the Third Point, namely 77, and then the Number will be 1977, and the work will ſtand thus,

```
. . . . . .
548777476 (23
4
---
148
129
---
 1977
```

Then double the two figures in the *Quotient*, and they make 46, for which number 46 make a *Tariffa* (as is the *Second Tariffa* beforegoing); and in that *Tariffa* find the neareſt number leſs than 1977, which you will there find to be 1856, againſt which ſtands the *Digit* 4; put 4 in the *Quotient*, and ſubſtract 1856 from 1977, and the Remainder will be 121, to which bring down the two figures belonging to the fourth Point, namely 74, and then the Number will be 12174, and the work will ſtand thus,

```
. . . . . .
548777476 (234
4
---
148
129
---
 1977
 1856
 ----
  12174
```

Then double the three Figures in the *Quotient*, and they make 468; for which Number make a *Tariffa* (as is the *Third Tariffa* beforegoing), and in that *Tariffa* find the neareſt Number, which is leſſer, to 12174, which I find there to be 9364, againſt which ſtands the Digit 2; put 2 in the *Quotient*, and ſubſtract 9364 from 12174, and the Remainder will be 2810, to which bring down the two figures belonging to the laſt Point, namely 76, and then the number will be 281076, and then the work will ſtand thus,

```
. . . . . .
548777476 (2342
4
---
148
129
---
 1977
 1856
 ----
  12174
  9364
  -----
   281076
```

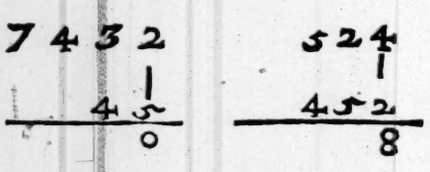
This done, Double the four figures in the *Quotient*, and they make 4684, for which number make a *Tariffa*, as is the *Fourth*, in the foregoing *Examples*; and in it find the neareſt number to 281076, and in it you ſhall find the very number to ſtand againſt the Digit 6: Wherefore put 6 in the *Quotient*, and ſubſtract 281076, from 281076, and there will remain nothing; which ſhews that the given number 548777476 is an exact *Square Number*, and that 23426 is the *Root* thereof; and ſo the whole Work being ended, it will ſtand as followeth.

```
. . . . . .
548777476 (23426
4
---
148
129
---
 1977
 1856
 ----
  12174
  9364
  -----
   281076
   281076
  -------
   000000
```

CHAP. XII.

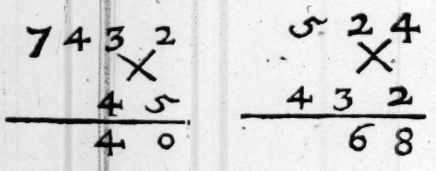
A Brief and Compendious Way of Multiplication, *wherein the* Product *is given at once, without several Workings, or writing down any other* Figures *but the* Product *it self.*

I. *Example.* LET it be requred to multiply 7432, by 45. Set down the Numbers as in the *Margent.* Then,

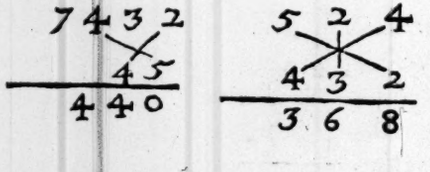


1. Multiply as the line directs you; say 5 times 2 is 10, place 0 under Unites, and carry 1 in mind.

2. Say 5 times 3 is 15, and 4 times 2 is 8, which added together, with 1 in mind, makes 24, set down 4 in the next place on the left hand, and carry 2 in mind.

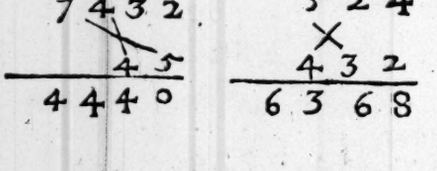


3. Say 5 times 4 is 20, and 4 times 3 is 12, which added together, with 2 in mind, makes 34, place the odd 4 in the next place, and carry 3 in mind.



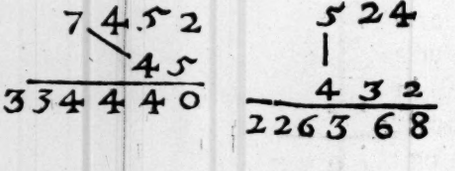
4. Say 5 times 7 is 35, and 4 times 4 is 16, which added together, with 3 in mind, makes 54, set down 4 in the next place, and carry 5 in mind.

Lastly, Say 4 times 7 is 28, and 3 in mind is 33, which set down, and the work is at an end: And the Product of 7452 multiplied by 45, is 33444o: As in the Margent.



II. *Example*, Multiply, 524 by 452.

1. Set the Numbers down, as in the Margent, and say (as the line directs) 2 times 4 is 8; set 8 in the place of Unites under 2.



2. Say 2 times 2 is 4, and 3 times 4 is 12, which added together make 16, place 6 under 3, and bear 1 in mind.

3. Say 2 times 5 is 10, and 3 times 2 is 6, and 4 times 4 is 16, which added together with 1 in mind, makes 33, place 3 under 4, and bear 3 in mind.

4. Say 3 times 5 is 15, and 4 times 2 is 8, which added together, with 3 in mind, makes 26, place 6 before 3, and bear 2 in mind.

Example I. *Example* II.

Lastly, Say 4 times 5 is 20, and 2 in mind makes 22, which set down, and the work is at an end: And the Product of 524 multiplied by 432 will be 226368, as in the Margent.

CHAP. XIII.

Of Ceres *and* Virginum.

IN ſome Ancient, and in ſome late Writers of *Arithmetick* alſo, I find a *Rule* called *Ceres* and *Virginum*, which teacheth only how to reſolve *Merry* or *Sporting Queſtions*, to puzzle young *Practitioners* in *Numbers*: And in regard that this *Book* is fraught with matters *Recreative*, I ſhall here inſert it, with ſome Reflections upon it: And this *Rule* will be made appear beſt, by the Solution of certain *Queſtions* as properly belong to it; as ſuch as here follow.

Queſtion I.

A Lady's Caterer *bought* Eight Birds *of two ſorts, namely* Geeſe *and* Hens, *for* 20 Shillings: *The* Geeſe *coſt* 4 ſhillings *apiece, and the* Hens 2 ſhillings *apiece: How many did he buy of each ſort?*

MUltiply the whole number of *Birds*, viz. 8, into the *Leaſt Price*, 2 *s.* the *Product* will be 16, which take from the whole number of *Birds*, viz. 20, there will remain 4 for a *Dividend*, which muſt be divided by the difference of the *Prices* of the two ſorts of *Birds*, namely, by 2, and the *Quotient* will be 2 for the number of *Geeſe*; then muſt the *Hens* be 6. And ſo

2 Geeſe at 4 *s.* is	8 *s.*
6 Hens at 2 *s.* is	12 *s.*
In all 8	In all 20.

Queſtion II.

There were One and Twenty perſons in Company, ſome Men, *ſome* Women, *and ſome* Children; *and amongſt them they ſpent* 26 Shillings, *and ſo, that every* Man *ſpent* 2 *Shillings, every* Woman 1 *Shilling, and every* Child 6 *Pence: How many muſt there be of each ſort?*

To reſolve this or the like, this is the RULE:

Multiply the Number of Perſons by the leaſt Expence, and take the Product of it from the whole Expence, the reſt ſhall be the Dividend; which divided by the difference between the Greateſt and Leaſt particular Expence, the Quotient is a Number, which the number of Men (or they which ſpend moſt) comes near to, but cannot exceed. ——Or if the ſaid Dividend be divided by the Sum of the Greateſt and Leaſt Expence, the Quotient is a Number, than which the number of men, (or thoſe which ſpend moſt) cannot be much leſs.

SO in this *Queſtion*, 21 *Perſons* multiplied by 6 Pence (the leaſt Expence), the Product is $10\frac{1}{2}$, which taken from 26 (the whole *Expence*) the Remainder is $15\frac{1}{2}$ for the Dividend; and then taking $\frac{1}{2}$ from 2, there reſts $1\frac{1}{2}$ for the Diviſor, and the *Quotient* is $\frac{11}{3}$, which is ſomething more than 10; the number of men therefore muſt be but 9.

Then turn the *Dividend* and the *Diviſor* both into whole Numbers, by multiplying them into the common *Denominator* 2, ſo they being reduced, will be 31 and 3, as before is ſeen in the Quotient.

Multiply the *Diviſor* 3 by 9 (which is the number of *Men*) the *Product* is 27, which taken from 31 (which is the reduced *Dividend*) the remainder is 4 for the Number of *Women*; and then the *Children* muſt be 8; as in the firſt Example.

			s.
First	9	Men, at 2 *s.* each	18
	4	Women, at 1 *s.* each, is	4
	8	Children, at 6 *d.* each	4
	21	Perſons.	26 *s.* expended.

But the *number* of *Men* may be alſo but 8, which multiplied by the reduced Diviſor 3, it produceth 24, which taken from 31 the remainder is 7 for the *Women*; and then the *Children* muſt be 6: As in this ſecond Example.

			s.
Second	8	Men, at 2 *s.* each	16
	7	Women, at 1 *s.* each	7
	6	Children, at 6 *d.* each	3
	21	Perſons.	26 *s.* expended.

Or the number of Men may be 7, which multiplied by 3, produceth 21, which taken from 31, there remains 10 for the Women, and then the Children muſt be 4, as in this third Example.

			s.
Thirdly	7	Men, at 2 *s.* each	14
	10	Women, at 1 *s.* each	10
	4	Children, at 6 *d.* each	2
	21	Perſons.	26 *s.* expended.

By what is here delivered in this ſecond Queſtion, it is plain that it is capable of ſeveral Solutions, and all of them true.

But further, The number of Men may be 10, and not more; for if you put them 11, that multiplied by 3 produceth 33, which is greater than 31 from whence it ſhould be taken: But they may be 10, and then there is only One Woman and 10 Children; and this confirms the former part of the Rule.

Now for the later part of it: If the Dividend 31, be divided by the Sum of the two Extreams (reduced by doubling as the Dividend is) 4, the Quotient will be $7\frac{1}{4}$, and the Men may be 7 as hath been ſhewed; but they may be alſo but 6, and fewer they cannot be: As 6 Men, 13 Women, and 2 Children: For if you put the Men Five, that multiplied by 3, produceth 15, which taken from 31, there remains 16 for the *Women*, and ſo there ſhould be no *Children*, which is contrary to the Suppoſition.

And further, Becauſe the Quotient was $7\frac{1}{4}$, the number of Men might be ſo (if pure Arithmetical Diviſion be only regarded.) And then the number of Women are $7\frac{1}{4}$, and the Children $5\frac{1}{2}$, as in this Example.

$7\frac{1}{4}$ Men,

		s.	d.
7	Men, at 2 *s.* each	15	6
7 ¼	Women, at 1 *s.* each	7	9
5 ½	Children at 6 *d.* each	2	9
In all 21	Persons	In all 26	0 expended.

Question III.

There is 900 l. *to be distributed by the Will of a Deceased Friend, to* 30 *Persons of three several Qualifications, as* Ministers, Lame Soldiers, *and* Poor Tradesmen; *so that each* Minister *may have* 60 l. *each* Soldier 40 l. *and each* Tradesman 20 l. *How many Persons of each Qualification must (or may) there be?*

MUltiply (according to the *Rule*) 30, the *Number of Persons*, by 20, the *least Share*, the Product is 600, which taken from 900, the whole *Legacy*, there remains 300 for the Dividend; and 20, the least share, taken from 60, the greatest share, leaves 40 for a *Divisor*; and so the *Quotient* will be 7 ½, and more the *Ministers* cannot be: Also add 60 and 20 (the *greatest* and *least Shares* together) their Sum is 80, by which divide 300, and the Quotient will be 3 ¾, and much fewer the *Ministers* cannot be.

Not to stand upon the *Fractions* (in the dividing of Men)

The Ministers, Soldiers, Tradesmen may be either	7.	6.	5.	4.	3.
	1.	3.	5.	7.	9.
	22.	21.	20.	19.	18.
			30		

That the *Ministers* cannot (in whole Numbers) be more than 7 or less than 3, may thus be proved. ——First, Let them be 8, then 8 times 40 is 320 which is more than 300, from which it should be taken. ——Secondly, Let them be 2, then 2 times 40 is 80, out of 300 there remains 220, which divided by 20, gives in the Quotient 11 for the *Soldiers*; so the *Ministers* and *Soldiers* being 13, the *Tradesmen* must be 17.

But,

	l.
2 Ministers, at 60 *l.* each, is	120
11 Soldiers, at 40 *l.* each, is	440
17 Tradesmen, at 20 *l.* each, is	340
	In all 900

Question

Question IV.

If there be Ten Persons of Four several Countries, English, Dutch, French *and* Spaniards, *to pay a Debt of* 1000 *l. so that every* English-man *pays* 50 l. *every* French-man 70 l. *every* Dutch man 130 l. *and every* Spaniard 150 l. *How many is there of each* Country?

THE *Dividend* (according to the former Rule) is 500: Now to find the *Divisor*, take his Sum that pays least (namely 50 *l.*) out of each of the other three, 150, 130, and 70, and the remains will be 100, 80 and 20.

And the *First* and *Last*, viz. 120 for the *Divisor*; the Quotient will be $4\frac{1}{12}$, and the *Spaniards* cannot be more.

Secondly, Add the first and second together, *viz.* 180, the Quotient is $2\frac{14}{18}$; and the *Spaniards* cannot be less. That is,

The *Spaniards* cannot be much more than 4, or less than 2: And therefore, seeing any one Solution will serve,

Let the *Spaniards* be 3, and by that multiply 100, and take the Product out of 500, there remains 200 for a second *Dividend*, which divided by the second remain, 80, the Quotient is $2\frac{1}{2}$, therefore the *Dutchmen* are 2, which multiplied by 80, makes 160; take that out of 200, there remains 40 for a third *Dividend*, which divided by the third remain, 20, the *Quotient* is 2 for the *French-men* also, and consequently the *English-men* must be 3, because all of them are 10: But the *Spaniards* may be also 4 or 2.

Example.

		l.
4	Spaniards, at 150 *l.* each	600
1	Dutch-man, at 130 *l.*	130
1	French-man, at 70 *l.*	70
4	English-men, at 50 *l.* each	200
In all 10		In all 1000

Again,

2	Spaniards, at 150 *l.* each	300
3	Dutch, at 130 *l.* each	390
3	French-men, at 70 *l.* each	210
2	English, at 50 *l.* each	100
In all 10		In all 1000

The Reason why the *Spaniards* and *English*, as also the *Dutch* and *French*, are equal in number, is, because their Payments differ equally from 100, which is the *Mean Sum* with which 10 Men should pay 1000 Pound.

Question

Question V.

If one should buy 12 *Loaves of Bread for* 12 *Pence; so that some might be Twopenny, some Penny, some Halfpenny, and some Farthing Loaves; and it be required to know how many he must buy of each sort?*

NOW, Because of 12 *Loaves* for 12 *Pence*, the *Mean Price* is 1, but one of the Particulars being also 1, there should be no *Penny Loaves*, because there is no difference betwixt the *Mean Price* and *One Penny*.

But it may be found by this Rule to be either

4	Twopenny Loaves,	8	Pence
2	Penny Loaves,	2	Pence
2	Halfpenny Loaves,	1	Penny
4	Farthing Loaves,	1	Penny
In all 12	Loaves	In all 12	Pence

Or else it may be

3	Twopenny Loaves,	6	Pence
4	Penny Loaves,	4	9
3	Halfpenny Loaves,	1	2
2	Farthing Loaves,	0	2
In all 12	Loaves.	In all 12	0 Pence

And so much for this Rule.

CHAP. XIV.

Of Numerical Devices.

If any Person write down any three Digits (as 6, 8, 7*) and under them make Nine other Digits set in Rank and File; and under them you set Nine other Digits in the same order; how to know, and set down the Agregate or total Sum of all the* 18 *Digits and the three first Figures, being added together; before any of the* 18 *Digits be set down.*

LET the three Figures first set down be 6, 8, 7, under them draw a Line, and under the Line make nine Pricks; and under them draw another Line, and under it make nine other Pricks, and a Line under them, as is done in the Margin: This done (always) substract 3 from the Digit standing in the place of Unity, in this case 7, and the remainder is 4; then, upon a piece of Paper (by the bye) write down 6, 8, 4 instead of 6, 8, 7, and to the left hand of it set 3, instead of the 3 which you abated from the 7; so will your Number be 3684; and that will be the Aggregate or Sum of the 18 Digits, and the three uppermost Figures, all being added together: As in the following Example.

6	8	7
·	·	·
·	·	·
·	·	·
·	·	·
·	·	·
·	·	·

Having

687
748
631
254
251
368
745
3684

Having ſet down the three Figures 687, and drawn under them a Line, and made 18 Points and Lines as in the Margin above, bid any Perſon about you write any nine Digits upon the nine upper Pricks, and you will write nine other upon the nine other Pricks below; all which being added together, ſhall make 3684.— Suppoſe (as in the Margin) on the uppermoſt nine Pricks be written 748—631—254; then do you, upon the nine Pricks under, write the Complements of thoſe nine Figures to Nine; as for his three firſt 748 do you write 251, for 631 write 368, and for 254 write 745; all which added together, will make 3684, the Sum you firſt ſet down before any of the 18 figures were written.

422
325
769
134
674
230
865
3419

Example 2. But if the Digit in the place of Unity be leſs than 3, as in theſe three figures 422, you muſt take 3 from 12, and there will remain 9, which ſet down; and (becauſe you borrowed 10) take 1 from 2, and there remains 1, and 4 is the ſame 4, before which put 3 for the 3 which you borrowed, and then the Sum is 3419, which you tell beforehand will be the Aggregate or Sum of all the Addition, as in the Margent is plain.

The like may be done by 4, 5, 6 figures, by obſerving the ſame method, by abating of ſo many Unites from the place of Unity, and reſtoring again in the place of Thouſands or Ten thouſands, *&c.* As in theſe Examples,

6785	25798
2368	20368
7521	42600
8372	53411
6154	27069
	83502
7631	
2478	79631
1627	57399
3845	41588
	72930
46781	16497
	525793

If Three of a ſort of the Five Odd Digit Numbers be ſet in Rank and File, as in the Margent, and it be required of any Five of theſe Odd Digits to make the juſt Number 20, *How may that be done?*

1 1 1
3 3 3
5 5 5
7 7 7
9 9 9

In the Performance of this there is a Falacy; for no five odd Numbers taken howſoever, can make up that Number; wherefore they do invert the Numbers, by turning of the Paper upon which they are written, upſide down, and then the three Nines become three Sixes; and ſo 3 times 6 is 18, and two of the Ones makes 20, as in the other Margent.

6 6 6
L L L
S S S
E E E
1 1 1

GEO-

Geometrical RECREATIONS.

CHAP. I.

Of Geometrical *Definitions*, and Practical *Problems*.

PROBL. I.

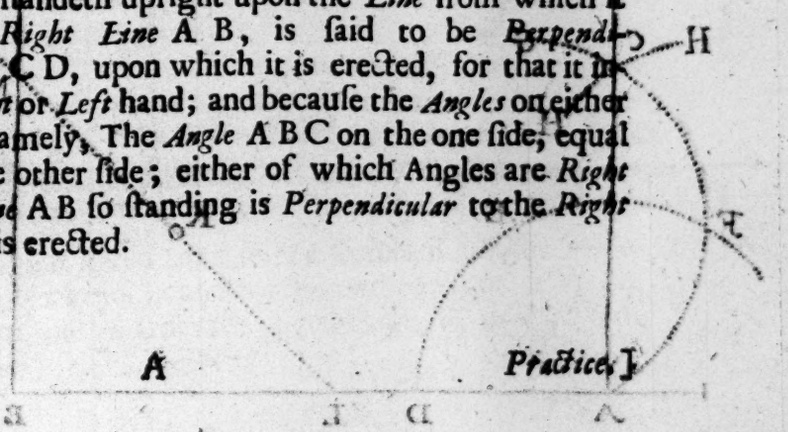
From a Point, *in a* Right Line *given, to erect another* Right Line *which ſhall be* Perpendicular *to the* Right Line *given.*

Definition I.] A *Point* is that which hath no Parts, and is the leaſt imaginary thing that can be conceived in the mind: As this *Point* or *Prick* noted with Z. Z

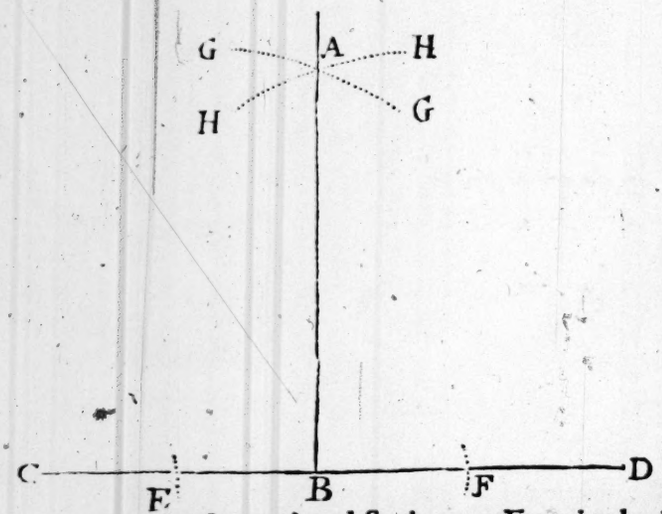
Definition 2.] A *Right Line* is a Line drawn equally between two given *Points*, and is the ſhorteſt diſtance between them, as is this *Line* X Y, which is the ſhorteſt diſtance between the two *Points* X and Y.

X ——— Y.

Definition 3.] A *Right Line* is ſaid to be *Perpendicular* to another *Right Line*, when it maketh the *Angles* on either ſide of the erected Line equal; that is, ſo that the *erected Line* inclines not either to the *Right hand*, or to the *Left*, but ſtandeth upright upon the *Line* from which it is erected: As in the *Right Line* A B, is ſaid to be *Perpendicular* to the *Right Line* C D, upon which it is erected, for that it inclineth neither to the *Right* or *Left* hand; and becauſe the *Angles* on either ſide thereof are equal; namely, The *Angle* A B C on the one ſide, equal to the *Angle* A B D on the other ſide; either of which Angles are *Right Angles*, and the *Right Line* A B ſo ſtanding is *Perpendicular* to the *Right Line* C D upon which it is erected.



Practice.]



Practice.] LET the Right Line given be C D, and let it be required to erect another Right Line which ſhall be Perpendicular thereunto, from the Point B.

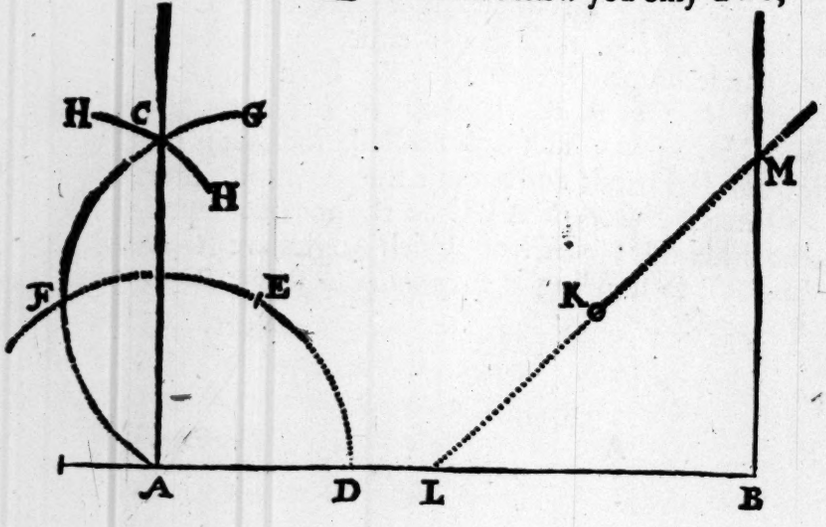
Open your Compaſſes to any convenient ſmall diſtance, and ſetting one Foot in the Point B, with the other make the two ſmall marks E and F, on either ſide, equidiſtant from the given Point B. —— This done, Open the Compaſſes again to any convenient diſtance (greater than the former) and ſetting one Foot in the Point E, with the other deſcribe the obſcure Arch G G (over the given Point B as near as you can gueſs). —— Again; (The Compaſſes being ſtill open at the ſame diſtance) ſet one Foot in the Point F, and with the other deſcribe another obſcure Arch H H, croſſing the former in the Point A: So is A a Point found, through which if you draw a Right Line from the given Point B, that Right Line A B, ſhall be Perpendicular to the given Right Line C D, and from the Point B, which was required to be done: And the Angle A B D, on the one ſide thereof, is equal to the Angle A B C, on the other ſide; and both of them are Right (or Square) Angles.

¶ *Note*, An Angle is always ſignified by Three Letters, as a Point is by One; the middlemoſt of which Three repreſenteth the Angular Point, as in this Caſe the Letter B. —— B being the Angular Point, and the Lines A B and B C the ſides containing the Angle B.

PROBL. II.

How to erect a Perpendicular, when the given Point is in (or near) the end of the given Right Line.

Practice.] THERE are ſeveral ways to effect this; of which I will here ſhew you only Two, as being the beſt.



Let A B be a Line given, and from the Point A, towards the end thereof, let it be required to erect the Perpendicular A C. —— Firſt, Open the Compaſſes to any ſmall diſtance, and ſetting one Foot in the given Point A, with the other deſcribe an Arch (or part) of a Circle, F E D, —And (keeping

ing the Compasses still at the same distance) set one Foot in D, and make a mark in the Arch at E, and setting one Foot in E, with the other describe another Arch of a Circle A F G, crossing the first Arch in F. Again, Set one Foot in F, and with the other describe the small Arch H H, crossing the former in the Point C, through which Point C, draw a Line from the given Point A, and that Line shall be Perpendicular to the given Line A B, and drawn from the Point A, as was required.

A Second Way.

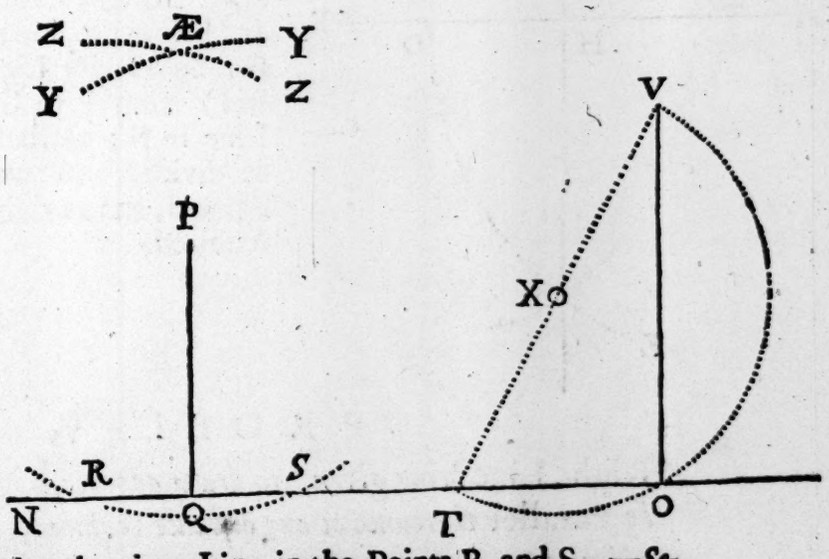
Let B (the extreme end of the given Line) be the Point given. ——Open the Compasses to any convenient distance, and setting one Foot in B, pitch down the other Foot at adventure in the Point K; so one Foot resting in K, turn the other about till it cross the given Line A B in L, and draw the right Line L K at length, and set the same distance K, L, (at which the Compasses already stand) from K to M; so a Line drawn from B through M, shall be Perpendicular to A B, and from the given Point B, as was required.

PROBL. III.

From a Point *above, to let a* Perpendicular *fall upon a Right Line under it.*

IN this there are two Cases, ——*First*, when the given Point above, is over (or near) the *middle* of the given Line. And —— *Secondly*, When the given Point is just (or near) over the *End* of the given Line.

Practice.] IN the first Case, Let N O be a right Line given, and from the Point P (over it) let it be required to let fall the Perpendicular P Q. *First*, Open the Compasses to any convenient distance greater than the distance between P and Q; and setting one Foot in P, with the other draw an obscure Arch of a Circle, cutting the given Line in the Points R and S. —*Secondly*, Divide the Space between R and S into two equal parts in the Point Q; so a Right Line drawn from the given Point P, to the Point Q, shall be Perpendicular to the Line N O.

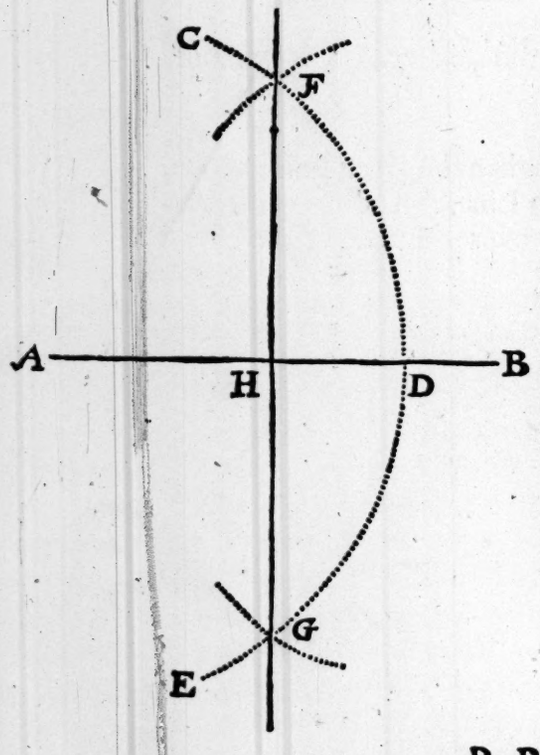


Note, To avoid the dividing of the ſpace R S into two equal parts, to find the Point Q; if you have room (either above or beneath your Line) you may ſet one Foot in S, and opening the Compaſſes to any convenient diſtance, make the Arch Y Y, and removing the Compaſſes to R, make the Arch Z Z, croſſing the former in Æ, ſo a Line drawn through Æ and P, ſhall be Perpendicular to N O.

In the ſecond Caſe, Let V be the Point given; —— *Firſt*, From any part of the given Line N O, as from T, draw a Right Line to the given Point V, which divide into two equal parts in X. —— Secondly, Set one Foot of the Compaſſes in X, and with the diſtance X T deſcribe the Arch (or Semicircle) V O T, cutting the given Line N O in O, ſo ſhall O be the Point, to which from the given Point V, if you draw a right Line, it ſhall be a Perpendicular to the Line N O, and from the Point V, as was required.

PROBL. IV.

To divide a Right Line into Two equal Parts, and at Right (or Square) Angles.



LET A B be a Line given, to be ſo divided. —— Take in your Compaſſes the Length of the given Line A B; or (if that be too long) to any other diſtance greater than half the length thereof; and ſetting one Foot in the end A, with the other draw the Arch C D E. —— Secondly, (the Compaſſes being open at the ſame diſtance) ſet one Foot in B, and with the other croſs the former Arch (both above and below the Line) in the Points F and G: So a Right Line drawn (or a Ruler laid) from F to G, ſhall cut the given Line in H; ſo ſhall the given Line A B be divided into two Equal Parts in the Point H, and at right Angles, which was required.

PROBL. V.

A Right Line *being given, to draw another* Right Line *which ſhall be* Parallel *thereunto at any diſtance required, or through any Point aſſigned.*

Definition.] *Right Lined Parallels* are ſuch Right Lines, which being drawn upon the ſame Plain, and infinitely extended on either ſide, would never concur or meet; but always, in all parts, retain

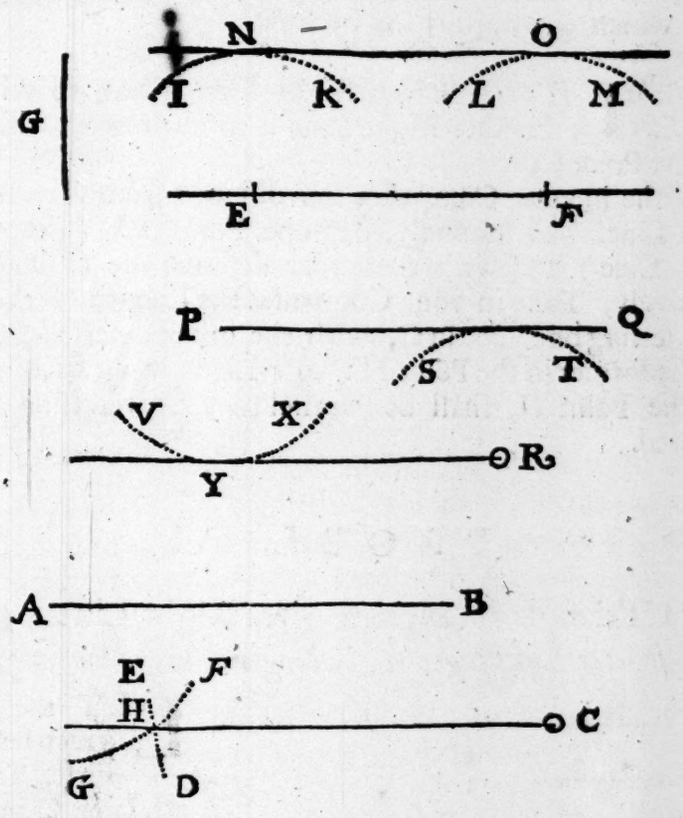
retain an equal distance; and such are these two Right Lines, B C, and B D.

In the describing or drawing of *Parallel Lines*, there may fall out *Two Cases*, or *Varieties*. As,

1. It may be required to draw a Line *Parallel* to another Line, at a certain given distance. Or,
2. The *Parallel* may be required to be drawn through an assigned *Point*.

And of this kind there are two *Varieties*. For,

1. The given *Point* may be *over* or *under* the given Line. Or,
2. It may be *Oblique* to the Line given.



Practice of the First Case. LET E F be a Right Line given, and let it be required to draw another Right Line Parallel thereunto at the distance of the Length of the Line G.

First, Take in your Compasses the Length of the given Line G, and set one Foot in the Point E, (or in any other part of the given Line towards the end thereof) describing the small obscure Arch I K.——Then move the Compasses to F, (near the other end of the Line) and describe another small and obscure Arch L M. ——Lastly, Lay a Ruler to the very top of these two *Arches*, so that the Ruler do not cross, but justly *touch* either of them: Then if by the edge of the Ruler you draw a Right Line N O, it shall be Parallel to the given Line E F, and at the distance of the Length of G; which was to be done.

Pra-

Practice of the Second Case. LET P Q be a Right Line given, unto which you are to draw another Right Line Parallel, which *Parallel* muſt paſs through the given Point R.

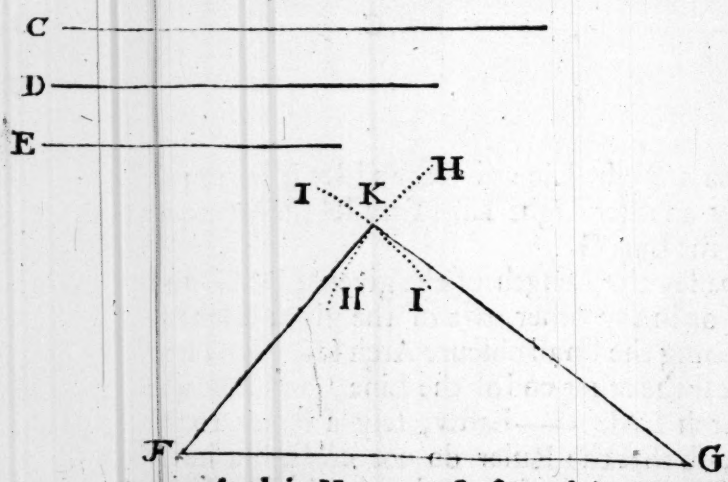
Firſt, Set one Foot of your Compaſſes in R, and with the other take the neareſt diſtance to the given Line P Q, which is done by opening or ſhutting of the Compaſſes, till the moveable Point of them do only touch the given Line P Q, and there deſcribe the Arch S T.—— *Secondly*, The Compaſſes at the ſame diſtance, ſet one Foot in P, (or any Point towards the other end of the given Line) and with the other deſcribe the Arch V X. ——*Laſtly*, Through the given Point R, and the very top of the Arch V X at Y, draw the Right Line R Y, and it ſhall be Parallel to the given Line P Q, and ſhall alſo paſs through the given Point R; which was required to be done.

Practice of the Third Caſe. LET A B be a Right Line given, to which another Parallel Right Line is to be drawn, which ſhall paſs through the Point C.

Firſt, Take in your Compaſſes the diſtance from C to B, the end of the given Line. ——Secondly, Set one Foot in A (the other end of the given Line) and with the other deſcribe the obſcure Arch F G. ——Thirdly, Take in your Compaſſes the Length of the given Line A B, and ſetting one Foot in C, with the other deſcribe the Arch D E, croſſing the former in the Point H: So a Line drawn from the Point C, through the Point H, ſhall be Parallel to the given Line A B; which was required.

PROBL. VI.

To make a Triangle *of any three given* Right Lines, *provided that the two ſhorter Lines together, be longer than the longeſt given Line.*

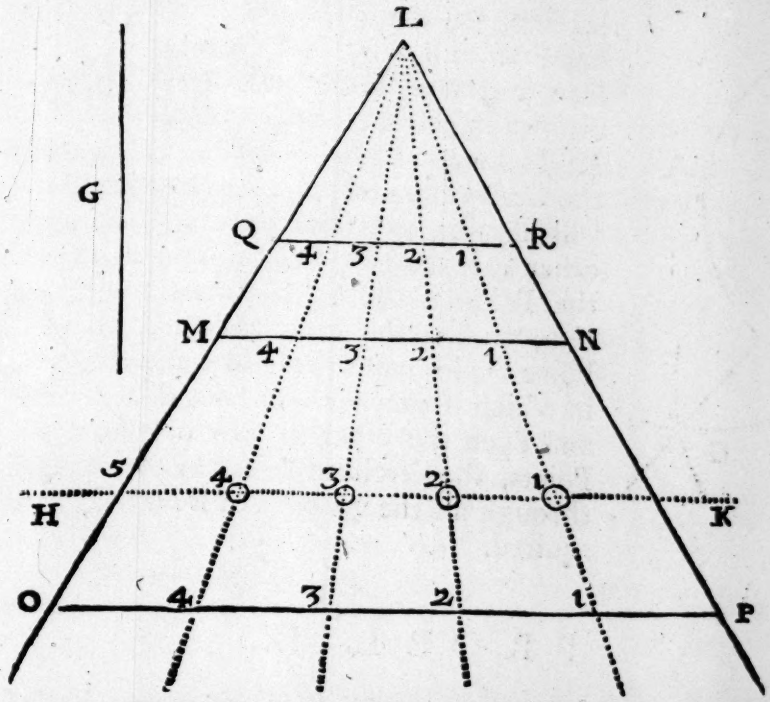


LET the three Lines given be C, D, and E. ——Firſt, Take the longeſt Line C in your Compaſſes, and make the Line F G equal thereunto. Secondly, Take in your Compaſſes the Line D, and ſetting one Foot in G, deſcribe the Arch H H. ——Thirdly, Take the Line E, and ſetting one Foot in F, deſcribe the Arch I I, croſſing the former Arch in K. ——Laſtly, draw the Right Lines K F and K G, and they ſhall conſtitute a Triangle, whoſe three ſides ſhall be equal to the three given Lines, C, D, E; which was required.

PROBL.

PROBL. VII.

To divide a Right Line given, into any number of Equal Parts.



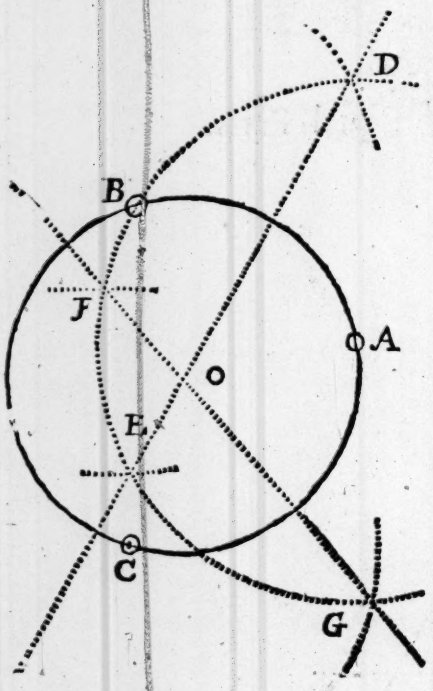
Practice.] LET the Right Line G be the Line given, to be divided into 5 equal parts. First, Upon any Line drawn at pleasure, as the obscure Line H K, with your Compasses opened to any small distance, run along the same Five times at the figures 0, 1, 2, 3, 4, 5. (because the Line given is to be divided into 5 parts). —— Then take in your Compasses the distance from 0 to 5, and with that distance make the *Equilateral Triangle* 5 L 0. —— Again, Take in your Compasses the given Line G, and set that distance from L to M and N, drawing the Line M N (which will be equal to the given Line G.) —— Lastly, Lay a Ruler from L, to the Points 1, 2, 3, and 4, in the Line H K, and the Ruler will cut the Line M N in the Points 1, 2, 3, 4; dividing the same Line, equal to G, into 5 equal parts; as was required.

¶ *Note*, This *Equilateral Triangle* thus made, is capable of dividing any other Line into 5 equal parts, be it either greater or lesser than this given Line G, if the length thereof be set from L on both sides of the Triangle, and drawing a Line from either side: As the Line Q R shorter than M N; or the Line O P longer than M N: All which Lines are divided into Five Equal Parts.

PROBL. VIII.

To any three Points *given, which lye not in a Right* Line, *to find the* Center *of a* Circle, *whose* Circumference *being described, shall pass through those three given* Points.

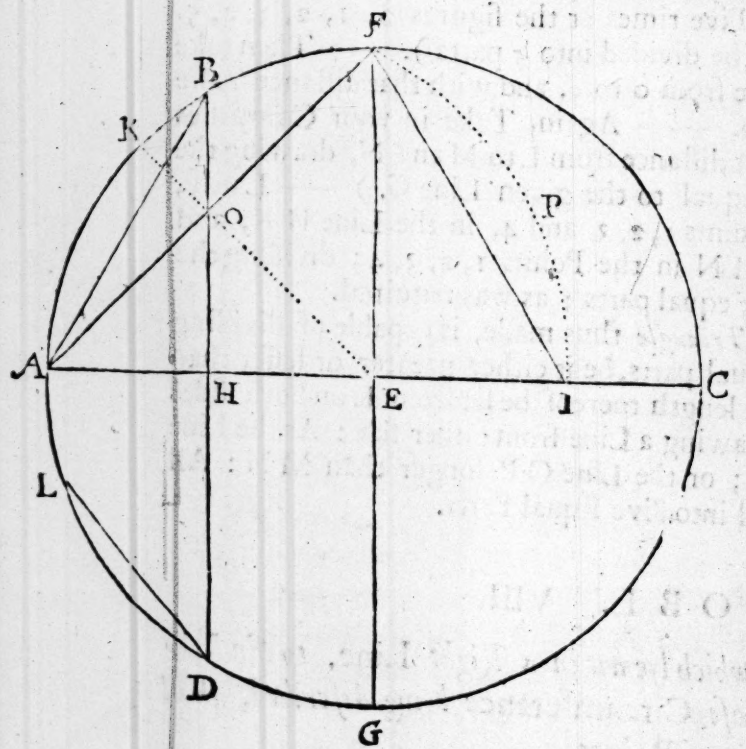
Let



LET the three given Points be A B C. *First*, Set one Foot of your Compaſſes in one of the given Points A, and extend the other Foot to B, another of the given Points, and draw the Arch of a Circle G E D. ——*Secondly*, (the Compaſſes being kept at the ſame diſtance as before) Set one Foot in B, and with the other Foot croſs the former Arch G E D, with two ſmall Marks or Arches in the Points D and E, and draw the Right Line D E. ——*Thirdly*, Set one Foot of the Compaſſes in the third given Point C, (they ſtill keeping the ſame opening) and with the other Foot croſs the firſt drawn Arch G E D, in the Points F and G, and draw the Right Line F G, croſſing the former Right Line D E in the Point O, ſo ſhall O be the Centre of a Circle; in which if you ſet one Foot of the Compaſſes, and open the other to any of the three given Points, the Circle ſo deſcribed ſhall paſs directly through all the 3 Points A B and C, as was required.

PROBL. IX.

To divide the Circumference of any Circle into any number of equal Parts, not exceeding Ten.



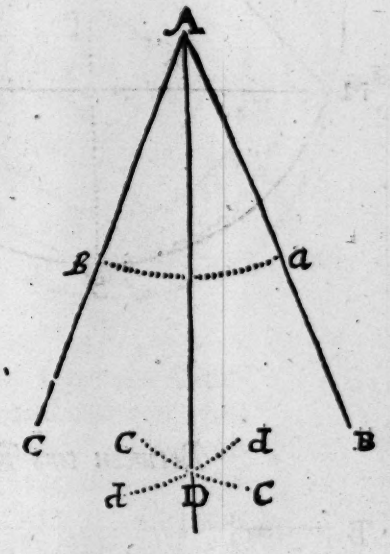
THE Circle being deſcribed upon the Centre E, ——*Firſt*, Draw the Diameter A E C, which divides the Circumference into 2 equal parts. ——*Secondly*, Take the diſtance A E in your Compaſſes, and ſet it from A to B, and to D, and draw the Line B D, which ſhall divide the Circumference into 3 equal parts. ——*Thirdly*, Divide the Diameter A C in two equal parts at right Angles, by the Diameter F G, and draw the Line A F, which ſhall divide the Circle into 4 equal parts. ——*Fourthly*, Set one Foot of the Compaſſes in H, and extend the other to F, and there deſcribe the Arch F P I, and draw the Line F I, which ſhall divide

vide the Circle into 5 equal parts. —— *Fifthly*, The lines E A, E C, E F, or E G, will either of them divide the Circle into 6 equal parts. ---*Sixthly*, The lines H B or H D will divide it into 7 equal parts. —— *Seventhly*, Through the point O, (where the lines H B and A F do interſect) draw the line E K, and a line drawn from A to K ſhall divide the Circle into 8 equal parts. —— *Eighthly*, Divide the Arch B A D into three equal parts, and ſet one of them from D to L, ſo ſhall D L divide the Circumference into 9 equal parts. —— *Laſtly*, The Line E I will divide the Circle into 10 equal parts; and ſo is the Problem performed as was required.

PROBL. X.

To divide a Right Lined Angle into two equal Parts.

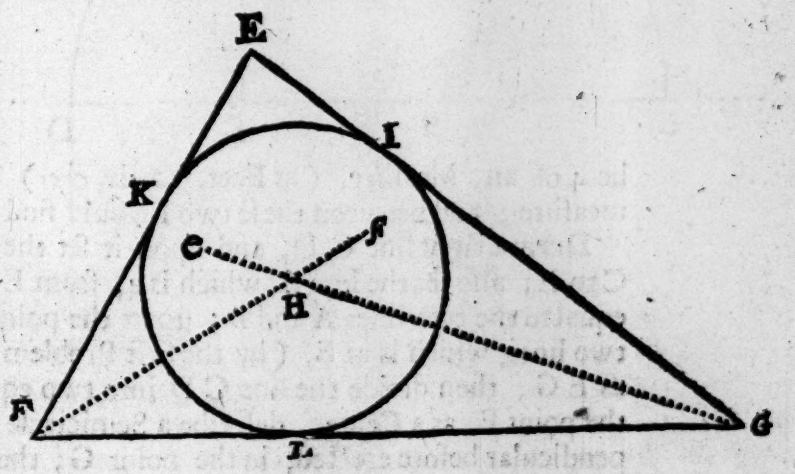
LET the Angle given be C A B; upon the Angular Point at A, with any diſtance of the Compaſſes, deſcribe an Arch a b, then ſet one foot in a, and with the other deſcribe the Arch c c, and ſet one foot in b, and with the other deſcribe the Arch d d, croſſing the former in the point D, through which draw the Line D A, ſo ſhall the Angle C A B be divided into two equal parts by the Line D A.



PROBL. XI.

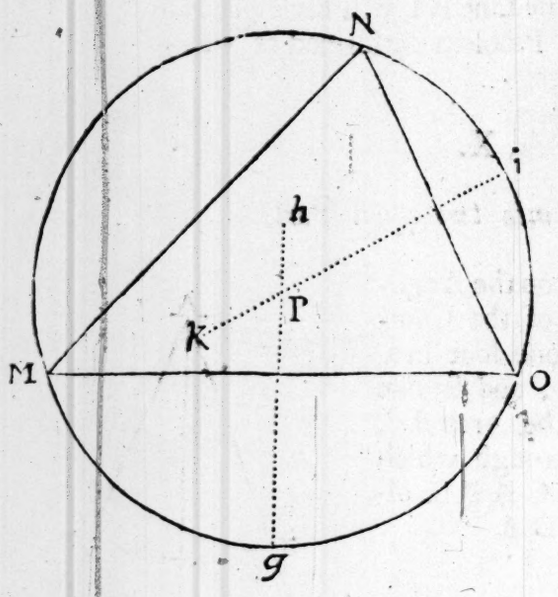
How to deſcribe a Circle within a Triangle, ſo that the Circumference of the Circle ſhall touch all the three ſides of the Triangle.

BY the laſt Problem divide any two of the Angles of the given Triangle into two equal parts, as the Angles E F G, by the line F f, and the Angle E G F by the line G e, cutting each other in H; ſo is H the Center, upon which if you deſcribe the Circle, it ſhall touch the three ſides of the Triangle in the Points I K and L.



PROBL. XII.

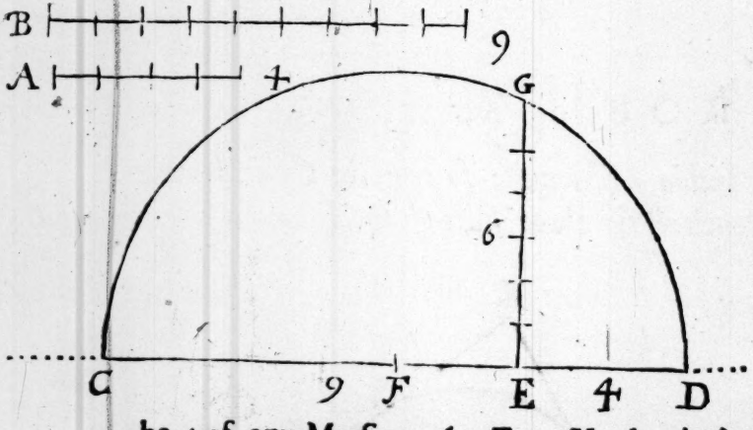
To describe a Circle about a Triangle, the Circumference whereof shall touch all the Angles of the Triangle.



LET the Triangle given be M N O, by the fourth Problem divide any of the two sides of the Triangles into two equal parts, at Right Angles, as the side M O, by the line g h, and the side N O, by the line i k, crossing each other in P; so is P a Center, upon which if you describe a Circle, it shall touch all the Angles of the Triangle.

PROBL. XIII.

Between two Right Lines given, to find a Mean Proportional Line.



Definition. A *Mean Proportional* Line, is such a Line, whose length being multiplied in it self, the Product shall be equal to both the Product of the other two Lines, between which it is a *Mean Proportional.*

Let A and B be two lines, of which let A be 4 of any Measure, (as Feet, Yards, *&c.*) and let B be 9 of the same measure; and between these two I would find a Mean Proportional.

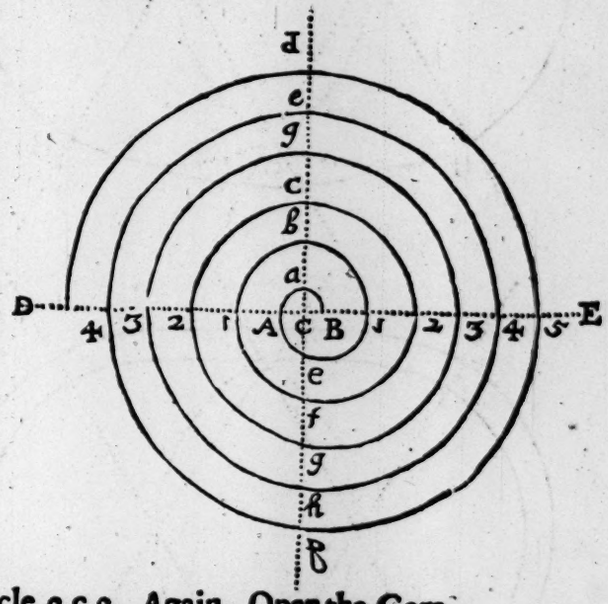
Draw a right line C D, and upon it set the line B, which is 9, from C to E; also set the line A, which is 4, from E to D; so is the line C D, equal to the two lines A and B; upon the point of the joining of these two lines, which is at E, (by the first Problem) erect a Perpendicular, as E G; then divide the line C D into two equal parts in F, and upon the point F, as a Center, describe a Semicircle C G D, cutting the Perpendicular before erected, in the point G; then is the line E G a Mean Proportional Line between the two lines A and B, and being measured, will

will be found to be 6; which being multiplied in it self produceth 36; and so will 9 multiplied by 4 produce the same.

PROBL. XIV.

How to make a Serpentine or Spiral Line, whose thrids shall be distant one from the other, the length of a Line given.

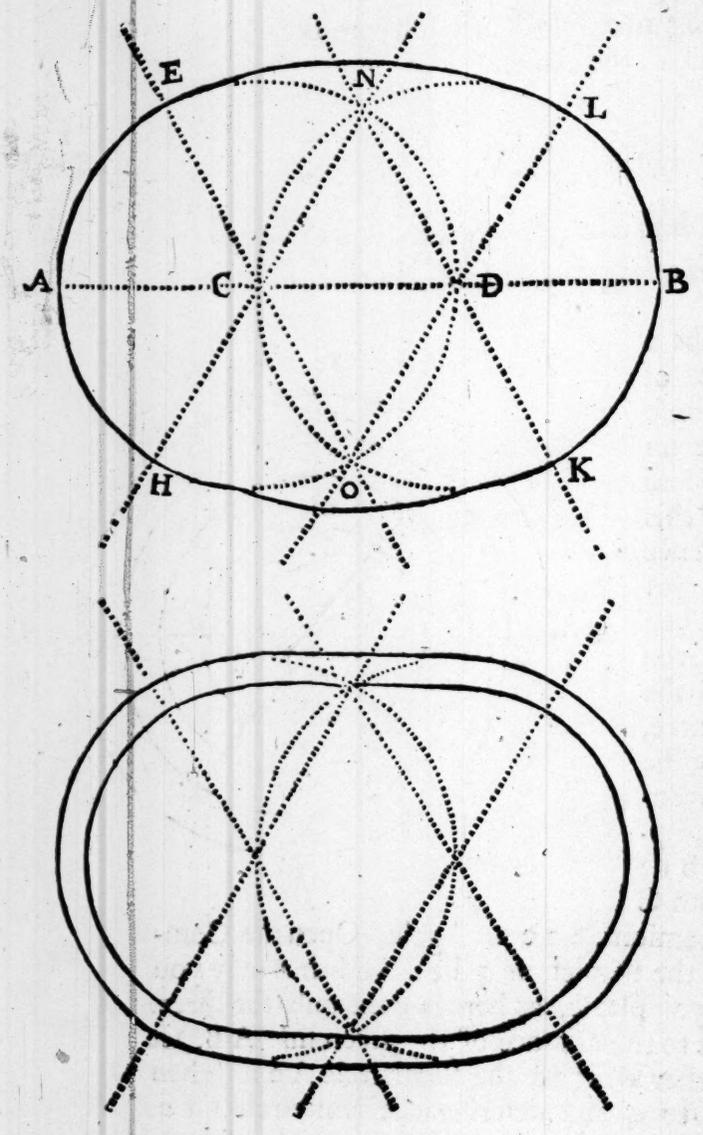
LET it be required to describe a Spiral Line, at the distance of the given Line A B. —— Continue the given line A B at pleasure on both sides, from A towards D, and from B towards E; and upon this continued Line (with the distance A B) set off the distances 1, 2, 3, *&c.* on either side from A and B. Then (by Probl. 1.) divide the given line A B into two equal parts in the point C, and upon C, as a Centre, at the distance C A or C B, describe the Semicircle A a B: Then opening the Compasses from C to 1, upon C describe the Semicircle 1 b 1; also extend the Compasses from C to 2, and upon C describe the Semicircle 2 c 2. Again, Open the Compasses from C to 3, and describe the Semicircle 3 d 3: And thus may you do for as many Revolutions as you please, as here is done only for three.



Again, Open the Compasses to the distance of the given line A B, set one foot in B, and with the other describe the Semicircle A e 1. Then extend the Compasses from B to 1, and describe the Semicircle 1 f 2. Lastly, Extend the Compasses from B to 2, and describe the Semicircle 2 g 3. So is your Spiral Line finished, consisting of Three Revolutions only, but may be continued to as many as you please.

PROBL. XV.

How to describe an Oval, the length thereof being given.

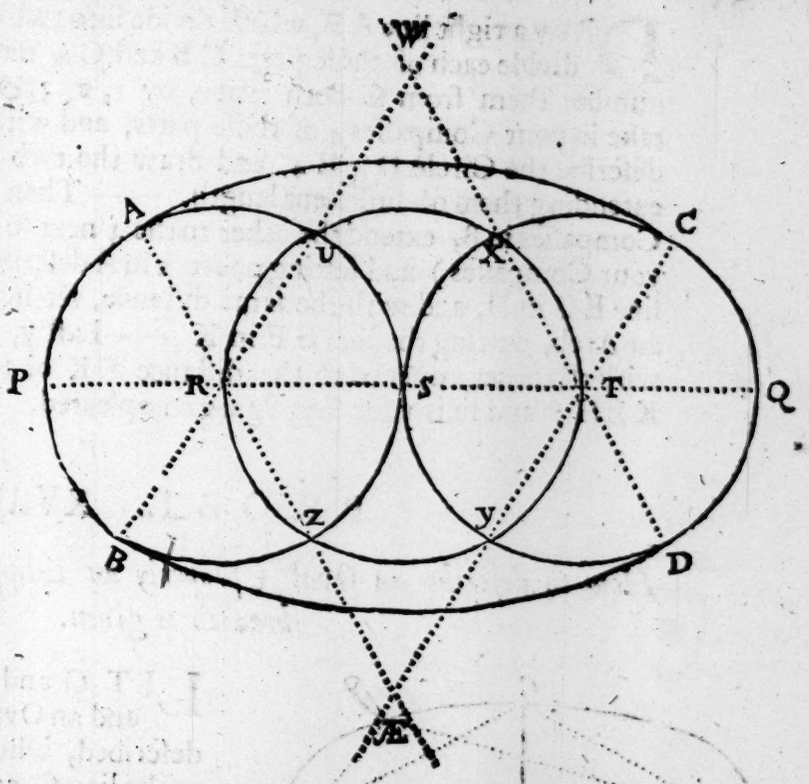


LET A B be a line given, and of that length let it be required to describe an Oval. —— Divide the line A B into three equal parts in the Points C and D. Then upon C as a Centre, at the distance C A, describe the obscure Circle E A H D; and upon D, at the distance D B, describe another occult Circle L B K C, crossing the former in the points N and O. This done, through O and C draw an obscure line cutting the Circle in E: Also through O and D draw another obscure line, crossing the other Circle in L. In like manner, through the points N and C, draw an obscure line cutting the Circle in H, and through N and D another cutting the Circle in K. Laltly, Upon O, at the distance O E or O L, describe the Arch E L, and (with the same distance) upon N, describe the Arch H K. The two ends of the Oval H A E, and L B K are parts of the two Circles first of all described; and so is your Oval compleated.

If you would draw an Oval Parallel to an Oval, you must describe it upon the same Centres, and to touch the lines O E, O L, *and* N H, N K, *extended as is done in the Figure.*

PROBL. XVI.

At the length of a line given, to describe an Oval, which shall be in proportion to the length thereof, narrower than the former.

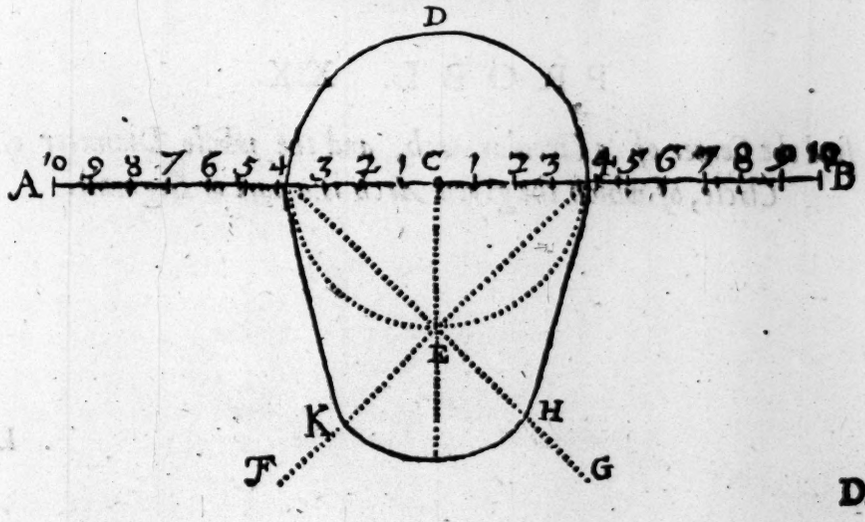


LET the given line be P Q —— Divide the line P Q (by the 13th. Probl.) into four equal parts in the points R S and T, and upon those three points, as three Centers, at the distance R P, or T Q, describe three Circles, the middlemost of which, described upon the Center S, cutting the other two in the points U X Y Z. This done, through the points R Z draw an obscure line at length, and cutting one of the Circles in A; also draw a line through T and Y, crossing the other Circle in C, and the line A Z extended to Æ. —— Again, through T and X draw an obscure line at liberty, cutting one of the Circles in D, and another obscure line through R and V, cutting the other Circle in B, and being extended through the former occult line in W. Lastly, Upon the Point Æ, at the distance Æ A, describe the Arch A C, and (with the same distance) one foot placed in W, with the other describe the Arch B D, compleating the Oval; the ends A P B, and C Q D, being parts of the two outermost Circles first described.

If you would have an Oval Parallel to this, it must be described upon the same Centres R T W *and* Æ.

PROBL. XVII.

How to make an Egg Form.

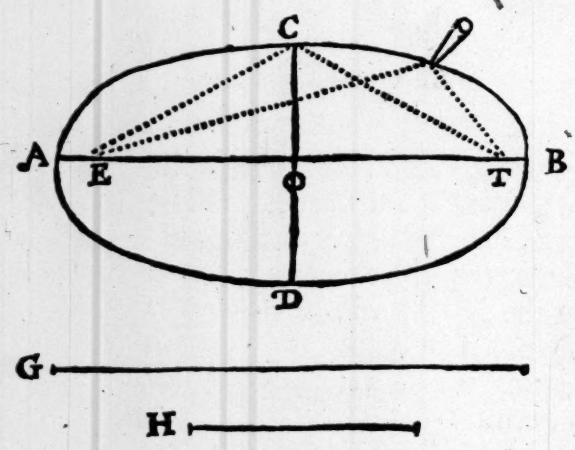


Draw

DRaw a right line A B, which divide into two equal parts in C. Then divide each of thoſe parts C B and C A into 10 equal parts, and number them from C both ways, by 1, 2, 3, *&c.* to 10. —— Then take in your Compaſſes 4 of thoſe parts, and with that diſtance, upon C, deſcribe the Circle D 4 E 4, and draw the two lines 4 E F, and 4 E G, extending them of ſufficient length. —— Then ſetting one foot of the Compaſſes in B, extend the other to the 4 next to A (or take 14 parts in your Compaſſes) and ſetting one foot in A deſcribe an Arch till it cut the line E G in H, and with the ſame diſtance, ſetting one foot in B, deſcribe an Arch, cutting the line E F in K. ——Laſtly, Set one foot in E, and with the other, opened to the diſtance E K or E H, deſcribe the Arch K L H; and ſo is your Egg figure compleated.

PROBL. XVIII.

How to deſcribe an Oval (properly an Ellipſis) whoſe length and breadth is given.



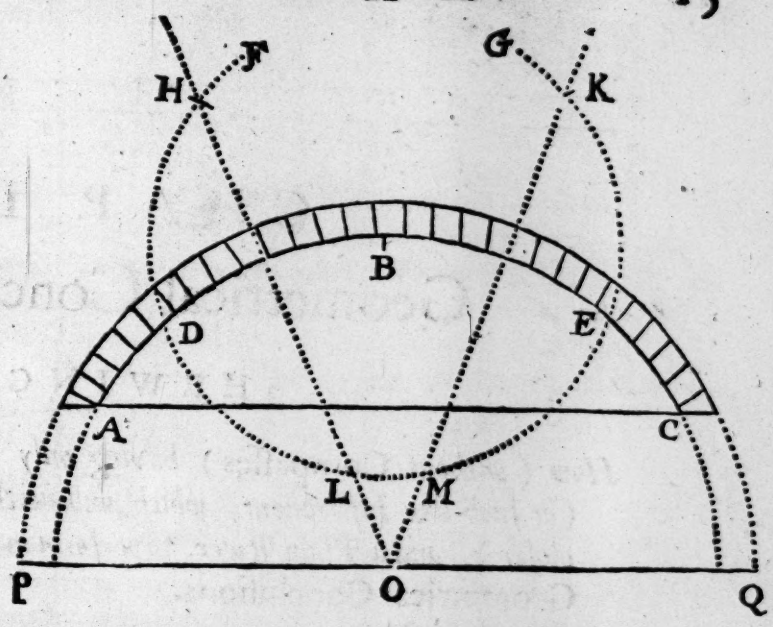
LET G and H be two lines given, and an Oval (or Ellipſis) is to be deſcribed, whoſe length is to be equal to the line G, and its breadth to the line H. ——Draw a line A B equal to the line G, and (by Probl. 1.) divide it into two equal parts at Right Angles in the point O, and make the line C D equal to the given line H. This done, take in your Compaſſes the length O A or O B, (half the length of the longeſt Diameter) and ſetting one foot in C, with the other croſs the Diameter A B, in the points E and F; which two points are the two Centers of the Oval, (properly the two Focus of the Ellipſis): Upon which two points, let two Pins be faſtned, and about them put a String, whoſe ends faſten together in the point C; this ſtring being moved about the two Pins with a Point or Tracer, will deſcribe the Oval (or Ellipſis) A C B D, whoſe length and breadth ſhall be equal to the two lines G and H.

PROBL. XIX.

To find the Center of a Circular Arch, and the whole Diameter of the Circle, of which the given Arch is a part or Segment.

LET

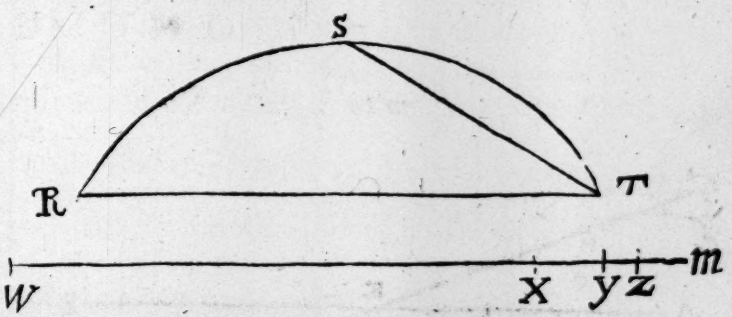
LET A B C be the inside of a Circular Arch, of which the whole Diameter and the Center is required to be found. —— Make choice of a Point towards the Top or Crown of the Arch, as B, in which set one foot of the Compasses, opening them to any competent distance, as from B to D or E, and with that distance describe an obscure Circle F D E G, crossing the Arch in D and E. Then (the Compasses being still open to the same distance) set one foot in D, and with the other make the marks F and L in the obscure Circle: Also set one foot in E, and with the other make the marks K and M in the same obscure Circle. This done, draw an occult line at pleasure, through the points H and L, and another through M and K, crossing the former in the point O: So shall O be the Center of the Arch, and O A, O B, or O C, the Semidiameter of the Circle of which the Arch is a part. The whole Diameter may be found by continuing the sides of the Arch from the Springs A and B, downwards towards P and Q: Then a line drawn through the Center O, parallel to A B, till it meet with the Arch continued on both sides at P and Q, shall be the Diameter of the Arch.



PROBL. XX.

The Arch or Segment of a Circle being given, to find a Right Line (the neareſt) *equal thereunto.*

THE Arch R S T is a Segment of a Circle, unto which a Right line is to be made equal. —From the two extream points of the Arch, draw a right line R T; then divide the Arch line into two equal parts in S, and draw the right line S T. This done, draw a right line at pleasure, as W m; upon which line set the length of the line R T, from W to X: Also take the line T S in your Compasses, which at twice will reach from W to Y; divide the line of difference X Y into 3 equal parts, and one of those parts set from Y to Z; so shall the right line W Z, be the neareſt right line that can Geometrically be found, equal to the Arch Line or Segment R S T.



CHAP.

CHAP. II.

Geometrical Concluſions.

SHEWING

How (without Compaſſes*) having only a common* Meat-Fork*, (or ſuch like Inſtrument, which will neither open wider, nor ſhut cloſer), and a Plain Ruler, to perform many pleaſant and delightful* Geometrical Concluſions.

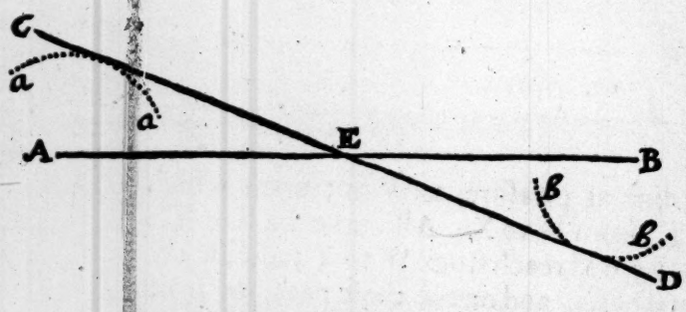
THE *Compaſſes* is an Inſtrument known to all men; and the Invention of them is attributed to *Talus* the Nephew of *Dædalus*; as appears by *Ovid Met. Lib.* 8. Where he ſays,

——*Et ex uno duo ferrea brachia nodo*
Junxit, ut equali ſpacio diſtantibus illis
Altera pars ſtaret, pars altera duceret orbem.
Which *Sandys* thus Tranſlates,
And two ſhankt *Compaſſes*, with Rivet bound,
Th'one to ſtand ſtill, th'other to turn round
In equal diſtance.

But one *John Baptiſt* an *Italian*, as alſo one *Jeronymus Cardanus*, a famous Mathematician, have performed and demonſtrated all (as is related) *Euclia*'s Elements, without Compaſſes; it is true, many Concluſions may be done without them, ſome whereof ſhall here follow:

CONCLUS. I.

How to divide a Right Line into two equal Parts.



LET AB be a right line given to be divided into two equal parts; ſet one point of the Fork in A, and with the other draw the ſmall Arch a a; then ſet one end of the Fork in B, and with the other deſcribe the Arch b b; then lay a Ruler to theſe two Arches a a and b b, ſo that the Ruler may only touch the tops of the Arches; then by the ſide of the Ruler draw the line C D, which will divide the given line A B into two equal parts in the point E; which was to be done.

CON-

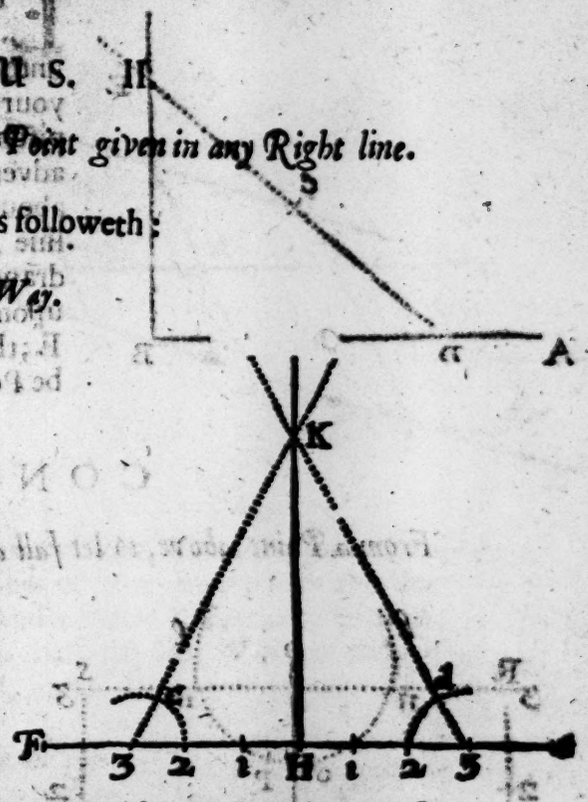
CONCLUS. II.

How to erect a Perpendicular from a Point given in any Right line.

THis may be done several ways; as followeth:

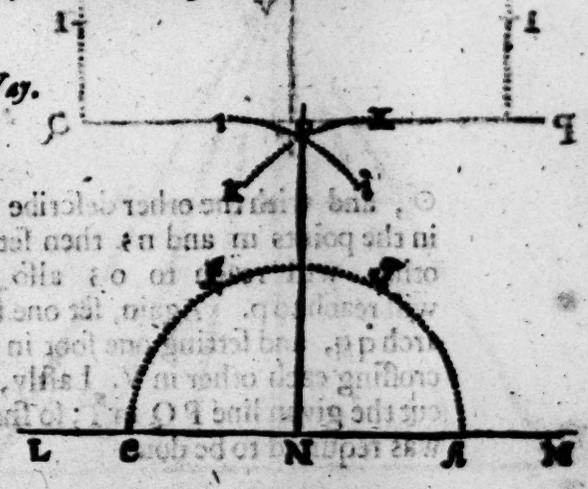
The First Way.

Let F G be a right line, and H a point given therein, from whence it is required to erect a Perpendicular: Set one point of the fork in the given point H, and run it over upon the given line two, three, or four times, on either side of the given point H, at the Points 3 2 1, and 1 2 3; then upon the points 3 and 3, describe the two Arches 3 c, and 3 d, setting the distance of the fork upon both the Arches from 2 to c, and from 2 to d: Then through the points 3 and c, and 3 and d, draw the two right lines 3 c, and 3 d, cutting one another in the point K; so a right line being drawn from H to K, shall be Perpendicular to F G; which was required to be done.



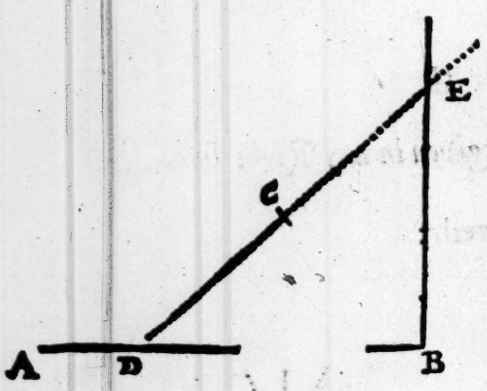
Another Way.

Let L M be a right line given, and N a point therein, from whence a Perpendicular is to be erected. Set one foot of the fork in the given point N, and with the other describe the Semicircle e f g h; then setting one foot in e, the other will reach to f; and setting one foot in h, the other will reach to g; then setting one foot in f, with the other describe the small arch i i, and and setting one foot in g, with the other describe the small arch k k, crossing the former arch in O, from the given point N draw the line N O, which shall be perpendicular to the line L M; which was to be done.



CONCLUS. III.

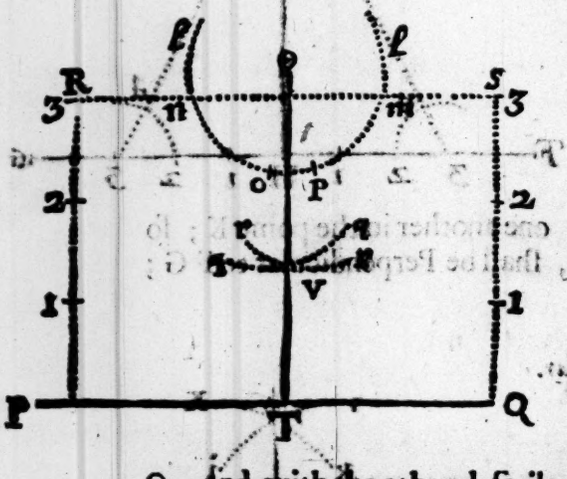
Upon the End of a Right Line given, to Erect a Perpendicular.



LET the given line be A B, and let it be required to erect a Perpendicular thereunto upon the end B. ——Set one point of your fork in the end B, and keeping it there, pitch the other down upon the Paper at all adventures in C, and upon C turn the fork about till the other point of it touch the given line A B in D; lay a Ruler from D to C, and draw an obscure line by the side thereof, and upon it set the distance of your fork from C to E; then a line drawn from B through E, shall be Perpendicular to the given line A B.

CONCLUS. IV.

From a Point above, to let fall a Perpendicular upon a ground line below.



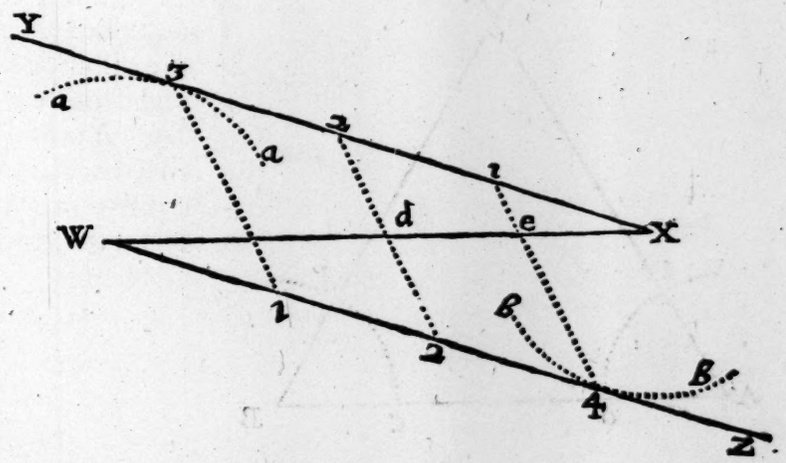
LET ⊙ be a point aloft, from which point it is required to draw a line which shall be Perpendicular to the right line P Q.

Upon the ends or towards the ends of the given line, erect two Perpendiculars as P R and Q S, upon which run over your fork as many times as you can, as at 1 1, 2 2, 3 3, *&c.* so far, till you come to have your given point to be nearer to the Parallel Line 3 3, than is the distance of your fork; Then set one foot of the fork in the given point ⊙, and with the other describe the Arch l l, cutting the Parallel 3 3, in the points m and n; then set one foot of the fork in m, and the other will reach to o; also set one foot in n, and the other will reach to p. Again, set one foot in o, and with the other describe the arch q q, and setting one foot in p, with the other describe the Arch r r, crossing each other in V. Lastly, draw the line ⊙ V, extending it till it cut the given line P Q in T; so shall ⊙ T be Perpendicular to P Q; which was required to be done.

CONCLUS. V.

To divide a Right Line into any number of equal parts.

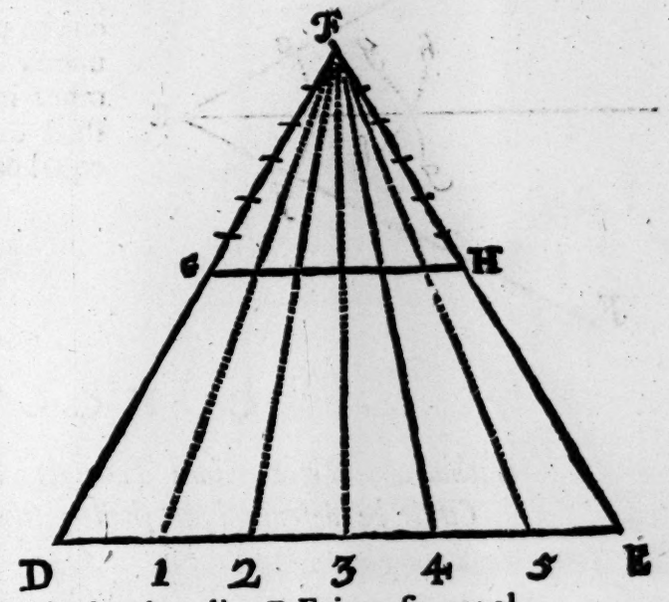
LET



LET W X be a right line given to be divided into 4 equal parts.

First, Upon the two points W and X, describe the two Arches a a above, the other b b below; then lay a Ruler to W, so as to touch the top of the Arch b b, and draw the line W Z, laying a Ruler to the top of the Arch a a, and the Point X, and draw the line X Y; then from W and X run the distance of your fork three times (which is one less than the number of parts into which the line is to be divided) at the points W 1, 2, and 3; and X 1, 2, and 3. Lastly, draw the lines 1 3, 2 2, and 3 1; so shall the line W X be divided into four equal parts, in the points c, d, and e.

Another Way to do the same.

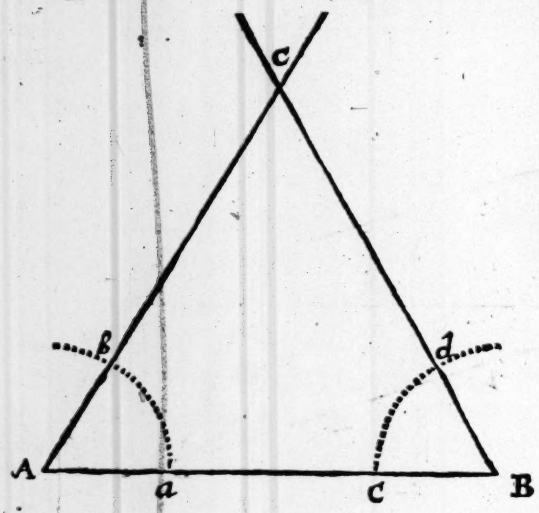


LET the given line be D E, and let it be required to divide the same into six equal parts.

First, By the following Conclusion, upon the given line D E, describe the Equilateral Triangle D E F; then with the distance of your fork run six times upon the sides F D and F E, at the points 1, 2, 3, 4, 5 G, and 1, 2, 3, 4, 5 H, and draw the line G H, upon which run also six times the distance of your fork, at the points a, b, c, d, e; then laying a Ruler upon F, and every one of the divisions a, b, c, d, e, draw the obscure lines F a, F b, F c, F d, F e, which extended, will divide the given line D E into six equal parts, in the points 1, 2, 3, 4, 5.

CONCLUS. VI.

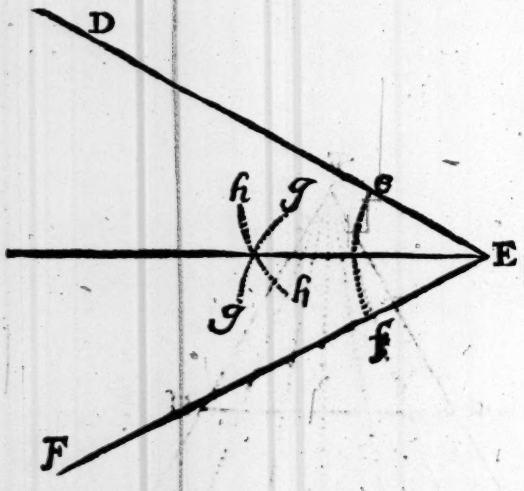
Upon a Right Line given, to make an Equilateral Triangle, or a Triangle of three equal sides.



LET A B be the line given, upon the points A and B; with your fork describe two Arches, and upon them set the distance of your fork from a to b, and from c to d; then draw the two lines A b and B d, extending them till they concur in C; so shall A B C be an equilateral Triangle, all whose sides are equal to the given line A B.

CONCLUS. VII.

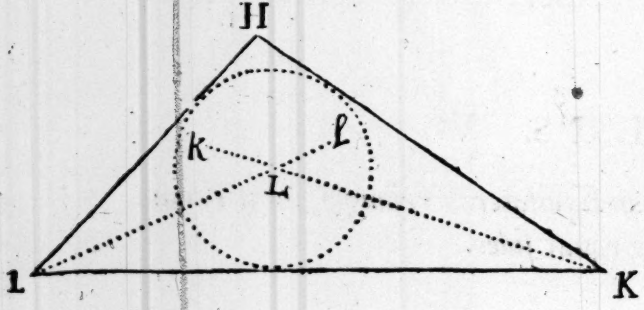
To divide a Right Lined Angle into two equal parts.



LET DEF be an Angle given, to be divided into two equal parts; upon the Angular Point E, with the distance of your fork, describe the Arch e f; and upon the point f describe the Arch g g, and upon e describe the Arch h h, crossing each other in G, draw the line E G, which shall divide the Angle D E F into two equal parts.

CONCLUS. VIII.

Within any Right Lined Triangle, to find a Point, upon which if a Circle be described, it shall justly touch all the three sides of the Triangle.



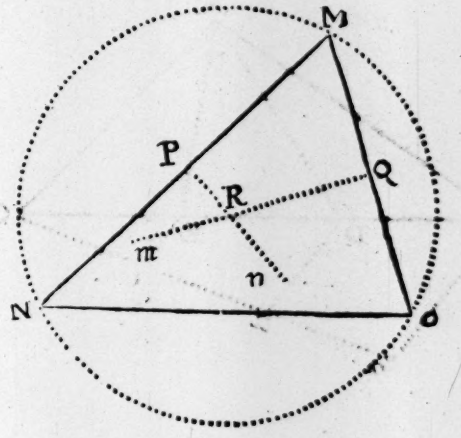
THE Triangle H I K is the Triangle given; by the foregoing Conclusion divide any two of the Angles into two equal parts, as the Angles I and K, by the Lines I l, and K k, crossing each other in L, upon which a Circle being described, it shall touch all the sides of the Triangle H I K.

CON-

CONCLUS. IX.

How to find a Point either within or without a Triangle, upon which, if a Circle be described, it shall touch all the Angular Points of the Triangle.

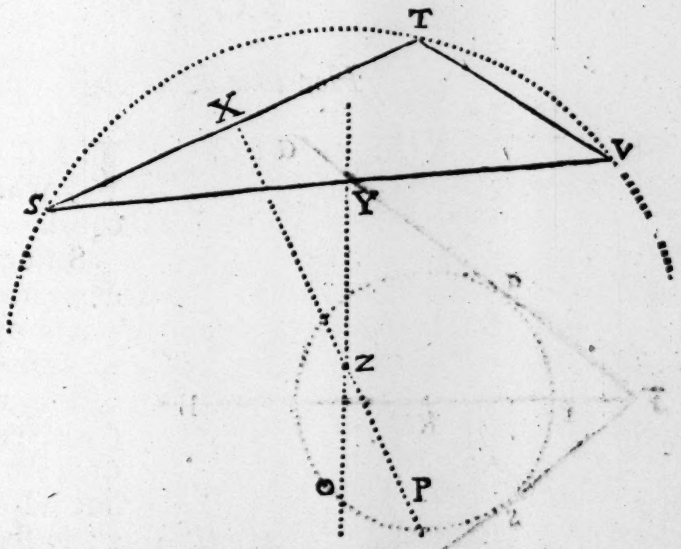
Fig. 1.



LET the Triangle given be M N O, and let it be required to find a Point, upon which a Circle being described, shall touch all the three Points M N and O.

By the second Conclusion divide any two of the sides of the Triangle, as the sides H O and M O, into two equal parts in the points P and Q; upon which points erect two Perpendiculars Q m and P n, crossing each other in R; so shall R be a Center, upon which a Circle being described, it will touch all the Angular Points of the Triangle M N O. In this case the Point R falls within the Triangle.

Fig. 2.



But if the Triangle had been that S T V, if the two sides thereof S T and S V had been divided into two equal parts of Right Angles, in X and Y, and from thence two Perpendiculars erected, as X p, and Y o, they would cross each other in the Point Z, without the Triangle, yet a Circle described upon that point, will pass by all the Angular Points of the Triangle.

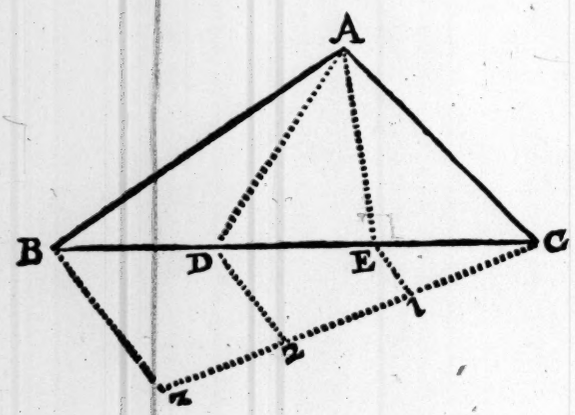
Note, If the Triangle given be an Acute Angled Triangle, the Center will fall within the Triangle; as in the first example, *Fig.* 1.

But if it be an Obtuse Angled Triangle, the Center will fall without the Triangle; as in the second example, *Fig.* 2.

And if it be a Right Angled Triangle, the Center will fall in the middle of the longest side.

CONCLUS. X.

How to divide a Triangle into any number of equal Parts, by Lines drawn from any Angle thereof.

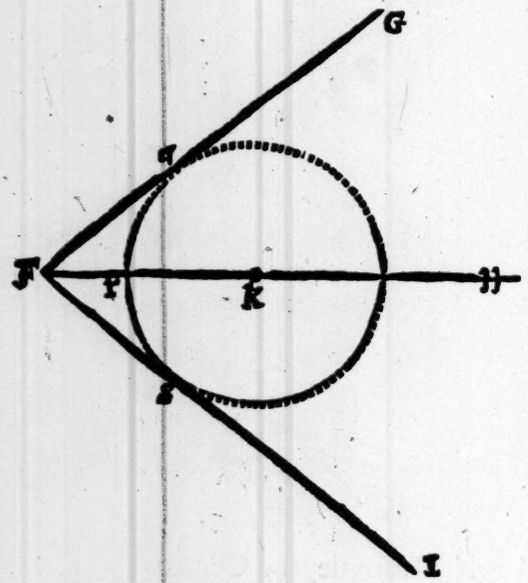


LET ABC be a Triangle given, to be divided into three Triangles by lines drawn from the Angle A.

First, By the 4*th*. Concluſion divide the ſide of the Triangle BC (which is oppoſite to the Angle A, from which the lines of diviſion are to be drawn) into three equal parts in the points D and E; then draw the lines AE and AD, ſo ſhall you have three Triangles, ABD, ADE, and AEB, all of them equal one to another, and all of them together equal to the given Triangle ABC.

CONCLUS. XI.

How to make an Angle equal to an Angle given.



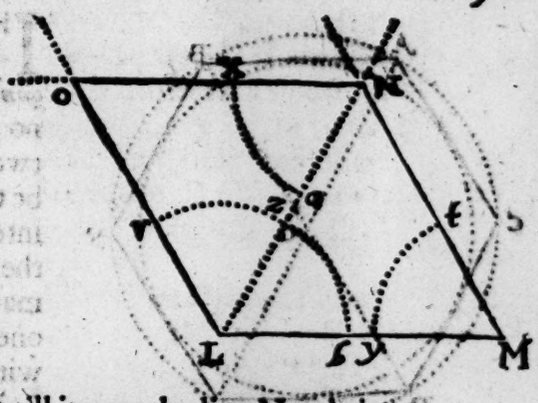
LET GFH be an Angle given, unto which it is required to make another equal.

Set one foot of your fork upon F, and move it along upon the line (or ſide) FH, till the other foot of the fork being turned about ſhall only touch the ſide FG, which will be at the point K: Upon K deſcribe (with your fork) the Circle pqrs, and from the Angular Point F draw the right line FI, ſo that it only touch the Circle in s; ſo ſhall the Angle HFI, be equal to the Angle GFH.

CONCLUS. XII.

Upon a line given, to make a Rombus, or Diamond-like Figure.

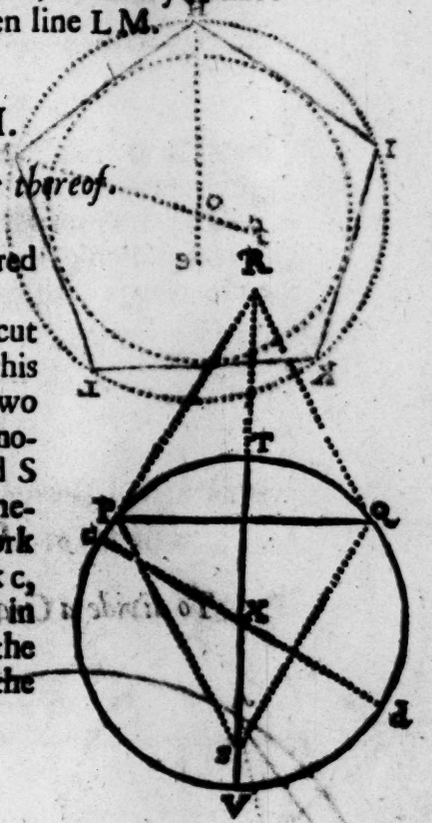
LET



LET L M be the line given; upon which, with your fork upon the end M, describe the Arch y t; also upon the end L describe the Arch b z v, and set the distance of your fork from y to t, from b to z, and from z to v; and draw the lines M t N, and L Z N, cutting each other in N; then upon N, with the distance of your fork, describe the Arch a x, setting the distance thereof from a to X, and draw the line N X at length: also from L draw the line L v, and extend it till it cuts the line N x, being extended in O: Lastly, draw the lines N O, and L O; so shall you have made a Rombus whose sides are all equal to the given line L M.

CONCLUS. XIII.

A Circle being given, to find the Center thereof.

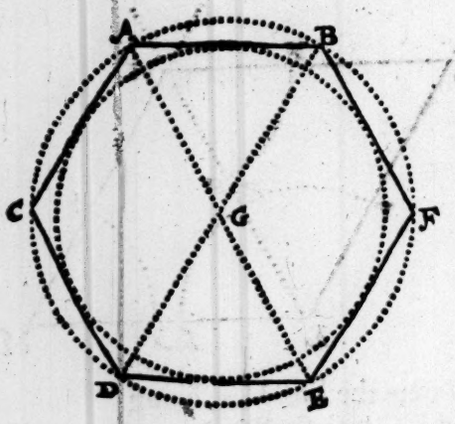


LET T P S Q be a Circle given, and it is required to find the Center thereof.

First, draw a right line within the Circle, to cut the Circle in two points, as the line P Q; upon this line P Q (by the *Vth.* Conclusion) describe two Equilateral Triangles, one above, as P R Q, and another below, as P S Q; through the points R and S draw the line R T S V; so shall T V be the Diameter of the Circle: Then with the distance of your fork set one foot in T, and with the other make the mark c, and setting one foot in S, make the mark d, (both in the Circumference of the Circle); then draw the line c d, cutting the diameter T S in X; so is X the Center of the Circle.

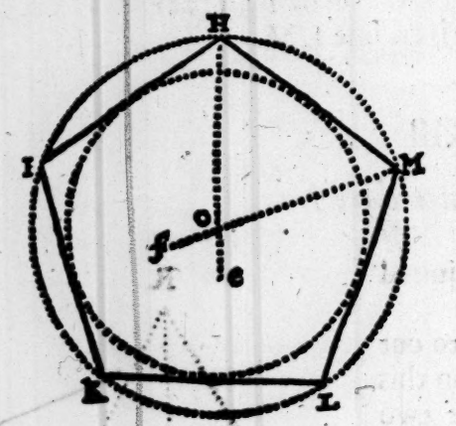
CONCLUS. XIV.

To find a Point within a Regular Poligon, *of any number of equal sides and Angles, upon which Circles may be drawn; which Circles shall either circumscribe the Poligon, touching all the Angles thereof without, or be inscribed to touch all the sides within.*



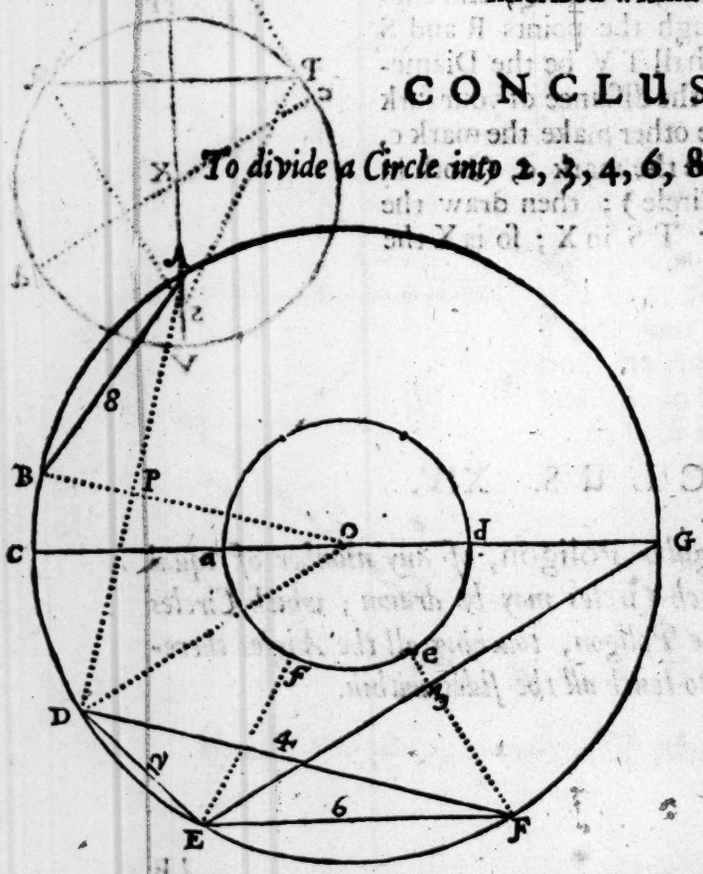
IF the *Poligon* be of an even number of sides, as 6, 8, 10, 12, *&c.* as the uppermost *Poligon* in the figure is of 6 sides; then you have no more to do, but to draw two lines from any two oppoſite ſides, and their concurrence ſhall be the Center; as the lines A E and B D do interſect each other in the point G, which is the Center of *Poligon*, upon which a Circle may be drawn to circumſcribe the *Poligon*, or one upon the ſame Center may be inſcribed within the *Poligon*, only to touch the ſides.

But if the *Poligon* conſiſt of any odd number of ſides, as of 5, 7, 9, *&c.* then (by the VI*th*. Concluſion) you muſt divide any two of the Angles thereof, into two equal parts, and the interſection of thoſe lines of diviſion ſhall give the Center either of the Inſcribing or Circumſcribing Circle about the *Poligon*; ſo in the lowermoſt *Poligon* of five ſides the Angle I H M is divided into two equal parts by the line H e, and the Angle H M L is divided into two equal parts by the line M f, cutting the line H e in O; ſo is O the Center of the *Poligon*, upon which a Circle may be deſcribed, either to be circumſcribed about, or inſcribed within the regular *Poligon*.



CONCLUS. XV.

To divide a Circle into 2, 3, 4, 6, 8, and 12 parts, or more.



THE Circle given is A B C D E F G H; then to divide it into two equal parts you have no more to do, but to draw a line through the Center thereof, as C O G; ſo is the Circle divided into two equal Parts or Semicircles, C B A H G, and C D E F G. But for the dividing it into the other parts,

Firſt, With your fork (upon the Center O) deſcribe a Circle, dividing it into Six equal parts (which the diſtance of your fork will do) in the points a b c d e f; then from the Center O, through the points e and f draw the right lines O f E and O e F, between

between which draw the line E F; which will divide the Circle into Six equal parts. Then (by the VI*th.* Concluſion) divide the Angle C O R into two equal parts, by the line O D, and draw the line D E, which ſhall divide the Circle into Twelve equal parts. Then draw a line from E to G, as E G, which ſhall divide the Circle into three equal parts. Then draw a line from D to F, which will divide the Circle into Four equal parts. Divide the line D A (which is equal to D F) into two equal parts in P, and draw the line O P, cutting the Circle in B; ſo ſhall the line B A divide the Circle into Eight equal parts.

And after this manner may the Circle be divided into ſeveral other equal parts, but not into any number.

CONCLUS. XVI.

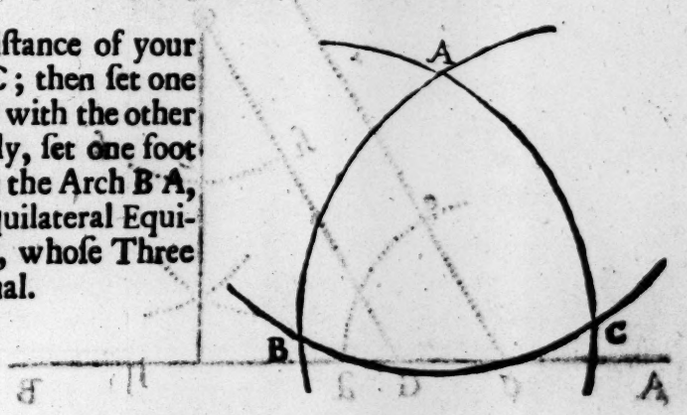
How with your Fork to deſcribe a kind of Oval Figure, by once turning it about upon one Center.

TAke a piece of Paper, and lay it upon a round Ruler, or Columb of Wood or Stone (if your fork be large), and the Paper lying ſtretched out upon the Columb, deſcribe thereon with your Fork a Circle, which when you have done, take the Paper from the Columb, and the Circle you there deſcribe, will now, the Paper lying flat, appear of an Oval Form.

CONCLUS. XVII.

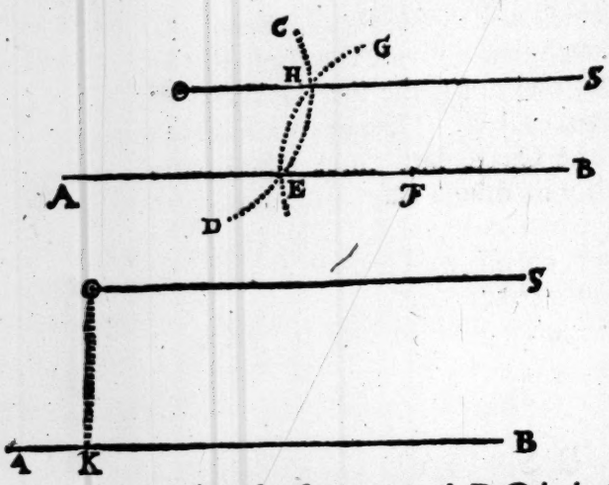
How with your Fork to deſcribe a Spherical Triangle, which ſhall have three Right Angles, and all the ſides alſo equal one to the other.

UPon a point A, with the diſtance of your fork, deſcribe an Arch B C; then ſet one foot in B, in the former Arch, and with the other deſcribe the Arch A C; and laſtly, ſet one foot in C, and with the other deſcribe the Arch B A, ſo ſhall you have deſcribed an Equilateral Equiangled Spherical Triangle A B C, whoſe Three Sides and Three Angles are all equal.



CONCLUS. XVIII.

Through a Point given, to draw a Line which ſhall be Parallel to a Line given.



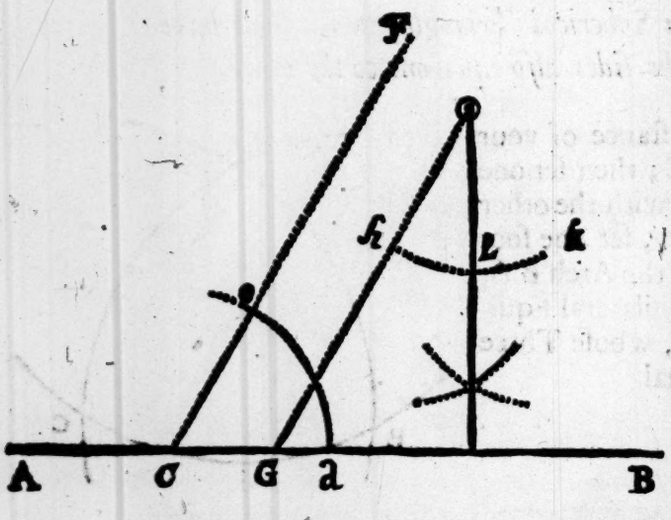
LET the Point given be ⊙, through which let it be required to draw a right line which ſhall be Parallel to the given line A B.

Firſt, If the diſtance between the Point ⊙, and the given line A B, be leſs than the diſtance of your fork, then ſet one foot in ⊙, and with the other deſcribe the Arch C D, cutting the Line A B in the point E: then ſet the diſtance of your fork from E to F, and upon F deſcribe the Arch E G, cutting the former Arch D C in in H, a line drawn from ⊙ through H, (as ⊙ S) ſhall be Parallel to the line A B.

Secondly, But if the diſtance between ⊙ and A B be greater than the diſtance of your fork, then (by Concl.) let fall a Perpendicular from ⊙ to A B (as ⊙ K) and upon the point ⊙ erect a Perpendicular (by Concl.) to ⊙ K, as ⊙ S, ſo ſhall ⊙ S be Parallel to A B, as in the lowermoſt Scheme.

CONCLUS. XIX.

From a Point above to let fall a Perpendicular upon a line below, otherwiſe than in Concluſion IV.



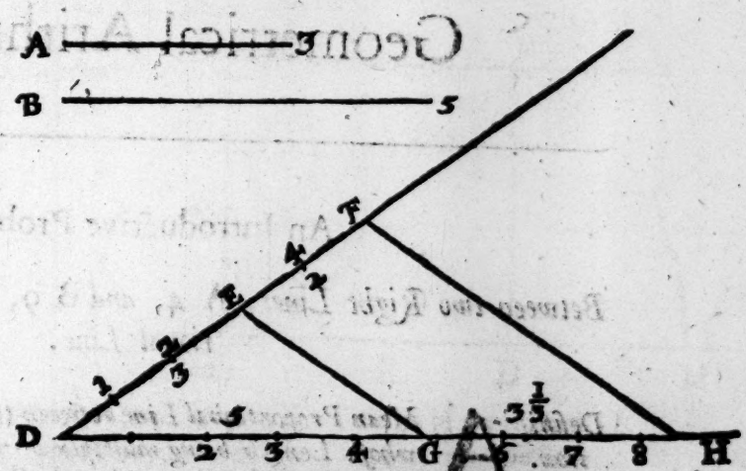
LET the given Point be ⊙, from whence a Perpendicular is to be let fall upon the Line A B.

Upon any part of the line A B, as C, with your fork deſcribe an Arch, and upon it ſet the diſtance of your fork from d to e, and draw the line C F: Then (by the laſt Problem) through the point ⊙, draw a line Parallel to the line C F, as ⊙ G; then upon the point ⊙ with your fork deſcribe an Arch, and ſet the diſtance of your fork upon it from h to k: Laſtly, (by Concluſion VI.) divide the Angle h ⊙ k into two equal parts in l, and through l draw the right line ⊙ M, which ſhall be Perpendicular to the Line A B.

CON-

CONCLUS. XX.

Unto two given Right Lines, A 3, *and* B 5, *to find a Third Line which shall be in proportion to them.*



THis is to perform the Rule of Three, or Golden Rule in Lines, (of which more hereafter in this Book). As if the Question were thus stated;

If 3 Yards of Cloath cost 5 Shillings;

What shall 5 Yards cost?

To perform this by this Artifice, draw two lines at length, making any Angle, as the lines D F and D H, making the Angle F D H: This done, with the distance of your fork, run three distances from D to E, upon the line D F, and 5 distances from D to G, upon the line D H, and draw the line E G; then run 5 distances (also) upon the line D F from D to F, and through F (by Conclusion XIX.) draw the line F H Parallel to E G, cutting the line D H in H; and then measuring with your fork from D to H, you shall find it to contain 8 distances of your fork, and one third part of a distance, which is 8 Shillings and 4 Pence, for the Price of Five Yards.

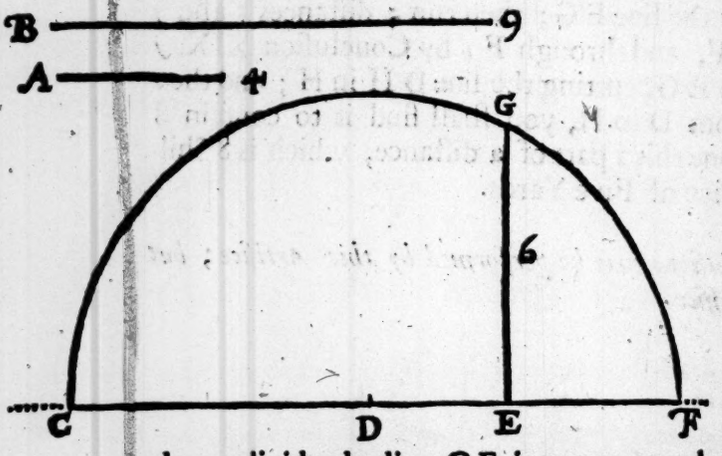
Many more of the like Conclusions may be performed by this Artifice; but for the present let these suffice.

CHAP. III.
Geometrical Arithmetick.

An Introductive Problem.

Between two Right Lines, A 4, *and* B 9, *to find a Mean Proportional Line.*

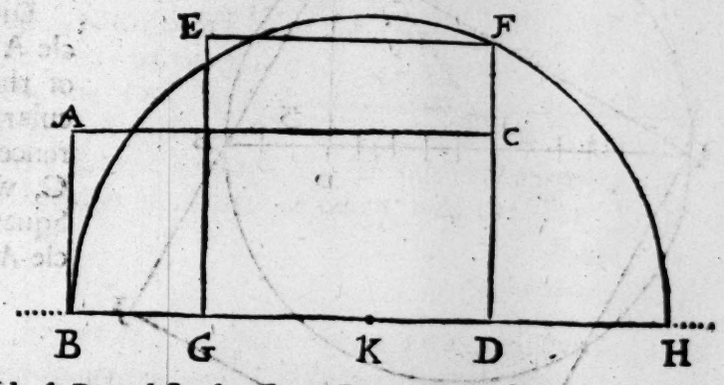
Definition. A *Mean Proportional Line between two other Lines, is such a Line whose Length being multiplied in it self, shall produce such a Number as shall be equal to the Products of the lengths of the two given Lines, they being multiplied one by the other.*



Practice. DRaw a Right line at pleasure, as the line C F, then take the length of the given line A 4 in your Compasses, and set it from F to B; also take the other given line B 9 in your Compasses, and set it from E to C; so is the line C F equal to both the given lines A 4 and B 9, (the point of joining them together being in the point E.) This done, divide the line C F into two equal parts in D, and upon D, as a Center, with the distance D C or D F, describe the Semicircle C G F: Lastly, from the point of joining of the two given lines A and B, namely, from the point E, erect the Perpendicular E G, cutting the Semicircle in G; so shall the line E G be a Mean Proportional between the two given Lines A and B, and will contain 6 such parts, as the whole line C F contains 13, that is, as A 4, and B 9: And so this *Mean Proportional Line* E G, 6, multiplied in it self will produce 36, equal to the line A 4, multiplied into the line B 9, for 9 times 4 is 36 also.

REDUCTION.

I. *How to reduce a long Square or* Parallelogram A B C D, *into a Geometrical Square* E F G D, *whose four sides are all equal.*



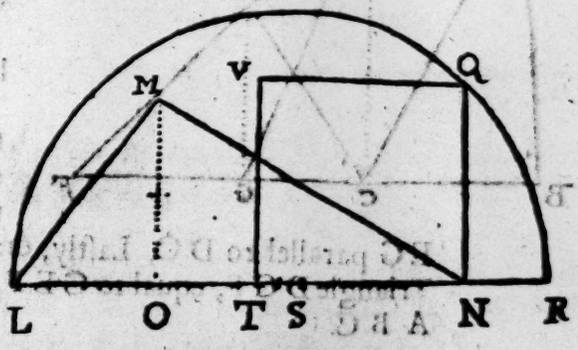
BY the Problem foregoing find a Mean Proportional Line between A B the shorter, and B D the longer side of the long Square A B C D, which will be found to be D F.

First, Draw a line as B H, and upon it set one of the longest sides of the Parallelogram, as A C, from B to D; also take the shortest side A B, and set that from D to H; and upon the point of joining at D, erect Perpendicular: Then divide the line B H into two equal parts in K, and upon K (as a Center) describe the Semicircle B F H, cutting the Perpendicular D F in F; so shall D F be the side of the Geometrical Square, G E F D, which shall be equal in quantity to the long Square A B C D.

For,

Suppose the lesser side of the Long Square A B to be 8 (of any measure) and the longer side A C to be 18, the Mean Proportional between them F D will be 12; and so 18 multiplied by 8, the Product will be 144; and so will 12 multiplied by 12, produce the same.

II. *To Reduce a Triangle into a Geometrical Square.*

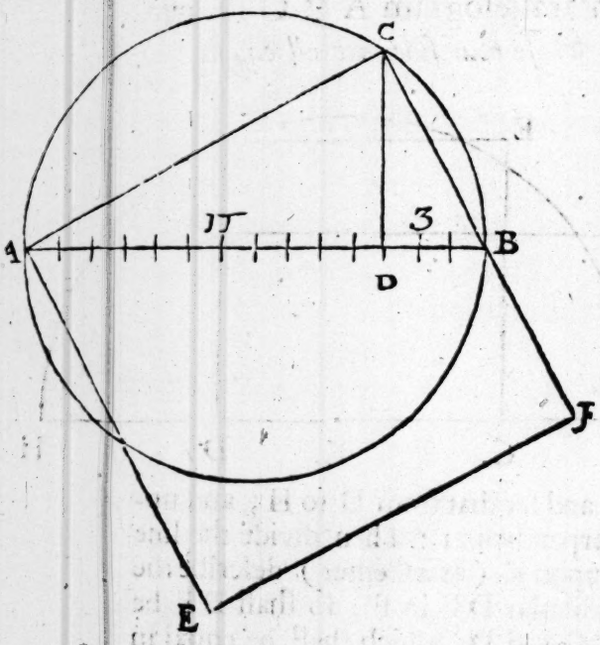


LET L M N be a Triangle given to be reduced into a Square.

Draw a line at pleasure, as L R, then take the length of the Base of the Triangle L N, and set it from L to N; then take half the length of the Perpendicular P O, and set it from N to R, and upon N erect the Perpendicular N Q; then divide the line L R into two equal parts at S, and upon S as a Center, with the distance S L or S R, describe the Semicircle L Q R, cutting the Perpendicular N Q in Q; so shall Q N be the side of a Geometrical Square, equal to the Triangle M L N.

III. *To*

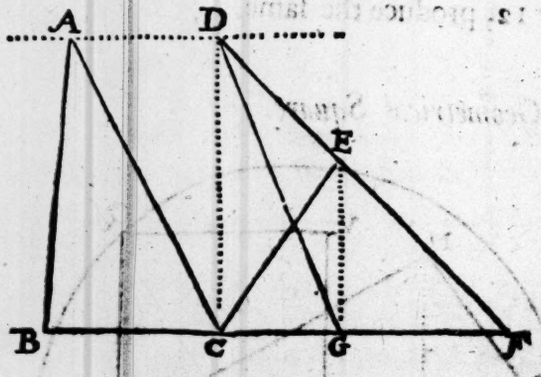
III. *To Reduce a Circle into a Geometrical Square.*



LET A B C be a Circle given to be reduced into a Geometrical Square.

Divide the Diameter of the Circle A B into 14 equal parts, and 11 of them, as at D, erect a Perpendicular D C, cutting the Circumference in C; then draw the line A C, which shall be the side of a Square A C E F, equal to the Circle A B C.

IV. *Two Triangles of different heights being given, to reduce them to one height.*



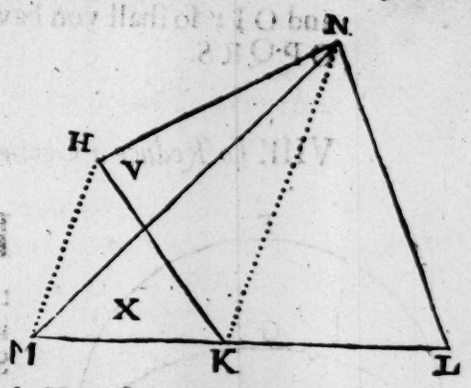
LET there be two Triangles A B C, and C E F, and let it be required to reduce the Triangle E C F into another Triangle that shall be equal in height with the Triangle A B C.

First, Through the point A draw a line A D, parallel to the bases of the two Triangles, *viz.* B F; then extend the side E F of the Triangle C E F, till it cut the Parallel line in D, and draw D C; then through the point E draw the line E G parallel to D C. Lastly, draw the line D G, so shall you have a new Triangle D G F, equal to C E F, and of equal height with the Triangle A B C.

V. *How to Reduce an irregular Figure of Four sides into a Triangle, from an Angle given.*

LET

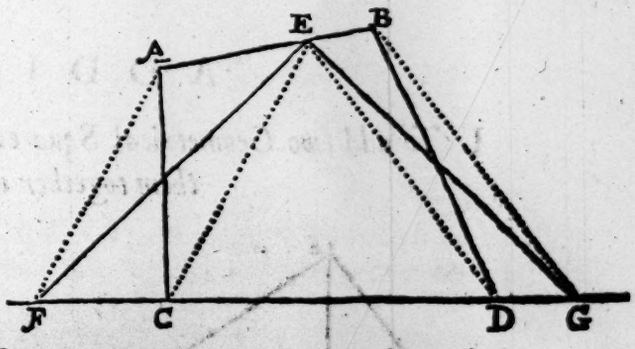
LTT HKLN be a *Tropezia*, or a Figure of Four unequal Sides and Angles, and let it be required to reduce the same into a Triangle, by a Line drawn from the Angle N.



Extend the side KL, which is opposite to the Angle N, towards M, and from N draw a line to the opposite Angle M, as NK; then through the point H draw the line HM parallel to NK: Lastly, draw the line NM, so shall you have a Triangle LMN, equal to the irregular Figure HKLM; for the Triangle V, left out in the irregular Figure, is equal the Triangle X, which is taken into the Triangle.

VI. *To reduce an irregular Figure of Four sides into a Triangle, by a line drawn from a Point limited in any of the sides thereof.*

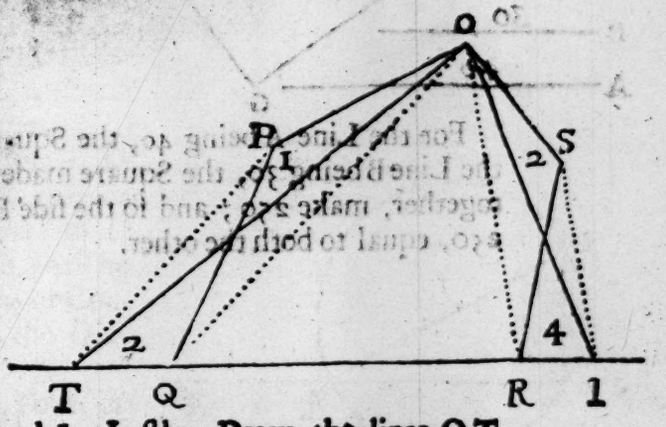
LET ABCD be an irregular figure, and it is required to reduce the same into a Triangle, by lines drawn from the point E, in one of the sides thereof.



From the given Point E draw two lines to the two opposite Angles, as the lines EC and ED, and extend the side CD, opposite to the given point E, out on both sides towards F and G; then through the point A draw the line AF, parallel to EC, and the line BG parallel to ED, cutting the side CD (extended) in the Points F and G: Lastly, draw the two lines EF and EG; so shall you have a Triangle EFG, equal to the irregular figure ABCD.

VII. *To Reduce an irregular figure of Five sides, into a Triangle, by lines drawn from any of the Angles thereof.*

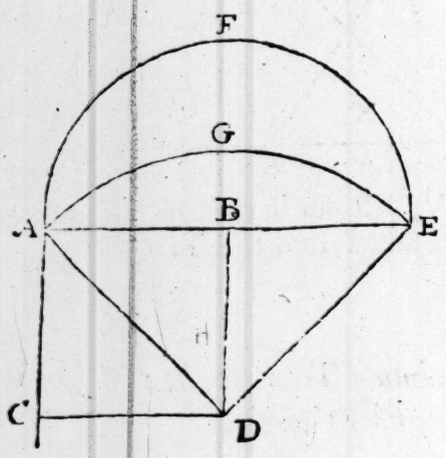
LET OPQRS be the figure given, and let it be required to Reduce it into a Triangle, by lines drawn from the Angle at O.



First, Prolong the side QR, opposite to the given Angle at O, both ways, towards T and I; and from the Angle O draw the lines OQ and OR; also through the Angular point P and S, draw the lines PT and SI, parallel to OQ and SR, cutting the side QR (extended) in the points T and I: Lastly, Draw the lines OT and

and O I; so shall you have a Triangle O T I, equal to the five sided figure O P Q R S.

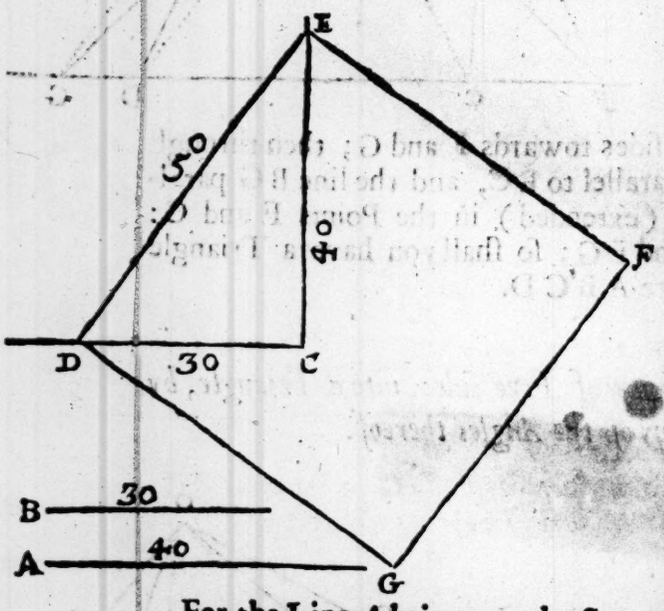
VIII. *To Reduce a Geometrical Square into a Figure of a Lunary Form.*



LET A B C D be the Square given; first draw the Diagonal line A C, and on the end thereof at C erect the Perpendicular E C, making it equal to A C; then continue the side of the Square A B to E; and on B, as a Center, with the distance B A or B E, describe the Semicircle A F E. Lastly, On the point C, and distance C A, describe the Arch A G E, leaving the Lunary Figure A F E G, equal to the Geometrical Square A B C D.

ADDITION.

I. *To add two Geometrical Squares together, and to give the Sum of them together in one Square.*



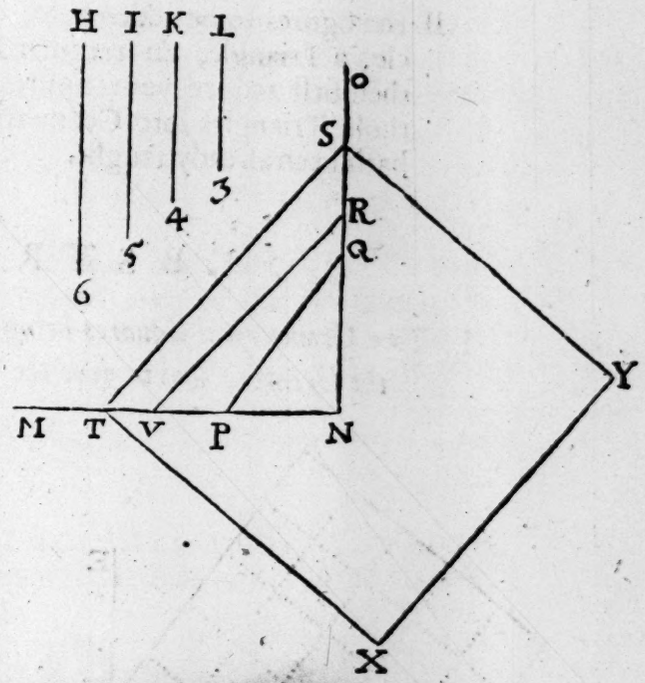
LET the two lines A and B be the sides of the two Geometrical Squares, the one being 30, the other 40, of any Measure.

First, Join two lines D C and E C together, making a right angle at C: Then take the line B in your Compasses, and set it from C to D; also take the line A in the Compasses, and set that from C to E, and draw the line E D, so shall E D, be the side of the Square D E F G, which shall be equal to both the Squares made of the lines A and B.

For the Line A being 40, the Square made thereof will be 160; and the Line B being 30, the Square made thereof will be 90; which added together, make 250; and so the side E D being 50, the Square thereof is 250, equal to both the other.

II. To

II. *To add Two, Three, or Four Squares together, and to give their Sum in one entire Square.*



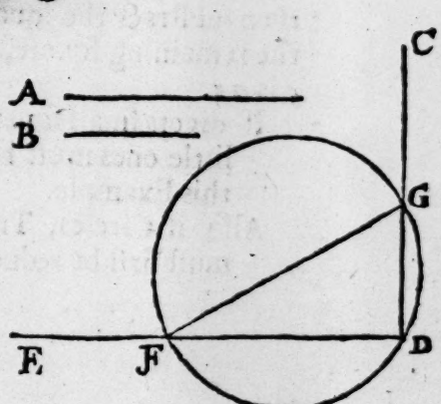
LET HIKL be four lines given, equal to the sides of four several squares.

First, Join two lines M N and N O together, so as to make a right Angle at N; then take the line L, and set it from N to P, and take the line K, and set it from N to Q, and draw the line Q P, which shall be the side of a square equal to the squares of L and K.—Then take the line Q P, and set it from N to V, and the line I and set it from N to R, and draw the line R V, which shall be the side of a square, equal to the squares made of the lines L, L and I. ——Then take the line R V, and set it from N to S, and the line H, and set it from N to T, and draw the line S T, which is the side of the square S T X Y, and is equal to the four squares made of the lines H, I, K, and L; which I shall prove by Numbers.

The Line	H	is	6	Square is	36.
	I		5		25.
	K		4		16.
	L		3		9.
				Their Sum is	86.

And the Square Root of 86 is $9\frac{27}{100}$, and somewhat more, for 9. 27 multiplied in it self, produceth for its Square 85. 93, which is too little; and the Square of 9. 28 will be, 86. 1[illegible] somewhat too much.

III. *To add two [illegible] together.*



LET the lines A and B be the Diameters of two Circles to be added together.

Make a right Angle by the two lines C D and E D; then take the line A, and set it from D to F, and take the line B, and set it from D to G, and draw the line G F, which shall be the Diameter of a Circle equal in Area or Content of two Circles described upon the Diameters A and B.

In this manner may you add Three, Four, or more Circles together.

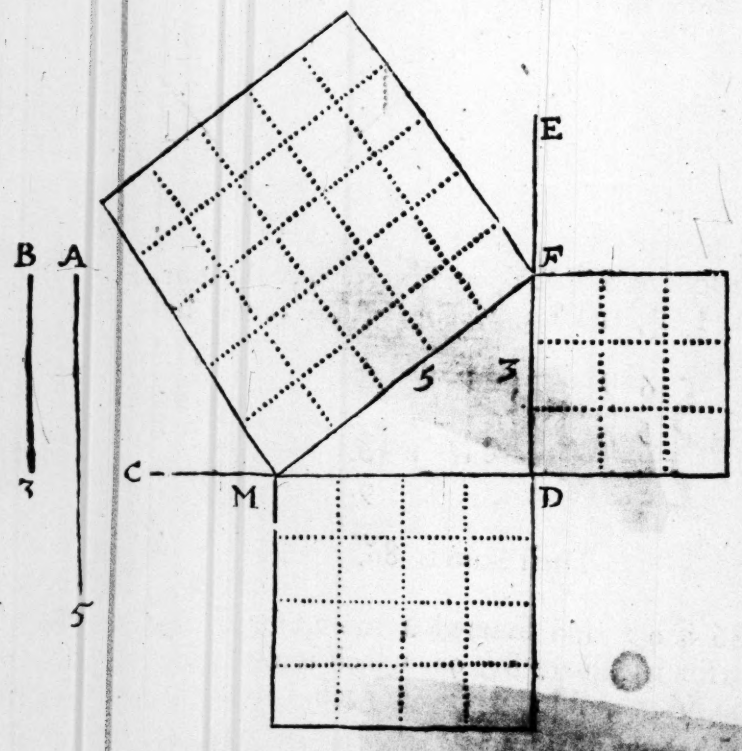
If you would add 2, 3, 4, or 5 Triangles together, you must first reduce them to Geometrical Squares, and then add them; as before is taught.

Alſo if they be Long Squares or Parallelograms, firſt reduce them to Geometrical Squares, and then add them.

If the figures to be added be of divers kinds, as a Long Square, a Circle, a Triangle, an Irregular Figure of four or five ſides, you muſt then firſt reduce your figures of 4 or 5 ſides into Triangles, and thoſe Triangles into Geometrical Squares, and then add them, as hath been already taught.

SUBSTRACTION.

I. *Two Geometrical Squares being given, to ſubſtract the Leſſer from the Greater, and to give the Remainder in a third Square.*



LET A and B be the ſides of two Squares, and let it be required to ſubſtract the ſquare of B from the ſquare of A.——Firſt, Join two right lines, C D and E D in a right Angle in D; then take the leſſer line B, and ſet it from D to F; then take the greater line A, and ſetting one foot of the Compaſſes in F, with the other mark the Point M in the line C D, and draw the line F M.

Then have you a Triangle F M D, of which the ſide D M, equal to the line B, is 3, and the ſquare raiſed thereon contains 9 little ſquares; and the ſide M F equal to the given line A, which is 5, and the ſquare raiſed thereupon contains 25 little ſquares; then ſubſtract the ſquare 9, from the ſquare 25, and there is left 16 for the remaining ſquare, for 4 times 4 is 16, as 3 times 3 is 9, and as 5 times 5 is 25.

If divers ſmall ſquares be to be ſubſtracted from one greater ſquare, the little ones muſt be all reduced into one, and then ſubſtracted, as in this Example.

Alſo, if Circles, Triangles, or other figures be to be ſubſtracted, they muſt firſt be reduced, and then dealt with as before.

MULTIPLICATION.

MUltiplication in Geometry, is nothing elſe but to find the Square that is made of any Line (or Number) multiplied in it ſelf.
Or to find the Rectangle or Parallelogram made of any two Lines or Numbers involved one into the other.

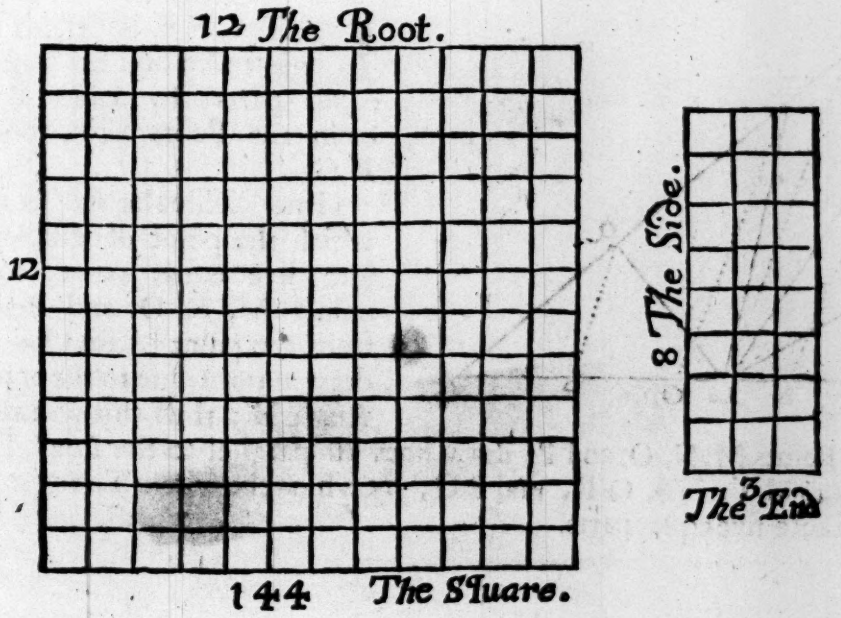
Example 1. *Let the Number* 12 *be given, to be multiplied in it ſelf.*

Now 12 multiplied in it ſelf (that is, by 12) produceth (or the Product is) 144; ſo that 144 is the Square, and 12 the Root thereof. And ſo of any other ſingle Number in it ſelf produceth a Square Number.

Example 2. *Let there be two Numbers* 8 *and* 3 *to be multiplied together.*

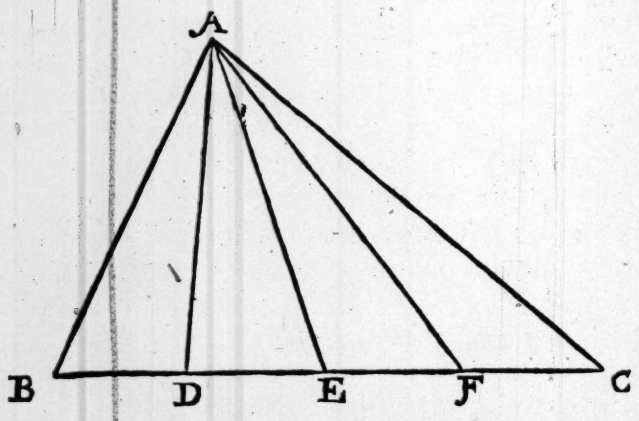
Now if you multiply 8 by 3, the Product is 24, which in Geometry is called a Rectangle, and 24 is the ſuperficial Content or Area thereof, and 8 and 3 are the two ſides thereof.

Which in Geometrical Figures will be as follows;



DIVISION.

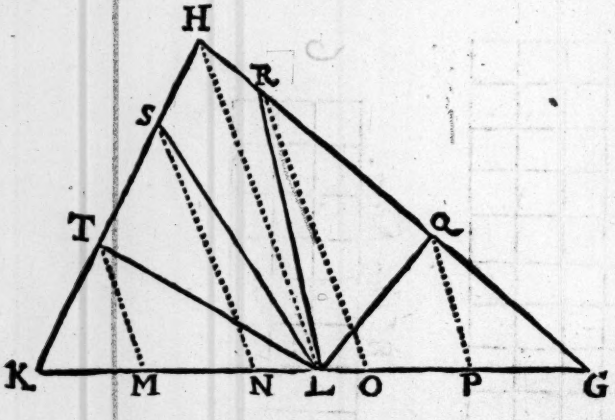
I. *To divide a Triangle into any number of equal parts, by Lines drawn from any Angle thereof.*



LET ABC be a Triangle to be divided into four equal parts, by lines drawn from the Angle at A.

First divide the ſide BC, which is oppoſite to the Angle A, (from whence the diviſion is to be made) into four equal parts, in the points D, E, and F; then if you draw the lines AD, AE and F, you ſhall have four Triangles ABD, ADE, AEF, and AFC, all of them equal one to another, and all of them together equal to the given Triangle ABC.

II. *To divide a Triangle into any number of equal parts, by lines drawn from any Point taken in any of the Sides thereof.*



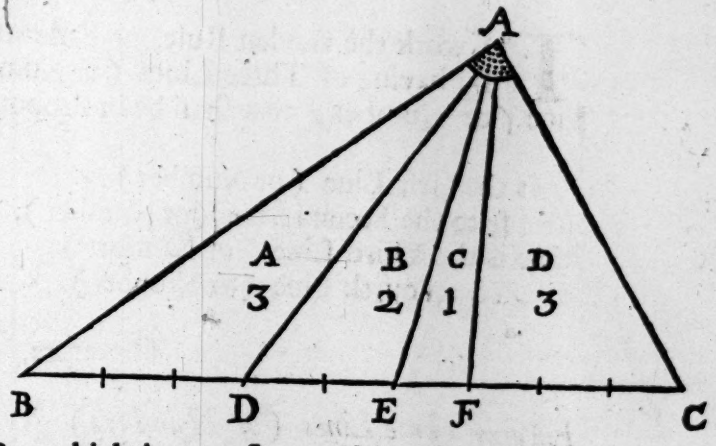
LET GHK be the Triangle to be divided into five equal parts, by Lines drawn from the Point L in the ſide KG.

First, divide the ſide GK, in which the point of diviſion is, into five equal parts, in the points M, N, O, and P; and from the point of diviſion at L, draw a right line to the oppoſite Angle H; then through the ſeveral Points M, N, O, and P, draw lines all parallel to the line LH, as the lines MT, NS, OR, and PQ; by which the whole Triangle is divided into five equal parts.

III. *How*

III. *How to divide a Triangle into Two, Three, or Four, (or more) unequal parts.*

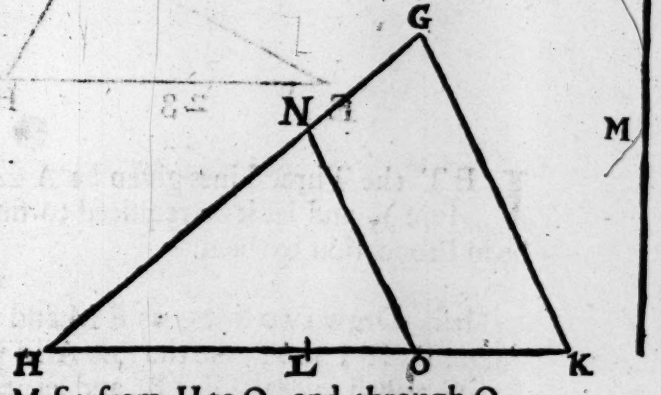
LET the Triangle A B C be a Piece of Ground containing Nine Acres, and it is let out to four men, A B C and D, of which A is to have 3 Acres, B 2 Acres, C 1 Acre about the middle, and D 3 Acres, and their lines of ſeparation are to be made from the Angle at A, there being a Pond to accommodate all the Tenants.



Firſt, Divide the ſide B C, which is oppoſite to the points from whence the diviſion is to be made, into 9 equal parts; then (becauſe A is to have 3 Acres) count three of the parts (or diviſions) from B to D, and draw A D, ſo is A B D 3 Acres, the ſhare for A. ——— Then count two parts more from D to E, and draw E A, ſo is A D E 2 Acres for B. ——— Then count one diviſion from E to F, and draw A F; ſo is E A F one Acre for C his ſhare; and the reſt, F A C, being three Acres, is the ſhare for D; in all, Nine Acres.

IV. *How to divide a Triangle into two or more equal or unequal parts, by a Line drawn Parallel to any ſide thereof.*

LET G H K be a Triangle given, to be divided into two equal parts, by a line drawn Parallel to any one ſide thereof, as the ſide G K.



Firſt, divide the ſide K H into two equal parts in L; then by the Introductory Problem find a Mean Proportional Line between the whole Baſe H K and H L, which line will be found to be the line M, which line M ſet from H to O, and through O, draw the line O N, parallel to G K; ſo ſhall the Triangle be divided into two equal parts, by the line N O.

The

The GOLDEN RULE,

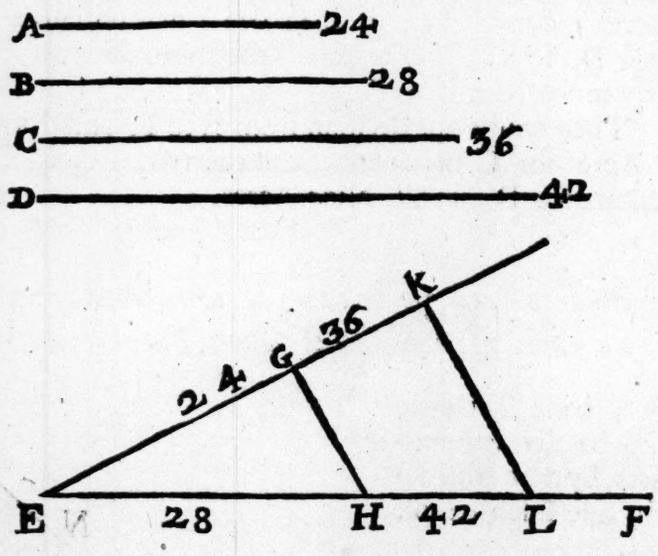
Or *Rule of Three.*

TO work the Golden Rule, or Rule of Three, Geometrically, is, by having of Three Lines (or Numbers) given, to find a Fourth Line (or Number) that ſhall be in Proportion to them; that is,

As the Firſt Line (or Number)
Is to the Second Line (or Number),
So is the Third Line (or Number)
To a Fourth Line (or Number).

Therefore,

Having Three Lines (or Numbers) given, let it be required to find a Fourth that ſhall be in Proportion to them.



LET the Three Lines given be A 24, B 28, and C 36, of any Meaſure), and let it be required to find a Fourth Line D, which ſhall be in Proportion to them.

Firſt, Draw two lines, as E M and E F, making any Angle, as the Angle D E F; then take the line A in your Compaſſes, and ſet it from E to G; then take the line B, and ſet it from E to H, and draw the line G H; then take the third line C in your Compaſſes, and ſet that from E to K, upon the line E M, the ſame where the firſt line A was ſet; and through the point K draw the line K L Parallel to G H, cutting the line E F in the point L; ſo ſhall the line E L be the Fourth Proportional Line required.

For,

For,

As the Line A, equal to E G 24,
Is to the Line B, equal to E H 28;
So is the Line C, equal to E K 36,
To the Line D, equal to E L 42.

And ſo,

If 24 Yards of any Commodity coſt 28 Shillings, what ſhall 36 Yards of the ſame coſt at that rate?

Thus, as in Vulgar Arithmetick, if you multiply the ſecond number 28, by the third number 36, and divide the Product by the firſt number 24, the Quotient will be 42 for the fourth number ſought; and ſo many Shillings will 36 Yards coſt.

CHAP. IV.

Of Altimetria, Longimetria, *and* Planometria.

Shewing how to take all manner of *Heights*, *Depths*, and *Diſtances*, whether Acceſſible or Inacceſſible, and to *Meaſure Land* Mechanically.

Sect. I. *Of* Altimetria, *or Meaſuring of Heights.*

PROBL. I.

How to take the Height of any Tree, Steeple, or other Upright Building, by the Shadow thereof.

FIG. I.

LET A B be the Wall of ſome Caſtle or Watch-Tower, and the Sun ſhining caſts the Shadow thereof upon the Level Ground to C; now having a Walking-ſtaff in your hand, ſet that upright at the end of the ſhadow of the Wall at C, and I find that the ſtaff caſts its ſhadow to E, where I make a mark, as alſo another at C; then meaſuring the length of my ſtaff, I find it 38 Inches, and the length of the ſhadow of it C E, to be 46 Inches: Then meaſuring the length of the ſhadow of the Wall of the Tower A C, I find that to be 30 foot, which is 360 Inches. Now for the Height of the Caſtle-Wall, you muſt work by the Rule of Proportion, thus:

As C E, the length of the ſhadow of the ſtaff, 46 Inches,
Is in Proportion to the length of the ſtaff C D, 38 Inches;
So is A C, the length of the ſhadow of the Wall, 360 Inches,
To 297. 4 Inches, for the Height of the Caſtle Wall.

For

For if you multiply 360, by 38, the Product will be 13680; which divided by 46, the Quotient will be 297. 4 Inches *ferè*, for the Height of the Caſtle Wall B A, which is 24 Foot and 9 Inches, and ſomewhat more.

PROBL. II.

How to take the Height of a Watch-Tower, *by the Shadow, when you cannot come to the bottom of it, to meaſure the length of the Shadow.*

FIG. II.

LET A B be a Watch-Tower, whoſe Height I would know by the ſhadow thereof, but there is a Moat about it, as B C, ſo that I cannot come to meaſure the ſhadow thereof: However,

I come near to the Moat-ſide, and there I find the ſhadow of the top of the Tower, to caſt at C, where I erect my ſtaff C G, and that caſts its ſhadow to H; I meaſure the length of my ſtaff, and I find it 4 foot, or 48 Inches; and the length of the ſhadow thereof C H, I find to be 32 Inches, theſe two I note down.

Then, ſome time after, (when the Sun is lower) I come again to the place, and find the ſhadow of the top of the Tower to caſt at D, where again I erect the ſame ſtaff of 4 foot long, and find that it caſts its ſhadow to E, and that the length of the ſhadow thereof, D E, is 4 foot 5 inches, or 53 inches, and ſomewhat better; this I alſo ſet down, and then I meaſure the diſtance between the two places where the Tower caſts its ſhadow, at the firſt and ſecond time of my obſervation, namely the diſtance C E, and find it to be 10 foot, or 120 inches.

And now having all theſe numbers ſet down, I come to find the Height of the Tower A B, by help of the Rule of Proportion, as followeth.

(1.) As D E, the length of the ſhadow of the ſtaff D F at the ſecond Obſervation, 53 Inches,
Is to 48 Inches, the length of the ſtaff;
So is 10 foot (or 120 Inches) the length of the ſhadow between the two places of Obſervation C and D,
To 108 Inches, or 9 foot.
Which number 9 foot, or 108 Inches, ſet down,
And ſay again by Proportion,

(2.) As 48 Inches the length of the ſtaff G C,
Is to 10 foot (or 120 Inches) the diſtance between the two places of Obſervation C and D;
So is 108 Inches, (the Number before found)
To 270 Inches, the Height of the Tower; which reduced into Feet is 22 Foot, 6 Inches.

PROBL. III.

How to take the Altitude of any upright Building, or the like, by a Bowl of Water.

FIG. III.

TRavelling along the Road I ſee a May-pole, as K L, the height whereof I would gladly know, but having no Geometrical Inſtrument, I procure a Bowl of fair Water, which I ſet down upon the ground,

ground at M. And then, when the Water is still in the Bowl, I go backward in a right line from the May-Pole, till I see the shadow of the top of the May-Pole in the middle of the Water, which I do when I come at N; and at N, I make a mark upon the ground; then do I measure the distance from the foot of the May-Pole at L, to the Bowl of Water at M, and find it to be 175 Inches: Also I measure the distance from the Bowl of Water at M, to the place of my standing at N, and find that to be 72 Inches: Then I measure the Height of my eye from the Ground O N, and find that to be 60 Inches: These things known, I say by the Rule of Proportion,

If 72 Inches distance M N, give 60 Inches Altitude N O;
What Altitude shall 175 Inches the distance L M give?
Answer $145\frac{60}{72}$ Inches.

For if you multiply 175 by 60, the Product will be 10500, which divide by 72, the quotient will be $145\frac{60}{72}$, that is almost 146 Inches, which is 12 foot 2 Inches for the height of the May-Pole K L, required.

PROBL. IV.

How to take the Height of any upright Building that is approachable, by two Sticks or Rulers joined together Square-wise.

FIG. IV.

LET P Q be some Structure, standing upright upon plain Ground, whose height you require.

Go unto some convenient *Court, Yard, Garden*, or other piece of level Ground adjoining to the building to be measured, then take your Square in both your hands, holding it perpendicular, which you may do, by having a Thread and Plummet as T V, hung upon a pin near the top of the Square at T; then keeping it in this posture, go backwards or forwards (as occasion requires) till your Eye being at S, you can see the other end of your Square at T, and the Top of the Building at P, all in one Right Line, which when you do, make a stand at S. Then measure the height of your eye from the Ground X S, with a String, and set that length upon the ground from the place of your standing at S, to R: Then measure the distance from R to Q, for that shall be equal to the height of the building P Q, and is here 210 foot.

SECTION II.

Of Longimetria, or Measuring of Distances.

PROBL. I.

How by the help of the Square, standing upon a Platform of a known height, to find the distance from the Platform to any Tree, Tower, or other Object that is remote from you.

FIG. V.

LET A B be a Platform, whose Perpendicular height is 180 foot, being upon the top thereof at A, I would know how far the Object C, is distant from the bottom of the Platform at B.

Upon the top of the Platform at A, I erect a Pike or Javelin 12 foot long,

For if you multiply 360, by 38, the Product will be 13680; which divided by 46, the Quotient will be 297. 4 Inches *ferè*, for the Height of the Caſtle Wall B A, which is 24 Foot and 9 Inches, and ſomewhat more.

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Which number 9 foot, or 108 Inches, ſet down,
And ſay again by Proportion,

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If 72 Inches distance M N, give 60 Inches Altitude N O;
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SECTION II.

Of Longimetria, or Measuring of Distances.

PROBL. I.

How by the help of this Square, standing upon a Platform of a known height to find the distance from the Platform to any Tree, River, or other Object that is remote from you.

FIG. V.

LET A B be a Platform, whose Perpendicular height is 160 foot, being upon the top thereof at A, I would know how far the Oak at C, is distant from the bottom of the Platform at B.

Upon the top of the Platform at A, I erect a Pike or Javelin 12 foot

long, more or leſs, upon which I hang the Angle of my Square: And I look with my eye at D, along the ſide of my Square, till I ſee the bottom of the Oak at C, and in this poſition I fix my Square, with a Screw or the like, to the head of thé Javelin: Then from D I extend a thread or Line by the ſide of my Square, till it touch the Platform at E, and then I meaſure the diſtance upon the Platform from A to E, and find it to be 24 foot, 6 inches; then by Proportion I ſay,

As 12 foot, the length of the Javelin, D A,
Is to 24 foot and a half, the diſtance meaſured upon the Platform A E,
So is 112, the height of the Platform and Javelin together, B D,
To 228 foot, 8 inches, for the diſtance B C.

PROBL. II.

How to take the diſtance from the place of your ſtanding upon level Ground, to any Tree, Tower, or other thing, remote from you, tho you cannot come near the ſame, by your Square.

FIG. VI.

STanding at F, I ſee a Conduit head at G, whoſe diſtance from F where I ſtand, I would know, but I cannot come near it for a River between F and G: However,

At F I erect a ſtaff of 4 foot high, (or 48 Inches) as F H, upon the end whereof I hang the Angle of my Square, and I look by the ſide thereof, till I ſee the foot of the Conduit-head at G, and fixing my Square there, I extend a line from H, by the ſide of the Square, till it touch the Ground at K: Then meaſuring the diſtance between F and K, I find it to be 3 foot, or 36 Inches: Then by the Rule of Proportion I ſay,

As 36, the diſtance K F,
Is to F H, the length of the ſtaff, 48 Inches;
So is 48 inches, the length of the Staff F H,
To 64 inches, for the Diſtance F G.
For as often as K F is contained in F H,
So often is F H contained in F G.

PROBL. III.

How to take the Breadth of a River by the Square.

FIG. VII.

THere is a River M P O, whoſe breadth I deſire to know: Upon the brow of the River at M, I ſet up my Staff M L, which is 60 inches (or 5 foot) long, and hanging my Square upon the end thereof at L, I look by the ſide thereof, till I ſee the Brow of the River on the other ſide at O, and there fixing my Square, I extend a Thread by the ſide thereof, from L to N, then meaſuring the diſtance L N, I find it to be 15 inches (or 1 foot 3 inches); then I ſay by Proportion,

As N M, the diſtance meaſured, 15 inches,
Is to L M, the length of the Staff, 60 inches.
So is L M 60 inches,
To M O, 240 inches, (or 20 foot) for the breadth of the River M O.

PROBL.

PROBL. IV.

How to take an inaccessible Distance, *by three flat* Sticks *or* Rulers *of equal* Length, *joined together, thereby making an* Equilateral Triangle: *As in*

FIG. VIII.

LET A be a Tree or other Object, whose distance you would know from the place of your standing at B, there being a River or the like Impediment between you and the Tree.

Place your Triangular Rulers horizontally upon a staff at B, and observe the Object at A by the side of the Triangle B D; it so resting, look at some other Object (or cause one to be set up) by the side of the Triangle B E. Then transfer your Triangle along the line B C, and place it upon divers parts of the same line, till at length you find a Point C, upon which placing the Triangle, you shall see the Point B, by the side C G, and the Object A, by the side C F; then I say the lines C B and C A are equal; so that by measuring the line B C, you have the Distance from B to A.

PROBL. V.

How to take the Distance between Two (or more) Places, without coming near any of them, by a Two Foot Joint Rule.

FIG. IX. and X.

LET the two remote Places given be A and B, whose distance I would know, but I cannot approach or come near either of them; and I have no other Instrument but my Two-Foot Joint Rule; however, I make choice of a Place at C, from whence I can see both the Places, A and B, and there I set up a staff whereon to rest my Rule, and opening it to a Square Angle, I look by one side of it, till I espy my first place at A, and there keeping it fast and level, I look by the other side of the Ruler, and cause a Mark to be set up in a right Line from C, at a competent distance from C, as at D, 150 foot; then close in your Rule, till by the side thereof you see your second place at B; keep your Rule at that Angle:

Then having a sheet of Paper, or upon a Board, as Figure IX. draw two Right Lines thereon, as K L, and L M, making a Right (or Square Angle) at L.

Then bring your Ruler, (it being still kept at the Angle it was when you looked to B) and lay the Center of your Ruler upon L, and by the side of it draw a line L M; and because your measured distance between C and D was 150 foot. take 150 quarters of Inches (150 of any equal parts that you have upon your Ruler) and set them down upon your Paper or Board, from L to M.

Then take your Rule and go to D, and set the Center of it upon the Staff, look by one side thereof to C, and by the other to A, then bring the Rule to the Board, and lay the Center thereof on M, and one side upon the line M L, and by the other side draw a line at length, as the line M O, crossing the line L K in O; so shall O upon your Board represent the Place A in the Field: Again, Take your Rule, and go to D, and there resting it upon the Staff, look by one edg to A, and by the other

to B, and keeping it at that Angle, bring it to the Board, and lay one ſide upon the line M O, and by the other draw the line M P, croſſing the line L N in the Point P, ſo ſhall P repreſent upon the Paper the ſecond Place B in the Field, and being meaſured upon the ſame Scale whereof L M was meaſured, it will be found to be 250 foot, and that is the diſtance from A to B. And by this means you may find the diſtances of all the Places in the Figure, if you meaſure them upon the ſame Scale as L M, or O P were meaſured, and ſo ſhall you find

The Diſtance	L O L P M O M P	to Contain	128 317 200 220

Foot.

PROBL. VI.

How to take the Diſtance between One or more Places, by a Ten Foot Rod (divided into Inches) only.

FIG. XI, and XII.

STanding at A, I would know how far it is to the Tree at B, tho I cannot come near it.

Standing at A, I meaſure in a right line from thence 30 foot, from A to *a*: And looking towards D, I meaſure out 30 foot more, as from A to C; and meaſuring the diſtance *a* C, I find it to be 25 foot, which laid down upon Paper, do make the Triangle A C *a*, of which draw the line A *a* out at length.

Then ſtanding at C, I meaſure in a right line towards B, 25 foot, from C to *b*, and the diſtance between *a* and *b*, I meaſure to be 20 foot, which makes the Triangle C *a b*, Draw the ſide C *b* at length, till it croſs the former line A *a*, extended in B; ſo ſhall the line A B (being meaſured by the ſame Scale that the other Lines were laid down by) be found to contain 82 foot, and ſuch is the diſtance between A and B.

And according to this Method may the diſtances from ſeveral Places be meaſured, As in Figure XI. Where ſtanding at G and H, you may find the diſtance between E and F; and alſo all the other intermediate diſtances, as from G or H, to E or F, as alſo the diſtances G E, G F, H E, H F, *&c.* as by *Figure* XII. is evident.

SECT. III.

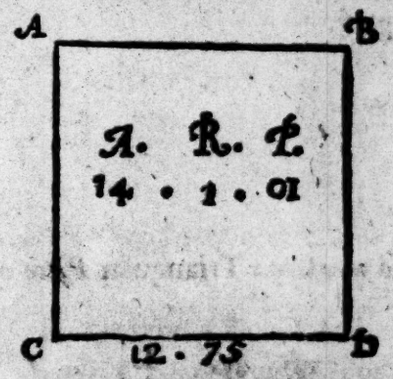
Of Planometria: *Or Land-Meaſuring Mechanically.*

FOR the Meaſuring of Land (by a Chain only without any other Graduated Inſtrument, which I call) Mechanichally: The beſt Chain that I can adviſe you to, is one that is commonly known by the name of *Gunter*'s Chain, and it contains in length four *Rods*, *Poles*, or *Perches* (which are all one), and each Pole contains 16 foot and a half: So that this Chain is 66 foot in length, and is divided into 100 equal Parts or *Links*, which are diſtinguiſhed by having a Braſs *Ring* at every Ten Links end: And one of theſe *Chains* in *Breadth*, and *Ten* of them in *Length*, do make One juſt *Acre* of Land according to the Statute: And

thus

thus much concerning the *Chain* it ſelf. —— And now, before I come to ſhew you how to take the Plot of a Field by help thereof, without any other Graduated Inſtrument, I think it convenient, firſt to ſhew you how any Piece of Land meaſured by ſuch a Chain may be caſt up; that is, to know how many *Acres*, *Roods*, and *Perches*, ſuch a Piece of *Ground* ſo meaſured doth contain. And in order thereunto I ſhall begin with the Menſuration of *Squares*, *Triangles*, &c.

I. *To Meaſure a* Square *Piece of Land.*



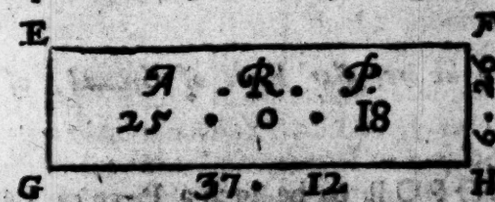
IF your Piece of Ground be exactly four Square, and have all the ſides of equal length, and at right (or ſquare) Angles, as the figure A B C D; and by your Chain you find that each ſide thereof contains 12 Chains, and 25 Links; you have no more to do, but multiply 12 C. 25 L. by 12 C. 25 L. as you ſee done in the Margent. And the Product is 1425625, from which (always) cut off five Figures towards the Right hand, by a Point or Line, and then the Product will be 14.25625; that is 14 Acres, and .25625 hundred thouſand parts of an Acre; and to know how many *Roods* and *Perches* that is, multiply .25625 by 4, (becauſe there are 4 *Roods* in one *Acre*) and the Product will be 1.02500; and five figures cut off, there is 1 beyond the Prick, which is 1 *Rood*, and .02500 remaining, which multiply by 40. (becauſe there are 40 *Perches* in a *Rood*) and the Product will be 1.00000; from which five figures being cut off, there will remain 1. which is one Perch: So that this Square Piece of Land thus meaſured, will contain 14 *Acres*, 1 *Rood*, and 1 *Perch*.

```
 12.75
 12.75
------
 6375
 8925
 2550
 1275
------
A 14.25625

 .25625
      4
------
R 1.02500

  .02500
      40
------
P 1.00000
```

II. *To meaſure a* Square *Piece of Land, whoſe Length and Breadth are unequal.*



LET CDEF be a Furlong in common Field, or the like; and let the length thereof G H be 37 Chains and 12 Links, and the breadth thereof

27.12
9.26

16272
5424
24408

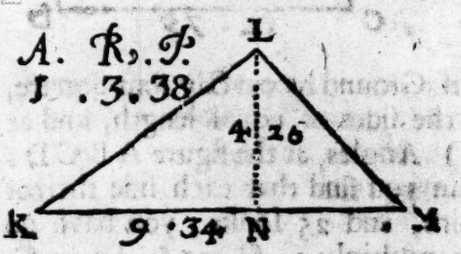
A. 25.11312
4

R. 0.45248
40

P. 18.09920

thereof F H 9 Ch. 26 L. Multiply 27. 12. by 9.26, and the Product will be, five figures being cut off 25. 11312, which is 25 Acres; and the remainder 11312 multiplied by 4, the Product (five figures being cut off), is 0. 45248, which is 0 Roods. And the remainder 45248 multiplied by 40, and five Figures cut off, will be 18. 09920, which is 18 Perches; so that this Piece of Land contains 25 *Acres*, 0 *Roods*, 18 *Perches*. As is seen in the Margent.

III. *To measure a* Triangular *Piece of Land.*



4.67
4.26

2802
934
1868

A 1.98942
4

R. 3.95768
40

P. 38. 30720

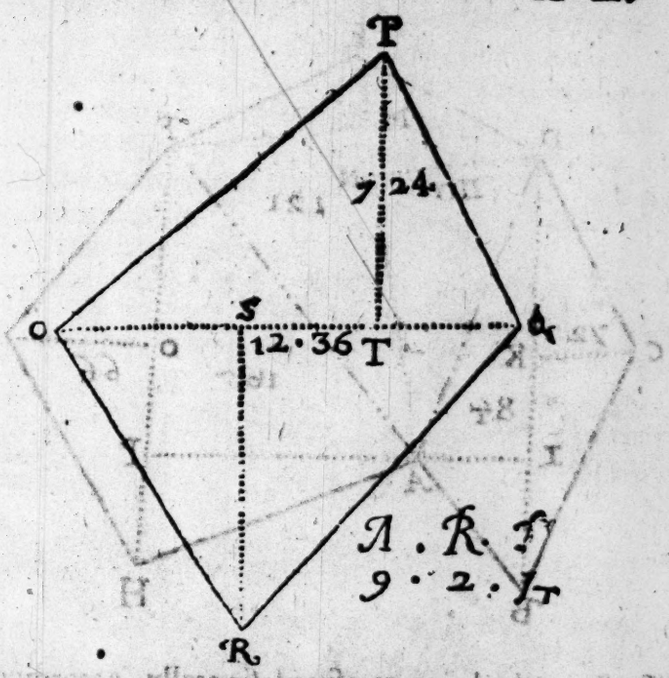
TO measure Triangular Pieces of Land, you must multiply Half the length of the *Base* by the *Perpendicular*, and that Product shall give the Contents of the Piece.

So in the *Triangular Piece* of Land K L M, the length of the line K M is 9 Ch. 34 L. the half thereof is 4 Ch. 67 L. this multiplied by 4 Ch. 26 L. the length of the Perpendicular L N, the Product (five figures being cut off) is 1 Acre, and .98942 remaining; which multiplied by 4, the Product is 3 Roods, and .95768 remaining; which multiplied by 40, it produceth 38 Perches, &c. as in the Margent: So that this Triangular Piece of Land contains 1 Acre, 3 Roods, and 38 Perches; which is 2 Acres wanting only 2 Perches.

IV. *How to measure an irregular Piece of Ground, of Four unequal Sides.*

SUppose the figure O P Q R to be such a Piece of Land to be measured by the Chain only.

First



First, measure the Length or Diagonal thereof O Q, finding it to contain 12 Chains, 36 Links, which set down; then measure the Perpendiculars P T and R S, finding one to be 7 C. 24 L. the other 8 C. 28 L. which added together makes 15 C. 52 L. the half whereof is 7 C. 76 L. which multiplied by the length O Q, 12 C. 36 L. the Product is 9 Acres, and .59136, which multiplied by 4, the Product is 2 Roods, and .36544 remaining; and that multiplied by 40, produceth 14 Perches, &c. as in the Margent; and the whole Content of the Triangle is 9 *Acres*, 2 *Roods*, and 14 *Perches*.

12.36
7.76

7416
8652
8652

A. 9. 59136
4

R. 2. 36544
40

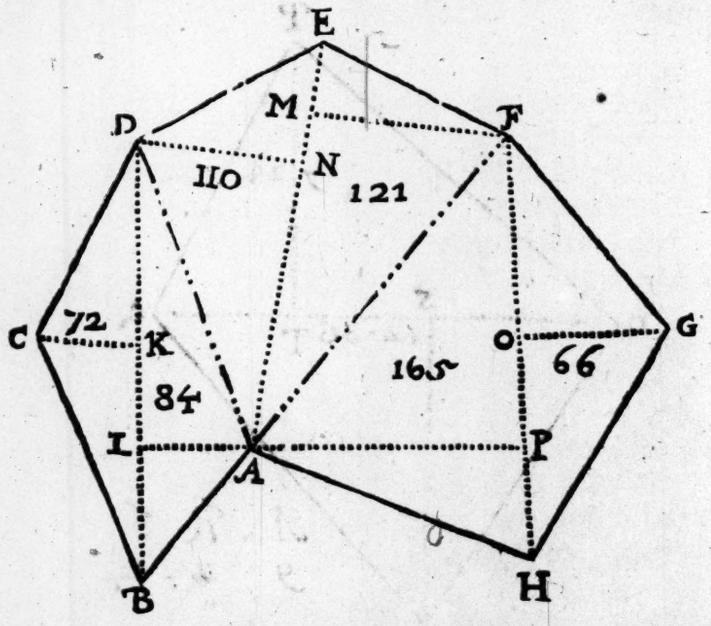
P. 14 61760

V. *Of Irregular Pieces of Ground of many Sides, how to reduce them into Triangles, and to measure them.*

LET A B C D E F G H be a *Field* to be measured: In regard that the Field is irregular, it must therefore be first reduced into *Triangles* by drawing (or imagining) lines to be drawn from one Angle to another, as the lines A D, D B, A E, and F H; by which lines so drawn (or imagined) the whole *Figure* will be reduced into Six *Triangles*, namely,

Into the Triangle
B C D
A D B
A D E
A E F
A F H
F G H

These



These ſix *Triangles* being meaſured ſeverally, according to the directions before given, the Contents of all of them added together into one Sum, will give you the Content of the whole *Piece* in Acres, Roods, and Perches.

			A.	R.	P.
Suppoſe the Triangle	B C D	Should be found to contain	1	1	16
	A D B		1	2	12
	A D E		2	0	10
	A E F		2	1	03
	A F H		3	0	15
	F G H		1	0	38
		The Sum	11	2	14

Which is the Content of the whole Field.

But for an Abbreviation of this Work, you need not to find the Content of every ſingle *Triangle*, but of every *Trapezia* or *Four-ſided Figure*, as is taught in the Fourth *Problem* next before going; for the *Figure* is as well divided into *Trapezias* as into *Triangles*, namely, into the Three *Trapezias* A B C D, A D E F, and A F G H; and ſo by this means you will abbreviate half your work; for if you meaſure the Three *Trapezias* ſeverally, you ſhall find

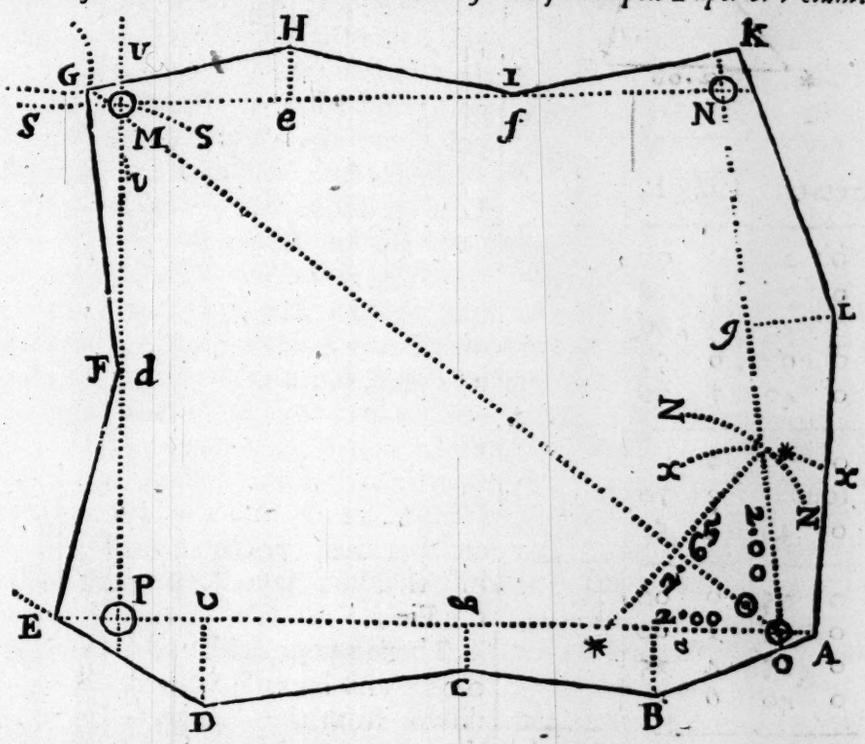
			A.	R.	P.
The *Trapezia*	A B C D	to contain	2	3	28
	A D E F		4	1	13
	A F G H		4	1	13
		The Sum	11	2	14

Note, That of what *Number* of *Sides* your *Figure* conſiſts of, the *Number* of *Triangles* into which it will be *reduced*, will (always) be leſs by *Two* than the Number of *Sides*: As in this *Figure*; the *Number* of *Sides* are *Eight*, and the *Triangles* are but Six; two leſs.

And

And thus have I done with the *Chain*, as to the deſcription of it, and how to caſt up any *Figure* or *Piece* of *Land*, *Regular* or *Irregular*, *meaſured* thereby, in *Acres*, *Roods*, and *Perches*: It reſteth now, that I ſhew you how to make uſe of it in the *Field*, in meaſuring of any Irregular Piece of Ground; and that ſhall be the Work of this next Paragraph.

VI. *How to take the true Plot of any Irregular Field of many Sides and Angles, by the forementioned Chain only, without any Graduated Mathematical Inſtrument; and to make a Plot of the ſame upon Paper or Velum.*



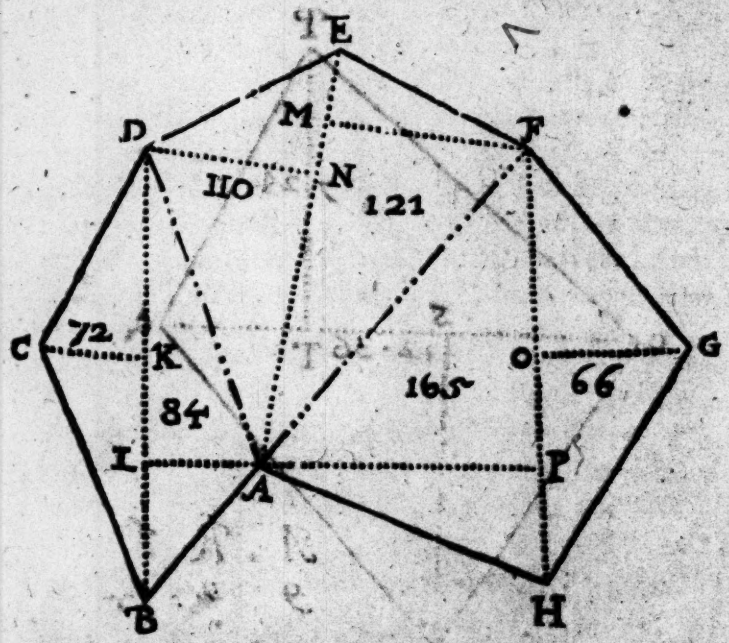
LET ABCDEFGHIKL, be an Irregular *Field* to be meaſured as aforeſaid.

When you firſt enter the *Field*, cauſe *Beacons* or other *Viſible Marks* to be ſet up near the principal *Angles* or *Corners* of the *Field*, as thoſe at M, N, O, and P.

Secondly, Conſider the beſt corner to begin at (tho any will ſerve) as I make choice of that Mark at O.

Thirdly, Take out your Chain, and holding one end at the Beacon at O, meaſure out 2 Chains (or more or leſs, as you ſee occaſion) from the Beacon at O towards the Beacon at N, and at the end of the 2 Chains ſet up a ſmall Bow or Stick at *. ——Then again, from your Beacon at O meaſure 2 Chains more in a right line towards P, and there ſet up another ſmall Bow or Mark as at *. ——And then meaſure with your Chain the diſtance between your two Marks * and *, which here is 2 Chains 65 Links.

Fourthly, Prepare a Book, or Sheet of Paper, Ruled as in the Margent, to ſet down your Meaſures as you go along, but firſt at the top of it make a Triangle anſwerable to that which you meaſured out in the Field, and ſet ſuch Numbers to it as you there meaſured, as 2 Chains from ⊙ to * both ways, and 2 Chains 65 Links from * to *.



These six *Triangles* being measured severally, according to the directions before given, the Contents of all of them added together into one Sum, will give you the Content of the whole *Piece* in Acres, Roods and Perches.

			A.	R.	P.
Suppose the Triangle	B C D	Should be found to contain	1	1	16
	A D E		1	2	12
	A D E		2	0	10
	A E F		2	1	03
	A F H		3	0	15
	F G H		1	0	38
		The Sum	11	2	14

Which is the Content of the whole Field.

But for an Abbreviation of this Work, you need not to find the Content of every single *Triangle*, but of every *Trapezia* or Four-sided *Figure*, as is taught in the Fourth *Problem* next before going; for this *Figure* is as well divided into *Trapezias* as into *Triangles*, namely, into the Three *Trapezias* A B C D, A D E F, and A F G H; and so by this means you will abbreviate half your work; for if you measure the Three *Trapezias* severally, you shall find

			A.	R.	P.
The *Trapezia*	A B C D	to contain	2	3	28
	A D E F		4	1	13
	A F G H		4	1	13
		The Sum	11	2	14

Note, That of what *Number* of *Sides* your *Figure* consists of, the *Number* of *Triangles* into which it will be *reduced*, will (always) be less by *Two* than the Number of *Sides*: As in this *Figure*; the *Number* of *Sides* are *Eight*, and the *Triangles* are but Six; two less.

And

And thus have I done with the *Chain*, as to the deſcription of it, and how to caſt up any *Figure* or *Piece* of *Land*, *Regular* or *Irregular*, *meaſured* thereby, in *Acres*, *Roods*, and *Perches*: It reſteth now, that I ſhew you how to make uſe of it in the *Field*, in meaſuring of any Irregular Piece of Ground; and that ſhall be the Work of this next Paragraph.

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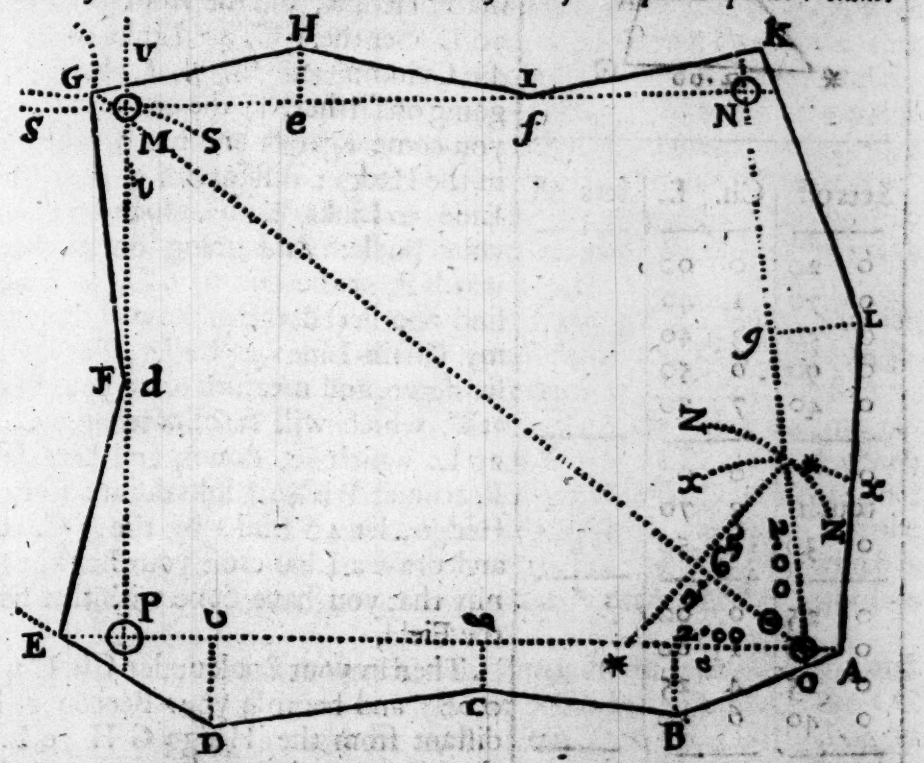

LET ABCDEFGHIKL, be an Irregular *Field* to be meaſured as aforeſaid.

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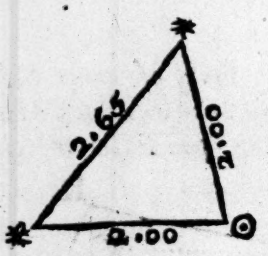
Secondly, Conſider the beſt corner to begin at (tho any will ſerve) as I make choice of that Mark at O.

Thirdly, Take out your Chain, and holding one end at the Beacon at O, meaſure out 2 Chains (or more or leſs, as you ſee occaſion) from the Beacon at O towards the Beacon at N, and at the end of the 2 Chains ſet up a ſmall Bow or Stick at *. ——Then again, from your Beacon at O meaſure 2 Chains more in a right line towards P, and there ſet up another ſmall Bow or Mark as at *. ——And then meaſure with your Chain the diſtance between your two Marks * and *, which here is 2 Chains 65 Links.

Fourthly, Prepare a Book, or Sheet of Paper, Ruled as in the Margent, to ſet down your Meaſures as you go along, but firſt at the top of it make a Triangle anſwerable to that which you meaſured out in the Field, and ſet ſuch Numbers to it as you there meaſured, as 2 Chains from ⊙ to * both ways, and 2 Chains 65 Links from * to *.

This done, go into the Field to your Beacon at O, and measure the distance of it from the hedge on your left hand, finding it to be 20 Links of your Chain, which set down on the left hand Column of your Book, against 0 Ch. 00 L. because the Hedge was on your left hand.



Sets off	Ch. L.	Sets off
0 20	0 00	
0 70	1 40	
0 50	3 40	
0 90	6 50	
0 40	7 20	
0 70	0 00	
touch	2 70	
0 30	5 50	
0 25	0 00	
0 60	1 80	
0 15	4 25	
0 40	6 50	
0 30	0 00	
0 95	2 70	
0 40	5 90	

Then with your Chain measure from the Beacon at O, towards that at P, in a right line, and as you go along, at the end of 1 C. 40 L. you find a Break or Bend in the Hedge, distant from your Chain 70 L. Set the 1 C. 40 Links in the middle Column, and the 70 L. by it; then going on farther, at the end of 3 C. 40 L. you come against another Break or Bow in the Hedge; distant from your Chain-Line 50 Links, both which set down in your Book: And going on farther towards P, at the end of 6 C. 50 Links, I find another Break or Bow distant from my Chain-Line 90 Links, both which set down, and measure on to your Beacon at P, which will terminate at 7 C. and 20 L. which set down, and because the Beacon at P is 40 Links distant from the Hedge, set 40 Links by the 7 C. 20 L. and draw a Line cross your Book, to signify that you have done with that side of the Field.

Then in your Book under Ch. L. write 0 00; and because your Beacon at P is distant from the Hedge G H 70 L. set that down against 0 00, and go on and measure towards the Beacon at M, and when you have measured 2 C. 70 L. you come to *touch* the Bow in the Hedge at F, wherefore set down 2 C. 70 L. and against it write *Touch*. Then measuring on to your Beacon at M, you find the length to be 5 Ch. 50 L. which set down, and by it 30 L. which is the distance of the Beacon M from the Hedge. Thus having finished this side of the Field, draw a Line cross your Book, as before.

And in this manner must you deal with the other two sides, M N and N O, till you come where you first began. And this way for measuring of any irregular piece of Ground, is as exact as is possible to be performed by the best Graduated Instrument that can be made.

And

And thus having ſhewed you how to meaſure in the Field, and take account thereof in your Book, it reſteth now to ſhew you how to lay the Plot or Figure of this Field down upon Paper or Velum, in order to the finding of the *Content* or *Quantity* of it in *Acres*, *Roods*, and *Perches*.

VII. *How to lay down upon Paper or Parchment any Piece of Ground taken by the Chain as before, in order to finding the Content or Quantity thereof.*

FOR the performance of this Work, you muſt provide for your uſe a Pair of very fine-pointed *Compaſſes*, and alſo a Ruler with ſeveral *Scales* of *Equal Parts*, ſuch as is mentioned in the next *Chapter*, with a figure thereof. This Plot here deſcribed, is laid down by a Scale, that half an Inch is equal in length to one whole *Chain* of *Four Pole*; and the half *Inch* in the *Scale* is divided by *Diagonals* into 100 parts, anſwerable to the 100 *Links* into which your *Chain* is divided. Being thus provided of *Scales* and *Compaſſes*, you may begin your *Plot* in this manner.

1. Upon a *ſheet of Paper* draw a right line at pleaſure, as the line P O; towards one end thereof, as at the *Station* or *Beacon* by O, make a Mark, as ⊙.

2. With your Compaſſes, out of your *Scale* take 2 Chains, (which in this Plot is one Inch) and ſetting one foot of the Compaſſes in ⊙, the other will reach to the Point *, upon the line O P, and one foot of the Compaſſes ſtill reſting in the point ⊙, with the other deſcribe the obſcure Arch x x. Then take out of your Scale 2 Ch. 65 L. (which was the diſtance you meaſured from * to *) and ſetting one foot of that diſtance upon the Point *, in the Line O P, with the other deſcribe another obſcure Arch z z, croſſing the former in the Point *, which is upon the line O N; and through the Point * draw another right line ⊙ * N at length.

3. Theſe two Lines being drawn, repair to your *Note-Book*; and therein finding your *Chain-Line* O P to be 7 Chains 20 Links, take 7 C 20 L. from your Scale, and ſet them from ⊙ to P. Alſo finding by your Book, that the Chain-Line O N did contain 5 C. 90 L. take that alſo out of your Scale, and ſet it upon the Line from O to N; then the Chain-Line P M being 5 C. 50 L. take them out of your Scale, and ſetting one foot of the Compaſſes in P, with the other deſcribe an obſcure Arch s s; and the Chain-Line M N being 6 C. 50 L. take them out of your Scale alſo, and ſetting one foot of the Compaſſes in N, with the other foot deſcribe the obſcure Arch v v, croſſing the former in the ſtation point ⊙ by M, through which Point draw the right lines P M and N M at length: And thus have you drawn upon Paper the Quadrilateral Figure M N O P, of the ſame Length and with the ſame Angles as you meaſured them with your Chain in the Field.

4. Having gone thus far, lay your *Note-Book* before you; and ſeeing that at your beginning at O, your Beacon at ⊙ did ſtand 20 L. diſtant from the Hedge, take 20 L. out of your Scale, and ſet them from ⊙ to o; alſo at 1 C. 40 L. at a, the bow of the Hedge was diſtant from the Chain-Line 70 L. take firſt 1 C. 40 L. and ſet them from O to a, and the 70 L. from a to B, and draw the line A B: Then take 3 C. 40 L. and ſet them from O to b, and 50 L. from b to C, and draw the line B C. Then take 6 C. 50 L. and ſet them from O to c, and 90 L. from c to D, and draw the line C D. Then at 7 C. 20 L. which is at the

Beacon P, ſet 40 Links to the Hedge, and through that Point draw the Line D E: So is this ſide of your Field finiſhed.

5. Begin again at the Beacon by P, where you find by your Book, that the Beacon was diſtant from the Hedge 70 L. ſet 70 L. from ⊙ to E. Then at 2 C. 70 L. you find that your Chain-Line did touch the Hedge; wherefore take 2 C. 70 L. from your Scale, and ſet them from P to F (or d) and draw the line E F; then your next length 5 C. 50 L. will reach from P to M, where the Beacon was 30 L. from the Hedge, ſet 30 L. from ⊙ to the Hedge, and through that point draw the line F G. And thus have you finiſhed your ſecond ſide: And in the ſame manner you muſt deal with the other two ſides; and in ſo doing you ſhall have compleated your Work, and have the exact *Draught* of your Field upon a ſheet of Paper, which you may caſt up into *Acres*, *Roods*, and *Perches*, by the Directions given in the preceding Third *Paragraph*.

And if you meaſure the ſmall Triangles, and other four-ſided Figures which are without the Chain-Lines, by themſelves, and afterwards the Quadrilateral Figure M N O P, by it ſelf, (which is the better way) you will find this Field to contain 4 Acres, 2 Roods, and 35 Perches.

CHAP. V.

Wherein is ſhewed how to take all manner of Heights and Diſtances; and to meaſure all manner of Land, of what kind ſoever, Inſtrumentally.

IN the foregoing *Chapter* you are taught how to take *Heights* and *Diſtances*, and alſo to *meaſure Land Mechanically*, whereby any perſon may very well (with more labour) perform many *Geometrical Concluſions* by ſlight and common *Tools* uſed by *Mechanick Artificers*, as by *Squares*, *Joynt-Rules*, *Rods*, &c. But in this *Chapter* I ſhall ſhew more Artificially how to perform the forementioned by *Graduated Inſtruments*: And altho I have handled this Subject at large in my *Compleat Surveyor*, yet I will here give a ſufficient and exact way to do the like by an *Inſtrument* both portable and exact, not there mentioned; namely, a *Semicircle*, with its Appurtenances.

A Deſcription of the Semicircle.

A *Semicircle* in *Geometry* is thus defined, *A* Semicircle *is the one half of a* Circle *cut off by the* Diameter, *and is contained under a right Line, which is the* Diameter; *and an Arch or crooked Line, which is called ſometimes the* Circumference, Perifery, *or* Limb.

The *Semicircle* which we here intend to deſcribe and ſhew the uſe of, is uſually made in Braſs, the *Diameter* whereof may contain in length about 10 Inches, the *Semidiameter* half as much; the *Diameter* and *Limb* are either of them about an Inch in breadth: The *Limb* of this *Semicircle* is divided into 180 equal parts, called *Degrees*, and every of thoſe *Degrees* is uſually divided into ſmaller equal parts, according to the bigneſs of the *Inſtrument*, (uſually into four) ſo is every ſmall diviſion or part, 15 *Minutes*. It is ſometimes divided by *Diagonals*, ſo that you may (the

the

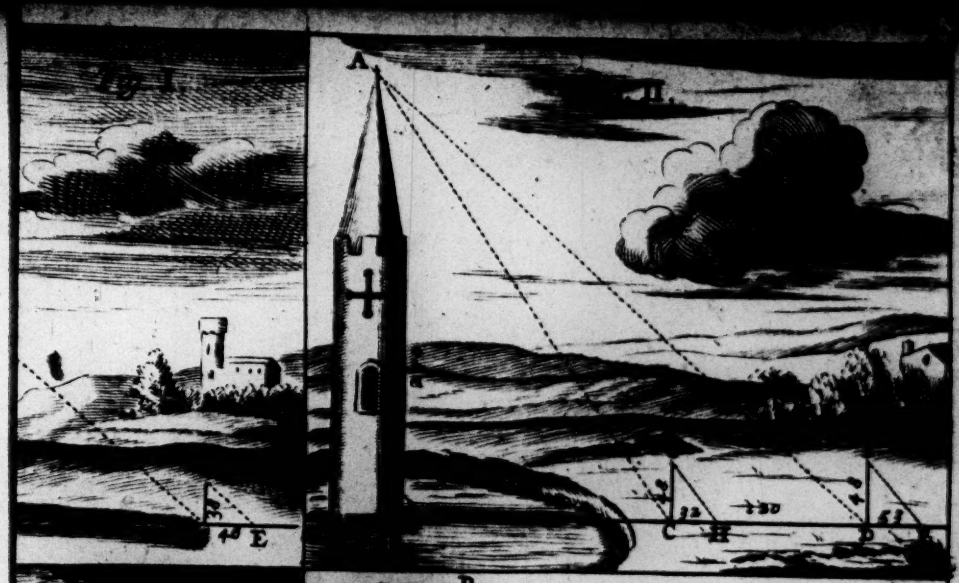


Fig. I.
A
E
C
H
D
120

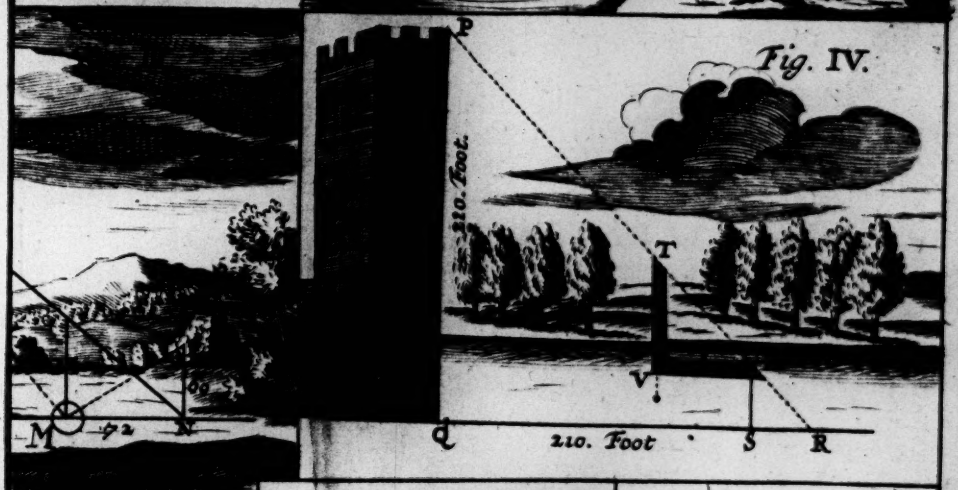


Fig. IV.
P
210. Foot.
T
V
M
N
Q
210. Foot
S
R

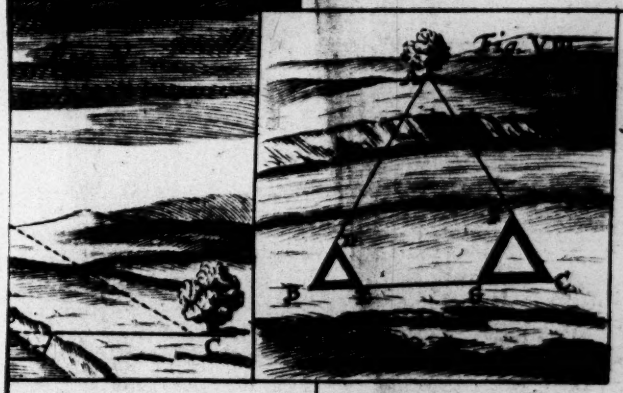


Fig. VIII.
E
C

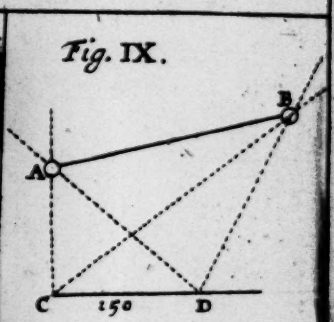


Fig. IX.
B
A
C
150
D

Fig. VI.

Fig. VII.
G

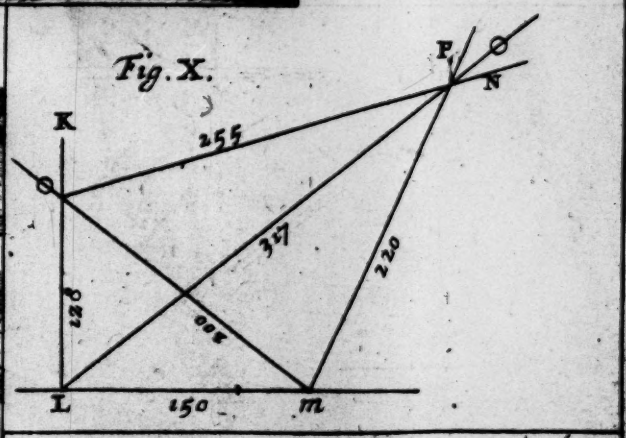


Fig. X.
F
N
K
255
347
220
120
085
L
150
m

B

Fig. XI.

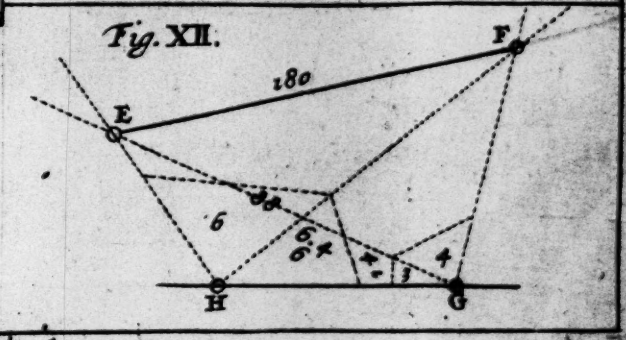
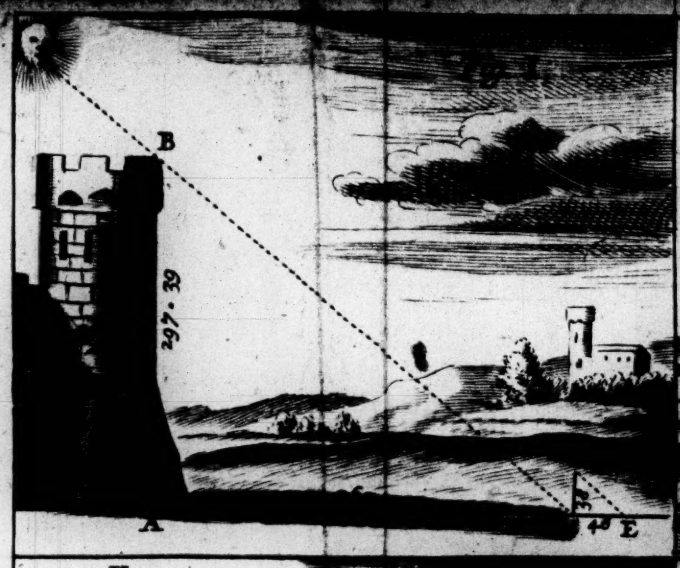
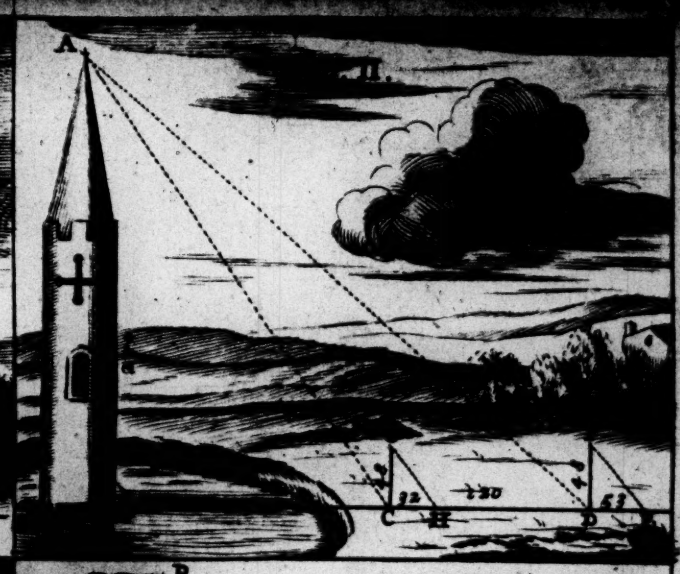


Fig. XII.
F
180
E
6
6
4
H
G





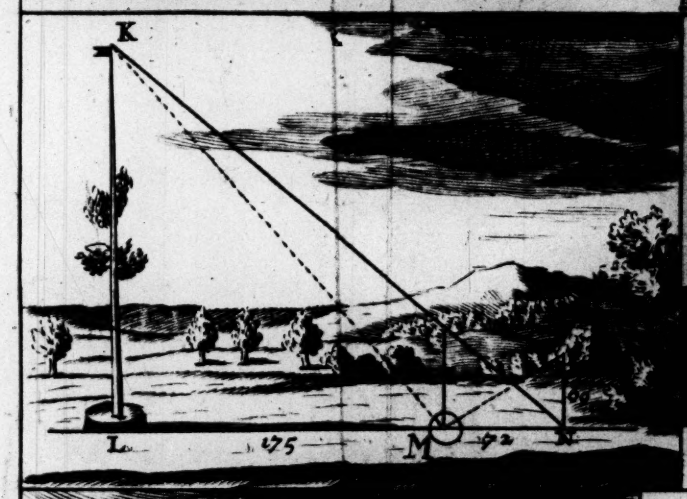
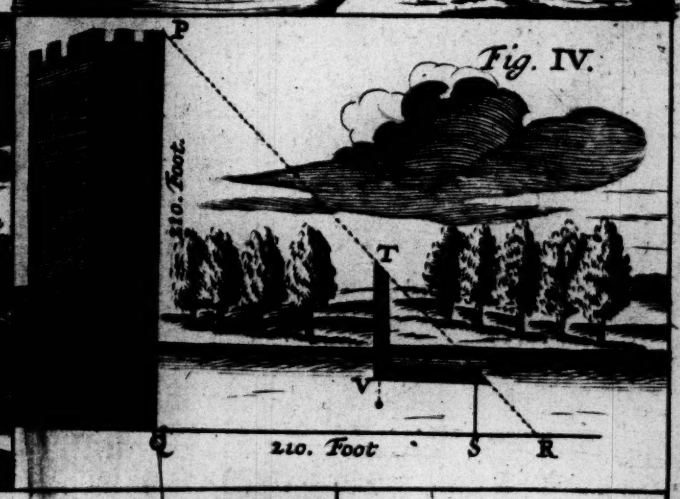


Fig. IV.



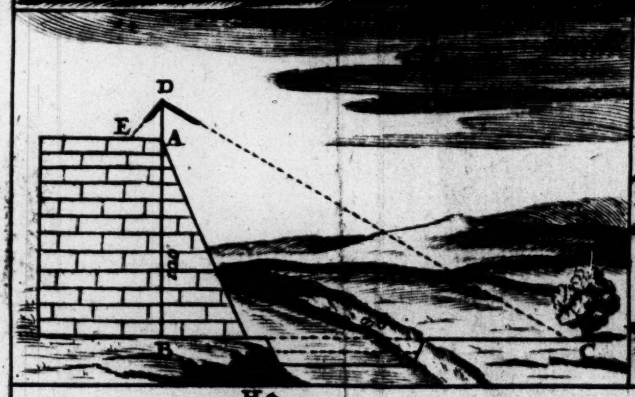


Fig. IX.

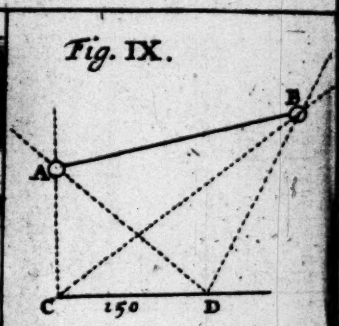


Fig. VI.

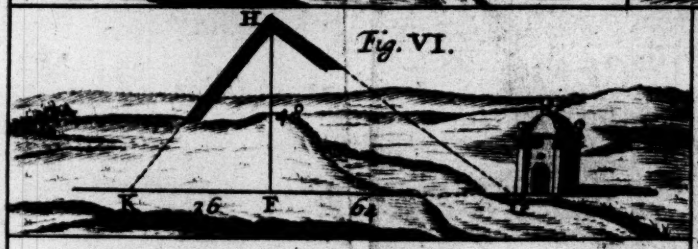
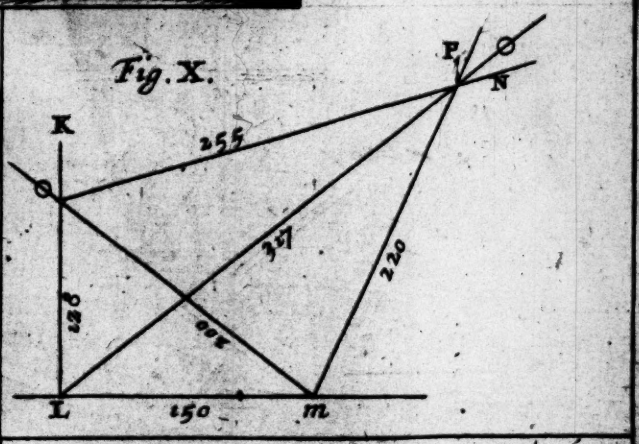


Fig. VII.

Fig. X.



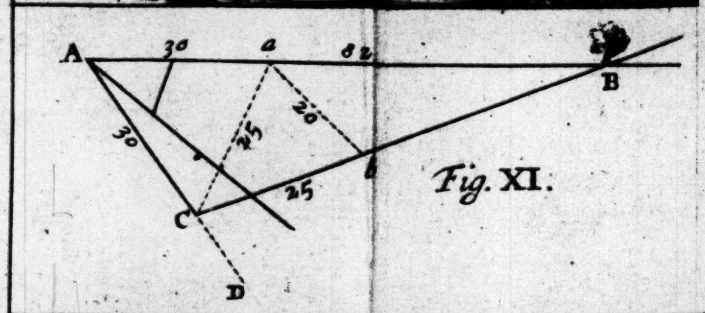
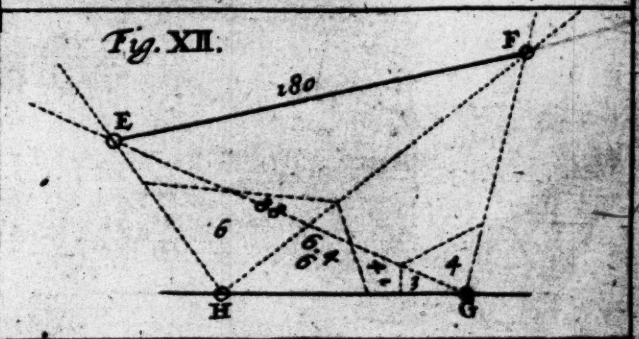


Fig. XI.

Fig. XII.



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the Inſtrument be ſmall) take off every third or fourth Minute. Theſe Degrees are numbred from the left hand towards the right, by 10, 20, 30 &c. to 180; and back again from the right hand towards the left, by 190, 200, 210, &c. to 360.

Upon the *Centre* of this *Semicircle* is placed a round *Box* of Braſs, in the bottom whereof is a *Chard*, having upon it the 32 points of the Mariners Compaſs, and over it a *Needle* covered with Glaſs; this *Needle* is at any time to ſet the *Diameter* of the *Semicircle* due *North* and *South*, or according to any other *Coaſt*.

Between the *Semicircle* and this *Box* is an *Index*, which moves upon the *Centre* of the *Semicircle*, and is of ſufficient length to cut at either end thereof the *Degrees* on the *Limb* of the *Semicircle*. It hath upon it two *Sights*, through which you may look either backward or forward, to any *Mark* or *Object*.

On the backſide of the *Semicircle* is ſcrewed a *ſocket*, into which goeth another *ſocket* called a *Ball-ſocket*, which ſerveth to ſet the *Semicircle* when it is upon the *Staff* in any poſition whatſoever, as *horizontal* or *level*, *vertical* or *perpendicular*, or *reclining*, or *inclining* to any Angle.

Into this *Ball-ſocket* there goes the head of a *three-legged ſtaff*, which ſupporteth the *Inſtrument* in the field.

Theſe are all the parts of the *Semicircle*, as it is to be uſed in the fields to make obſervations; but to lay down your Work at home, there belongeth,

1. *A Protractor.*
2. *A Scale.*
3. *A Pair of Compaſſes.*
4. *A protracting Pin.*

1. The *Protractor* is made of a thin piece of Braſs divided in all reſpects as the *Semicircle* was, and numbred in the ſame manner from 1 to 180 degrees, but from 180 degrees to 360 degrees; it ought to be numbred from the left hand towards the right, contrary to the numbering of your *Semicircle*. The uſe of this *Protractor* is to lay down the quantities of thoſe Angles upon *Velum* or *Paper*, which you obſerve in the field.

2. Your *Scale* is a Ruler of Braſs, upon which you may have what Scales you pleaſe, either *plain* or *diagonal*, as of 10, 11, 12, 16, 20, 24, 30 and 32 in an Inch; theſe *Scales* are to lay down the lengths of your Lines, as you meaſured them in the field with your Chain; and according to the largeneſs or ſmalneſs of your Scale, you may make your Plat or Draught of what bigneſs you pleaſe.

3. Your *Compaſſes* are to take your diſtances from your Scale, and to apply them to your Paper or Velum.

4. Your *Protracting Pin* is to fix in your Center Point, and alſo to mark the degrees on the edge of your *Protractor*.

The *Chain* which you meaſure your lengths in the field, may be of what length, and how you will divided; but the Chain which the Examples in this Book are wrought by, is 4 Pole in 100 Links.

the Inſtrument be ſmall) take off every third or fourth Minute. Theſe Degrees are numbred from the left hand towards the right, by 10, 20, 30 &c. to 180; and back again from the right hand towards the left, by 190, 200, 210, &c. to 360.

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2. Your *Scale* is a Ruler of Braſs, upon which you may have what Scales you pleaſe, either *plain* or *diagonal*, as of 10, 11, 12, 16, 20, 24, 30 and 32 in an Inch; theſe *Scales* are to lay down the lengths of your Lines, as you meaſured them in the field with your Chain; and according to the largeneſs or ſmalneſs of your Scale, you may make your Plat or Draught of what bigneſs you pleaſe.

3. Your *Compaſſes* are to take your diſtances from your Scale, and to apply them to your Paper or Velum.

4. Your *Protracting Pin* is to fix in your Center Point, and alſo to mark the degrees on the edge of your *Protractor*.

The *Chain* which you meaſure your lengths in the field, may be of what length, and how you will divided; but the Chain which the Examples in this Book are wrought by, is 4 Pole in 100 Links.

The Use of the *Semicircle*, in taking of *Heights*, *Depths*, and *Distances*.

PROP. I.

How to take the Height of any Tree, Tower, Steeple, *or other* Object, *which standeth upright, and being accessible at one Station.*

FIG. I.

LET AB be a *Castle-Wall*, whose height you would know; you standing at C, place your Semicircle at G, and turning it about by help of your Ball-socket, direct it to the Object; then fixing all your Sockets fast, by help of the screws for that purpose, hang a Thrid and Plummet upon the Center of your Semicircle, and move the Semicircle up and down till the Thrid and Plummet hang directly upon 90 deg. then laying the Index just upon the Diameter, there hold it, and looking through the Sights, mark what part of the *Castle-Wall* you see, for that part is in the true level with your eye, which point let be D; then, the Index still remaining fixed, move the Semicircle upwards or downwards, till (through the sights) you see the very top of the *Castle-Wall* at A, (the Semicircle still remaining immoveable) look what number of degrees and parts the Thrid cutteth, which degrees let be 55, which you must set down; lastly, measure the distance from G to B, which let be 200 Foot, to which add 5 Foot, the height of your Semicircle's Center from the ground C E, and it makes 205 Foot; by help of this distance, and the degrees before noted down, you may find the altitude of the *Castle-Wall*, as followeth.

Example.

Upon a piece of Velum or Paper, draw a line at length, as M N; towards one end whereof, as at B, erect a Perpendicular A B, representing the *Castle-Wall*; then from one of your *Scales* take with your Compasses 205 Foot, and set that distance upon your Paper or Velum, from B, the foot or bottom of the *Castle-Wall*, to C, so shall C upon your Paper represent the place on the ground where the *Semicircle* stood, and 5 Foot more being added (which is the height of your *Semicircle* from the ground) E C maketh 205 Foot, which taken from your Scale will reach from B to F.

This done, place the Center of your Protractor upon the Point F, and the Diameter thereof upon the line M N, then count 35 degrees (which is so much as 55 deg. which the thrid cut, wants of 90 deg.) upon the edge of the Protractor, and against it make a *mark* or *point* with your *protracting Pin*, through which point (or mark) and the point F, draw the line F A, cutting the line A B, representing the Castle-Wall in A; so shall A B be the height of the Wall, which being measured by the same Scale as B F 205 foot was taken from, will be found to be 143 foot; and so high is the Castle-Wall from the ground.

PROP.

PROP. II.

How to take the Height of an Object, which is not acceſſible at two Stations, by help of the Semicircle.

SUppoſe A B to be a Caſtle-Wall as before, and you cannot come nearer to the Wall than G, for that there is a Moat about the Wall ſo broad.

Firſt, Place your Semicircle at G, the edge of the Moat or Trench, and directing the ſights to A, the top of the Wall, you ſhall find the Thrid to cut 30 deg.

Secondly, Go backwards a competent diſtance of ground, in a right line, as to F, and there placing your *Semicircle* as before, direct the ſights to A, where you ſhall find the Thrid to cut 55 deg.

Thirdly, Meaſure the diſtance between G and F, which let be 125 Foot; theſe two Obſervations being made at G and F, and the diſtance between them meaſured by the Chain, the altitude of the Wall may be eaſily attained in manner following.

Example.

Firſt, Upon Paper, or the like, draw a line M N, upon which, towards one end thereof, as at F, place the Center of your Protractor; and becauſe the Thrid there cut 55 deg. make a mark againſt 35 deg. of your Protractor (which is ſo much as 55 deg. wants of 90 deg.) and through this point, and the point F, draw a right line at length upwards.

Secondly, For that the diſtance between G and F (your two ſtations) was 125 Foot, take 125 out of ſome one of your ſcales, and ſet off that diſtance upon your paper from F to G.

Thirdly, Lay the Centre of your Protractor upon G, and the Diameter thereof upon the line M N; and becauſe when you made Obſervation at G, the Thrid cut 30 deg. make a mark againſt 60 deg. of the Protractor, (which is ſo much as 30 deg. wants of 90 deg.) and through this mark, and the point G, draw a right line at length upwards, continuing it till it cut the former line drawn from F, which it will do in the point A.

Laſtly, From the point A let fall a perpendicular upon the line M N, which will fall upon the point B, ſo is A B the height of the Caſtle-Wall, which if you take in your Compaſſes, and meaſure it upon the ſcale from whence you took your diſtance F G, you ſhall find it to contain 143 Foot, and ſo high is the Caſtle-Wall.

PROP. III.

A Fort or Caſtle being beſieged, how the Beſiegers ſhall know of what length to make Scaling-Ladders that ſhall reach from the edge of the Moat or Trench to the top of the Wall.

LET A B be the Wall of a Fort or Caſtle, and that the Caſtle within were beſieged, the Beſiegers lying at F, and cannot come nearer

to

to the Wall than G, the edge of the Moat; wherefore ſtanding at G, make Obſervation with your Semicircle as you did before, and you ſhall find the Thrid to cut 30 deg. ——— Do the like at F, and you ſhall find the Thrid to cut 55 deg. and the diſtance between G and F meaſured will be found to be 125. —— Theſe two Obſervations being made, and the diſtance F G meaſured (all which are the ſame as in the laſt *Prop.*) you may protract or lay down the ſame upon Velum or Paper, in all reſpects as in the former *Prop.* and ſo meaſuring the line G A upon the ſame ſcale that G F was meaſured by, you ſhall find it to contain 166 Foot, and ſo long muſt Ladders be to ſcale the Walls of this Fort or Caſtle.

PROP. IV.

Standing upon a Wall or Tower of a known height, to find how far any Ship, Tree, &c. *is from you.*

LET A be the top of a Tower or Caſtle ſtanding by the Sea-ſide, and let F be a Ship lying at Anchor, and you would know how far that Ship is off the Caſtle-Wall.

Standing upon the Tower-Wall at A, with your *Semicircle*, direct the ſights (the Index lying upon the Diameter) to F, the Thrid will cut 55 deg. Then, the Caſtle-Wall being 143 Foot high, by help of theſe two you may find the diſtance that the Ship at F is from the Caſtle-Wall B, in this manner:

Upon Paper or Parchment draw a line A B, repreſenting the Caſtle-Wall, and upon it by help of your ſcale, ſet the height thereof 143 Foot, from A to B, and upon the point B erect the Perpendicular B F.

Then placing the Center of your Protractor upon A, turn it about upon that Point, till 55 deg. (which were the degrees cut by the Thrid) come to lye directly upon the line A B, then at 00 deg. (or the beginning of the Semicircle of the Protractor) make a mark cloſe by the edge thereof, through which point and the point A, draw a line at length downwards, which line being drawn, will cut the line B F in the point F, ſo the diſtance B F being taken in your Compaſſes, and meaſured upon the ſame ſcale from whence A B (the height of the Wall) was taken, it will be found to contain 205 Foot, and ſo far is the Ship F from the Caſtle-Wall B.

PROP. V.

How to take an inacceſſible Diſtance at two Stations by the Semicircle.

FIG. II.

SUppoſe you were ſtanding at G, and that it were required of you to know how far diſtant the Tree of A is from you, between which and you there is the River D, ſo that you cannot come near A.

Firſt, Place your *Semicircle* at C, laying the Index on the Diameter thereof, and turn the Semicircle about till through the ſights you ſee the Tree at A, and there fixing the Semicircle, turn the Index about till you ſee the Mark ſet up at B, and there note what degrees the Index cutteth, which let be 110 deg.

Secondly,

Secondly, Remove your Semicircle from C to B (setting up a Mark at G, where your *Semicircle* before stood) and laying the Index on the Diameter thereof, turn the *Semicircle* about till through the sights thereof you see the Mark at C, where your Instrument before stood, and there fixing the *Semicircle* turn the Index about till by the sights you see the Tree at A, and there also note what degrees the Index cutteth, which let be 40 deg.

Thirdly, Measure the distance between C and B, which let be 120 Foot; by help of these three you shall find the distance of A from either C or B, as followeth.

Example.

Draw a line upon Paper, as A C, and laying the Centre of your Protractor upon C, and the Diameter thereof upon the line C A, with your protracting Pin make a mark against 110 deg. and through that point and the point C, draw a line C B; then from your Scale take 120 Foot, and set it upon the line C B, from C to B.

This done, lay the Center of your Protractor upon the point B, and the Diameter thereof upon the line C B, and make a mark against 40 deg. through which mark and the point B, draw the line B A, cutting the former line C A in the point A.

Lastly, If you take in your Compasses the line C A, and measure it upon the Scale from which you took B C 120 Foot, you will find it to contain 154 Foot and about half a Foot, and such is the distance A C.

In like manner, if you measure A B by the same Scale, you shall find it to contain 224 Foot, and about half a Foot.

PROP. VI.

How to measure the Distance of several Places from you, as also of one from another, by measuring only of one Distance, *and observing of* Angles *by the* Semicircle.

FIG. III.

LET A, B, C, D, E, be several places, as Churches, in a Town or City, or the like, whose distance you require one from another.

First, Make choice of two places, from either of which you may see all the places, whose distance you require, which places let be F and G, distant one from another 100 Foot.

Secondly, Place your Semicircle at F, (and laying the Index on the Diameter, turn the Semicircle about till through the sight you see your other place of standing at G, and there fixing your Instrument, first direct your sights to A, noting what degrees the Index cuteth; then to B, and note what degrees are there cut by the Index, doing the like at C, D, and E; all which degrees note down in a Book or Paper.

Then removing your Semicircle to G, and setting up a Mark at F, lay the Index on the Diameter thereof, and turn the Semicircle about till you see the place of your former standing at F, and there fixing the *Semicircle*, make observation of every place, as you did at F; First, directing the sights to A, noting the degrees cut by the Index, likewise to B, C D, and E.

By these observations of Angles, made at either station F and G, and and the stationary distance, you may draw a perfect draught of all the places upon Paper or Velem, in their true position and scituation, in manner following:

Example.

First, Upon your Velem or Paper draw a line F G, containing 100 Foot of any Scale, which line shall represent the distance of your two stations.

Secondly, Place the Center of your Protractor upon F, and lay the Diameter thereof upon the line F G, and there keeping it steady, make a mark against those deg. which the Index cut upon the *Semicircle* when the sights were directed to A; also make a mark against those deg. of the Protractor which the Index cut at B, and the like at C, D, and E.

Thirdly, Through the point F, and every one of those marks you made against the side of the Protractor, draw lines, as F A, F B, F C, above the line F G, and F D, F E, below the line.

Fourthly, Lay the Center of your Protractor upon the point G, and the Diameter thereof upon the line F G, and there keeping it fast make marks against the edge thereof, at those degrees the Index cut upon the Semicircle, when the sights were directed to A, B, C, D, and E; and through those points and the point G, draw right lines G A, G B, G C, G D, and G E, cutting the former lines drawn from F, in the points A, B, C, D, and E; which points will stand in the same position and scituation as the Churches or other places in the City or Town do; and if you take in your Compasses the distance between any two of them, and measure that distance upon the same Scale that F G 100 was measured from, it will there shew you the distance of those two places.

The Use of the Semicircle in Measuring of Land.

PROP. I.

To take the Plat of a Field *by placing the* Semicircle *in any part thereof, from whence all the Angles of the* Field *may be seen.*

PLace your Semicircle upon its staff in some convenient place of the Field (whose Plat you would take) that from thence you may conveniently see all the Angles thereof, then setting the Semicircle level (by help of the Ball-socket) lay the Index upon the Diameter, and turn the Semicircle about till the Needle hang directly over the North and South Points of the Chard, and there fix the Semicircle, by help of the screw of the *plain Socket*, then doth the Instrument (the Diameter thereof) lye directly in the Meridian, and is fitted for your present use.

Your Instrument thus fixed, cause marks to be set up at every angle of the field (or rather let one go from angle to angle with a long staff having a white cloth or paper tied to the top thereof) and coming to your Instrument, the Diameter being next to you, and the Limb of the Semicircle from you, turn the Index about till through the sights thereof you espy the first of your marks (or man) which is on your right hand,

hand, and when you have directly found your mark, let the Index rest, and see what degrees and parts of a degree of the Semicircle are cut thereby; which degrees and parts note dote down in a book or paper ruled as that which followeth.

In this nature must you deal with every angle round about the field, beginning at that which is next towards your left hand, and proceeding gradually from that towards the right hand, till you find that end of the Index which cut the degrees, falls off of the Instrument, (which will always be when you come past 180 degrees) then must you look through the other sights, and the Limb of the *Semicircle* will be next to you, and you must count your degrees from 180 towards 360; to which, before you come, you will have passed all your Angles.

When you have taken observation of all your Angles, you must, with your Chain, measure from the place where your Semicircle stands, to every Angle (beginning with the first, and so proceeding from the left towards the right hand) and note how many Chains and Links each line contains, which lengths must be set down in your book or paper, in a column by themselves, against the degrees which the Index cut, when you made observation at that Angle, that is, the first length against the first degrees, and the second length against the second degrees; and in this order must you proceed till you have measured all your lengths.

The foregoing Precepts illustrated by Example.

FIG. IV.

LET *a*, *b*, *c*, *d*, *e*, *f*, *g*, be a field to be Surveyed. Place your *Semicircle* (it being a convenient place) at O, the diameter thereof, as also the Needle in the Box, hanging over the North and South Points in the Chard, then will they lye directly over the line N S in this figure; then fix your Instrument, and directing your sights to A (your first Angle) you shall find the Index to cut 12 *d* 15 *m*. which set down in your book or paper; then directing your sights to B, the Index cuts 37 *d*. at C, 94 *d*. 30 *m*. and the rest, as in the following Table, representing your book or paper in which you took your notes.

The Index cut when directed to the mark,	*d.*	*m.*	And the distance from O the place of the Semicircle, was	C.	L.
A	12.	15	OA	10.	35
B	37.	0	OB	9.	45
C	94.	30	OC	10.	30
D	169.	15	OD	6.	20
E	193.	0	OE	8.	93
F	270.	30	OF	8.	05
G	347.	0	OG	5.	20

Having gone through all your Angles, and noted them down in your book or paper, as is done in the foregoing Table, in the column having *d. m* at the top thereof (signifying Degrees and Minutes) you must then go to the measuring of your lines, where you shall find O A to contain 10 Chains 35 Links, O B 9 Chains 45 Links, and the rest as in the Table in that column which hath C. L. at the top thereof, C signifying Chains, and L Links.

When you have thus taken obſervation of all your Lines and Angles; and noted them down in a Book or Paper, you may at pleaſure draw the Plat of the Field upon Velum or Paper, by the Precepts which ſhall be delivered in the following Propoſition.

PROP. II.

How to lay down the Plat of the former Field upon Velum or Paper, by help of the former Obſervations of Lines and Angles.

UPon a piece of Velum or ſheet of Paper, draw a line for the Meridian or North and South Line, repreſented by the line in the Figure by N S. In ſome convenient place of that line (as at O) aſſign a point, which point O upon your paper, repreſents or ſignifies the place in the field where your Semicircle ſtood when you made obſervation of your Angles.

Upon this point O place the Center of your Protractor, laying the Meridian Line or Diameter thereof juſt along the line N S, drawn upon the paper, and there hold it faſt. Then having recourſe to your Table of Angles, which you obſerved in the field, you find your firſt to be 12 deg, 15 min. wherefore, cloſe to the edge of your Protractor, and againſt 12 deg. 15 min. make a mark with your protracting Pin; and ſtill keeping your protractor in the ſame poſition unmoved, make a mark againſt 37 deg. your ſecond Angle, alſo againſt 94 deg. 30 min. your third Angle, and againſt 169 deg. 15 min. your fourth Angle.

Then becauſe your fifth Angle at E exceeds 180 deg. it being 193 deg. you muſt turn the *Semicircle* of your *Protractor* downwards, laying the Center thereof upon the point O, and the Diameter upon the line N S as before, and againſt 193 deg. make a mark with your protracting Pin, do the like againſt 270 deg. 30 min. your ſixth Angle, and againſt 347 deg. your laſt Angle.

Theſe points being marked upon your Paper, through every one of them, and the point O, draw obſcure lines, as O A, O B, O C, O D, O E, O F, and O G. ——Then having recourſe to your Table of Lines which you meaſured in the field, you find that the firſt line O A, contained 10 Chains 35 Links, wherefore from ſome Scale (according to the bigneſs you would have your *Plat*) take 10 C. 35 L. and ſet it upon your paper from O to A, this point A upon the paper ſhall repreſent the firſt Angle *in* the field A; your ſecond length being 9 C. 45 L. take that from your Scale, and ſet it upon your paper from O to B. Do thus with all the reſt of the lengths; and when you have found out and marked upon your paper the ſeveral points A, B, C, D, E, F, and G, if you draw lines from point to point, as from A to B, from B to C, from C to D, *&c.* you ſhall conſtitute the figure A B C D E F G, which ſhall be the exact *Plat* of your *Field* O. And thus may you take the *Plat* of any *Field*, where you may ſee all the Angles from any one place.

PROP.

PROP. III.

How to take the Plat *of a* Field *or other piece of* Ground, *by placing the Semicircle in some one Angle thereof, from whence all the rest may be seen.*

YOur *Semicircle* being placed in the Angle where you intend to make your observation with the Index upon the Diameter thereof, and the Needle hanging over the Meridian Line in the Chard. It being thus fixed, direct your sights to the first Angle towards your left hand, noting the degrees and minutes cut by the Index, and note them down in a book or paper, in all respects as you did those in the former *Proposition.*

Then measure with your Chain from that Angle where your *Semicircle* standeth, to every one of the other Angles, noting the lengths of every of them, and write them also down in your book or paper, proceeding thus from the first line on your left hand, till you come round the field to that again.

Example.

FIG. V.

LET HKLMNOPQ be a piece of Ground to be measured in the manner here prescribed: The *Semicircle* placed in the Angle Q, direct the sights to H, where the Index cuts 15 deg. which note down, then direct the sights to K, where the Index cuts 31 deg. 30 m. likewise to L, M, N, O, P, where the Index cut the degrees and parts, as in the Table following:

The Index cut when directed to the Angle,	d.	m.	And the distance from the Angle Q, as	C.	L.
H	15.	0	QH	9.	90
K	51.	30	QK	10.	25
L	87.	14	QL	11.	60
M	106.	0	QM	12.	30
N	168.	15	QN	12.	0
O	196.	0	QO	10.	85
P	250.	0	QP	6.	88

PROP. IV.

By the former Observations of Lines and Angles, to draw the Plat or Figure of the Field upon Velum or Paper.

DRaw upon Velum or Paper a line N S, representing the Meridian Line, upon which line assign a point as Q, which denotes the place where your *Semicircle* stood in the field.

Upon this point Q, lay the Center of your Protractor, and the Diameter thereof upon the line N S; and there holding it fast, look in your Table what degrees the Index cut when you made observation at H, which were 15 degrees, make a mark against 15 degrees of your Protractor; likewise see by your Table what degrees the Index cut when

when you made obſervation at K, which were 31 deg. 30 min. make a mark againſt 31 deg. 30 min. of your Protractor; and in like manner do with every Angle, as at L, 87 deg. 15 min. at M, 106 deg. 0 min. at N, 168 deg 15 min. at O, 196 deg and at P, 250 deg. at all which numbers of degrees, make marks againſt the ſide of your Protractor.

Then removing your Protractor, draw right lines from the point Q through every one of thoſe marks, ſo ſhall you have upon your Velum or Paper the ſeveral Lines QH, QK, QL, QM, QN, QO, and QP.

Again repair to your former Table, where you ſhall find that the line QH, being meaſured by the Chain in the field, contained 9 Chains, 90 Links, take this diſtance from any of your Scales, and ſet it upon the line QH, from Q to H; and ſo have you the point H upon the paper.

In like manner, take 10 Chains 25 Links from your Scale, and ſet it from Q to K, ſo have you the point K upon paper. And in this manner deal with all the numbers of Chains and Links in your Table, and they will give you the points L, M, N, O, P, upon your Velum or Paper.

Laſtly, If you draw the lines QH, QK, QL, QM, QN, QO, QP; you ſhall thereby conſtitute the true Symetry or Proportion of the field you meaſured, the ſeveral Angles whereof are H, K, L, M, N, O, P, and Q.

PROP. V.

How you may by the Semicircle take the true Plat of any large piece of Ground, as Park, Wood, Marſh, *or other ſpacious Incloſures, by going round about the ſame.*

IN going round about a field (or other ground) to ſurvey, there are two ways, but both effected by the ſame artifice; for in going about a field, you may either go about it on the inſide of the field, or on the outſide thereof; and ſometimes you ſhall be conſtrained to go partly within and partly without; and if you go without the field, you may (if you will) take the quantity of the Angles within; and if you go on the inſide, you may take the quantity of the Angles without.

Of both which ways I ſhall give Examples.

I. *By going about the Field within the ſame.*

Let A B C D E F be a Field to be meaſured by going about the ſame on the outſide.

FIG. VI.

Firſt, Begin at any Angle thereof, as A, and there place your Semicircle, laying the Index upon the Diameter, and turning it about, direct the ſights to B, there fix the Semicircle, and turn the Index about, till through the ſights you ſee the Angle at F, and there note what deg. the Index cutteth, which you will find to be 300, for the quantity of the exterior Angle F A B without the field, or 60 deg. for the quantity of the interior Angle within the field; but (as I ſaid before) going with-

without the Field I make use of the exterior or outward Angles: Then measure the side F A, which you shall find to be 10 Chains 25 Links, and the side A B 7 Chains 50 Links, these distances with the quantity of the Angle note down in a Book or Paper, as you see in the following Table.

Secondly, Remove your Semicircle to B, and laying the Index upon the Diameter, turn the Instrument about, till through the sights you see the place where your Semicircle last stood at A, and then fixing it, turn the Index about, till by the sights you see the Angle C, and there note the *degrees* which the Index cutteth, which here are 145 *degrees*, and the distance B C measured is 7 Chains 10 Links; note both the *degrees* and distance down as before.

Thirdly, Carry your Semicircle to C, and placing the Index on the Diameter, look back to B, and there fix the Instrument, then turn the Index about till you see the Angle at D, where the Index cutteth 270 *degrees*, and the distance C D is 5 Chains 85 Links.

Fourthly, Remove the Semicircle to D, and the Index on the Diameter, look back to C, then fix the Semicircle and turn the Index about, till by the sights you see the Angle at E, the Index cutting 263 *degrees*, and the measured distance D E being 7 Chains, note these down in your book or paper.

Fifthly, Place the Semicircle at E, and the Index lying on the Diameter, look back through the sights to D, then fixing the Instrument there, turn the Index about till by the sights you see the Angle F, the Index cutteth 220 *degrees*, and the measured distance E F being 10 Chains 70 Links, both which you must note down. And,

Sixthly, Carry the Semicircle to F, and the Index lying on the Diameter, look back to E, and then fix the Instrument, then turn the Index about till by the sights you see your first Angle at A, the Index then cutting 210 *degrees*; these being noted down will stand as in this Table.

		d.	*m.*	C.	L.	
Angles.	A	300	0	10	25	Sides.
	B	145	0	7	50	
	C	270	0	7	10	
	D	263	0	5	85	
	E	210	0	7	00	
	F	210	0	10	70	

These Observations of Sides and Angles being made in the Field, and noted down as in the foregoing Table, a Plat thereof may be drawn upon Velum or Paper by help of the following Directions.

PROP. VI.

How to draw a true Plat *of the Field before measured, by help of the Table of Sides and Angles.*

UPON your Velum or Paper draw a Line A B, containing 7 Chains 50 Links of any Scale, and upon the end A place the Centre of the *Protractor*, laying the Diameter upon the line A B, then the exterior Angle A being 300 *degrees*, make a mark against 300, and through

through that point and the point A, draw a line downwards as A F, containing 10 Chains 25 Links.

Secondly, Place the Centre of the *Protector* upon B, and the Diameter upon A B, and againſt 145 *degrees* (the Angle at B) make a mark, and through that mark and B draw the line B C, containing 7 Chains 10 Links.

Thirdly, Lay the Centre of the *Protractor* upon C, and the Diameter upon B C, and againſt 270 *deg.* make a mark, through which, and C, draw the line C D, containing 5 Chains 85 Links.

Fourthly, Lay the Centre of the *Protractor* upon D, and the Diameter upon C D, making a mark againſt 263 *deg.* through which, and the point D, draw the line D E, to contain 7 Chains.

Laſtly, Lay the Centre of the *Protractor* upon E, and its Diameter upon D E, and againſt 220 *deg.* make a mark, through which, and the point E, draw a right line E F, which will cut your line A F in F; ſo have you upon you Velum or Paper, the true figure of your Field, A B C D E F.

In all the Work of theſe two laſt Propoſitions, *we have wrought by the exterior or outward Angles: If any have a deſire to work by the interior or inward Angles, it is but taking the exterior Angle from* 360 degrees, *and the interior Angle will remain. So the Angle at A being* 300 degrees, *that taken from* 360 degrees, *leaves* 60 degrees *for the interior Angle at A. So B being* 145 degrees, *that taken from* 360, *leaves* 215 *for the interior Angle.*

II. *By going about the* Wood *or* Park *on the* Outſide *thereof.*

THE foregoing way, where the Hedges are ſtreight, and not many; and the Ground clear of *Wood*, *Pools*, *Quags*, and ſuch like, is ſufficient to perform the Works there intended: But if the *Wood* be overgrown, ſo that you cannot ſee from Angle to Angle, nor meaſure along by the Hedge-ſide, then make uſe of the way hereafter preſcribed.

FIG. VIII.

Let A B C D E F G H I K L M N O, be a *Wood* to be *meaſured* and *Plotted*, and it is ſo overgrown with Wood that ye cannot by any means come within the ſame to meaſure any *Sides*, or ſee to take any *Angles*; wherefore you muſt go on the outſide thereof: And having made choice of a place, as at P, where to begin your Survey, here ſet up a viſible mark, then ſtanding at P, look along the Wood's ſide towards Q, where ſet up another viſible Mark: And at Q, look towards R, where ſet up another viſible Mark as at R: From which looking along the Wood's ſide (as near the ſide as you can) ſet up another Mark as S, from whence you may ſee your firſt Mark at P: and by this means you have encompaſſed the whole *Wood* within the Quadrilateral Figure P Q R S: And now to begin your Inſtrumental Work:

1. Set up your *Semicircle* at P, laying the Index upon the Diameter thereof, and turn the whole Inſtrument about, till through the ſights you

you see the *Mark* set up at Q, and then fix the Instrument, and turn the Index about till through the Sights you see the last Mark at S: where the Index will cut 89 deg. 00 m. Which set down in the first Column of a *Field-Book*, or Paper, ruled like this in the *Margin*: Then at the beginning of your Measure from P towards Q, you find that the corner of the Wood at A is distant from P, 1 Chain and 80 L set down 1 C. 80 L. in the next Column; and in the third Column say, from P to A: —— Then measuring along the *Chain-Line* from P towards Q, I find that from Q to a is 3 C. 55 L. which I set down in the first Column; and because *a* is distant from the Angle of the Wood at B 90 L. I set down 0 C. 90 L. and in the third Column, from *a* to B— Then measuring on till I come to *b*, 4 Ch. 60 Li. I find the Chain to touch the very corner of the Wood at C. wherefore, I write down 4 C. 60 L. in the first Column; *Touch.* in the second Column, and at C, in the third Column: And in this manner I proceed in *Measuring* and *setting down* (as in the Margin) till I come to the second Mark at Q. Then

d.	*m.*	C. L.	From
89	00		
0	00	1 80	P to A
3	55	0 90	a B
4	60	touch	at C
8	50	1 40	c D
11	15	0 50	d E
12	70	0 80	e F
15	00	touch	at G
18	80	0 0	at Q
87	30	C. L.	From
0	00	0 90	Q to G
3	00	touch	at H
6	35	0 60	h I
6	90	0 00	at R
91	00	C. L.	From
0	60	0 40	k to I
2	75	touch	at K
5	45	0 65	m L
8	40	0 70	n M
11	90	touch	at N
14	90	1 20	p O
15	65	0 0	at S
92	30	C. L.	From
0	00	0 0	at S
1	10	0 85	q to O
4	70	touch	at A
6	50	0 0	at P

2. Set up your *Semicircle* at Q, the Index upon the Diameter hereof; and turn the Instrument about till through the Sights you see your first Mark at P, and then fasten it: Then turn the Index about to R, where you shall find it to cut 87 Deg. 30 m. which set down in the first Column of your Book: And measuring from Q towards R, you find that at 3 Chains end, the Chain touches the Corner of the Wood at H, wherefore, write down 3 Chains in the first Column, *Touch* in the second Column, and at H in the third Column: Then measuring on towards R, till I come to *b*, 6 Ch. 35 L. I find that the Angle at *b*, is distant from I, 60 Links: wherefore, I set down 6 C. 35 L. in the first Column, 60 L. in the second, and, from *b* to I in the third Column: And lastly, 6 C. 90 L. at R.

3. In like manner set up your Instrument at R, and at S, measuring and setting down as before, and as you see done; And you shall find all the Sides and Angles to be such as they are noted to be in the figure of the Field-book in the Margin: From whence, by what hath been already said, and the sight of the *Figure* (which will give more light to the Work than a whole *Chapter* of Directions) you may draw a true *Plot* of the *Wood* upon *Paper* or *Parchment*.

And for the casting up of the *Content* thereof the directions formerly given, in *measuring* by the *Chain* only: And those which follow in the next *Proposition* will sufficiently inform you.

But, by way of *Advertisement*, In this *Example* where you went on the outside of the *Wood*, you must first find the *Content* of the whole *Quadrilateral Figure* PQRS, and then of the several small *Triangles* and *Trapezias* made without the Wood, as the *Trapezia*, PaBr, and the rest: Which being deducted from the Content of the whole *Quadrilateral* PQRS, will leave the *Content* of the whole *Wood* included therein.

PROP. VII.

The Plat of a Field being laid upon Paper, and the Scale by which it was laid down known, to find how much the said Field containeth in Acres, Roods and Perches.

THere are several ways to effect this; but I shall here deliver only one, which shall be general, and that which by all (or most) Surveyors is practised.

Every irregular Plat or Figure, before the quantity or content of it can be found, must first be reduced into such Regular Figures, for the mensuration whereof there are certain Rules: The most meet and convenient Figures into which Irregular Plats may be reduced, are Triangles and Quadrilaterals, called Trapezias, which is done by drawing of lines from Angle to Angle cross the Plat. And here note, That of how many sides soever your Plat consisteth, into so many Triangles, wanting two, will the Plat be reduced, and no less, as the Figure VII. denotes, where the Plat consisteth of 7 Sides, and it is reduced into 5 Triangles.

The manner of reducing the Plat into Triangles.

FIG. VII.

LET the Figure ABCDEFG be a *Plat* which contains seven sides; to reduce which into Quadrilaterals and Triangles, do thus:

First, Draw the line BF, so is part of the *Plat* reduced into the Quadrilateral ABFG.

Secondly, Draw the line FD, so is another part of the *Plat* reduced into the Trapezia BCDF; and the other part of the *Plat* is comprehended in the Triangle FDE.

Thus the whole *Plat* being reduced, contains two Trapezias, *viz.* K and L, and the Triangle M, in number five, (for every Trapezia contains two Triangles) which are less by two than the number of sides.

To

To cast up the Quantity or Content of the Plat.

First begin with the Triangle M, whose base, F B, is 13. 75. that is, 13 Chains 75 Links, and his Perpendicular E Q, 7 Chains 12 Links.

Now the Quantity, Area, or Content of every Triangle, is found, By multiplying the length of the base by half the length of the perpendicular.

So here the base 13. 75. and the perpendicular 7. 12. the half whereof is 3. 56. if you multiply 13. 75. by 3. 56. (as if they were whole Numbers, tho in reality the 75 and 56 are Fractions) the Product will be 4. 89500, and that is the Quantity of the Triangle F D E.

	13.75
	3.56
	8250
	6875
	4125
	4. 89500

Secondly, For the Trapezia or Quadrilateral L, in which Trapezia you may see that the line C F is a common base to the two Triangles B F C, and C D F, for the perpendiculars B P, and D O, of both Triangles, fall upon it.

Now the Quantity of any Trapezia is found, *By multiplying the common base* (here F C) *by half the Sum of the two Perpendiculars which fall upon it* (here B P, and D O.)

So in this Trapezia, the common base C F, is 14 C. 90 L. and the sum of the two Perpendiculars B P, and D O, is 13 Chains 80 Links, the half whereof is 6. 90. If you multiply 14. 90. by 6. 90. the Product will be 10. 28100.

	14.90
	6.90
	134100
	8940
	10. 28100

Thirdly, For the Trapezia K, multiply 12. 66. (the length of the base B G) by 4. 55. half the sum of the two perpendiculars A H and F R, the Product of that Multiplication will be 5.76030, which is the Quantity, Area, or Content of the Trapezia K.

	12.66
	4.55
	6330
	6330
	5064
	5.76030

Thus have you the Content of the two Trapezias and the Triangle. Now,

To find the Quantity of the whole Field, in Acres, Roods, *and* Perches.

Add the Products of the several Multiplications together, the Sum of them is the whole Quantity. So

The

The	Triangle M	containing	4. 89500
	Trapezia K		5. 76030
	Trapezia L		10. 28160
		Their Sum is —	20. 93630

Which is 20 compleat or entire Acres, and $\frac{21630}{100000}$ parts of an Acre; and to know how much that is, multiply 93630 by 4 (becauſe 4 Roods make an Acre) the Product is 3 compleat, and $\frac{74520}{100000}$ parts of a Rood; and to know how much that is, multiply 74520 by 40 (becauſe 40 Perches are contained in one Rood) the Product will be 29.80800, which is 29 compleat Perches, and $\frac{80800}{100000}$ parts of a Perch, which is inconſiderable, for 160 whole Perches make but one Acre.

93630
4
3.74520
74520
40
29.80800

Thus have you the Area, Quantity, or Content of this Field caſt up, and you find it to contain 20 Acres, 3 Roods, and 29 Perches. And in this manner may you caſt up the quantity of any Irregular Plat whatſoever, remembring that in all your Multiplications you cut off 5 figures towards your right hand with a prick of your Pen, as in this Work I have done all along.

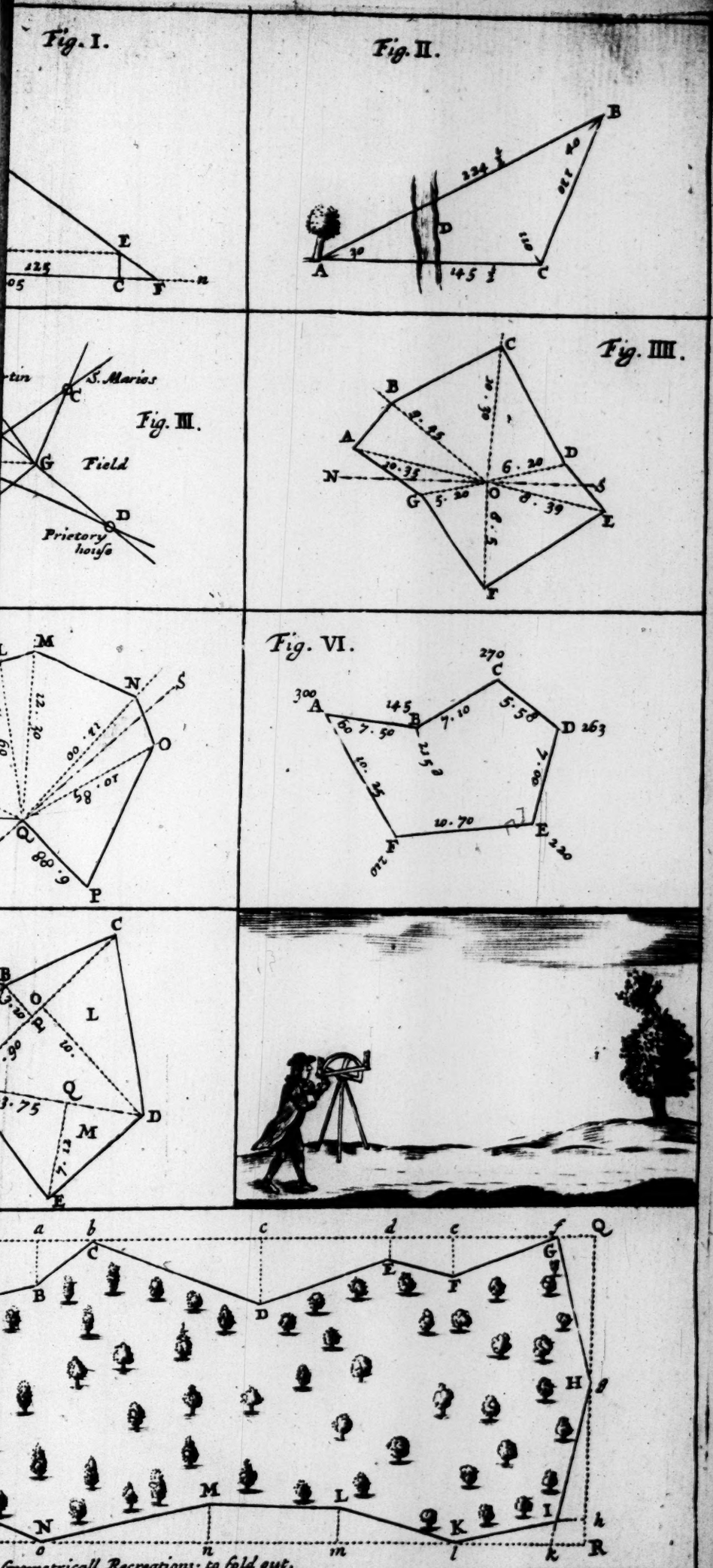
Fig. I.
Fig. II.
Fig. III.
Fig. IIII.
Fig. VI.
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Field
Prietory house
of Geometricall Recreations; to fold out.

Fig. I.

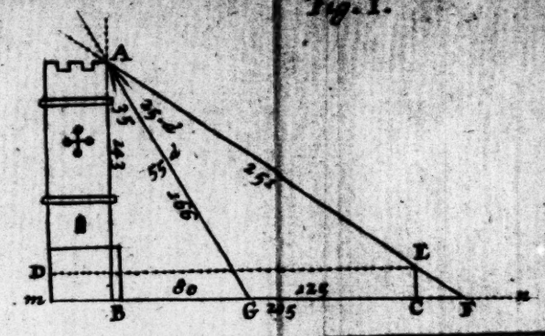
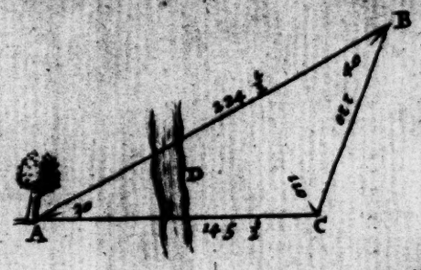


Fig. II.



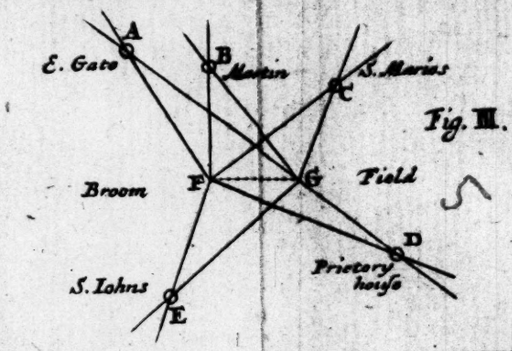


Fig. III.

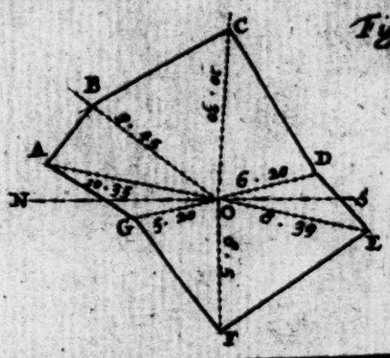


Fig. IIII.

Fig. V.

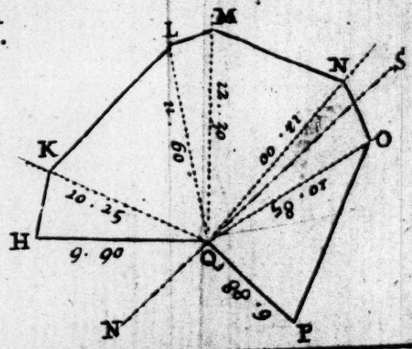
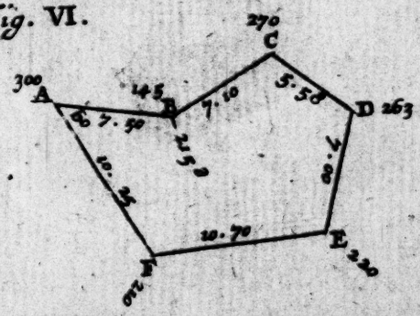


Fig. VI.



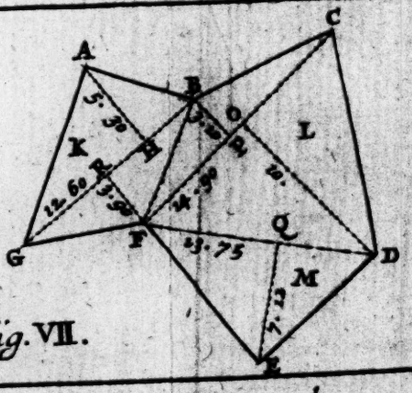


Fig. VII.

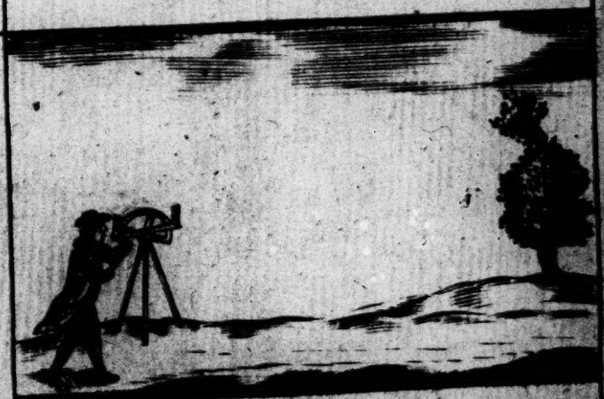
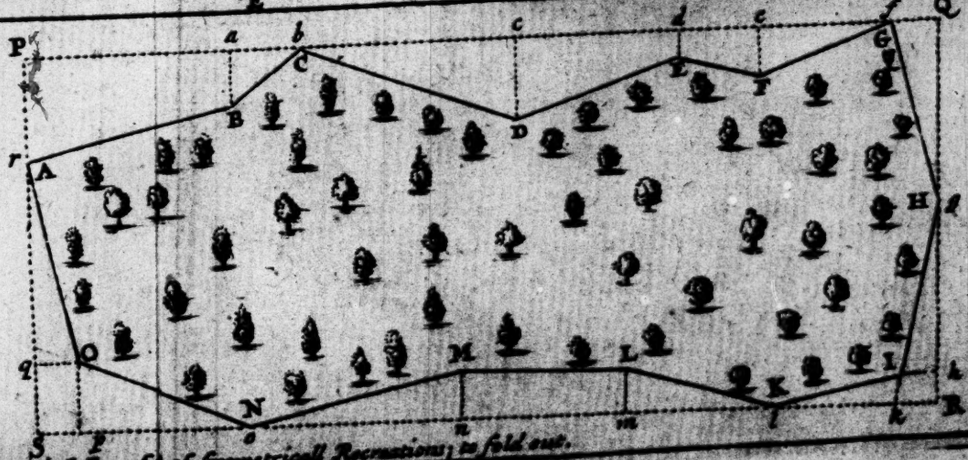


Fig. VIII.



Place this against Page 62 of Geometricall Recreations; to fold out.

Mechanical RECREATIONS.

CHAP. I.

Of Engines, by help of which we may Raise a very great Weight, with small Strength.

THE invention of all these Engines depend upon one Sole Principle; which is,

That, *The same Force that can lift up a Weight* (for Example of 100 Pounds) *to the height of one Foot; can lift up one of* 200 *Pounds, to the height of half a Foot: Or one of* 400 *Pounds to the height of a fourth part of a Foot*; And so of the Rest, be there never so much applyed to it.

And this *Principle* cannot be denied, if we consider that the *Effect* ought to be proportioned to the *Action* that is necessary for the Production of it: So that, if it be necessary to employ an *Action* by which we may raise a *Weight* of 100 Pounds, to the height of *two Foot*; for to raise one such to the height of *one Foot* only, this same ought to weigh 200 Pounds: For, it's the same thing to raise 100 Pounds to the height of *one Foot*; and again yet another 100 Pounds to the height of *one Foot*, as to raise one of 200 Pounds to the height of *one Foot*, and the same also, as to raise 100 Pounds to the height of *two Feet*.

Now the *Engines* which serve to make this *Application* of a *Force* which acteth at a *Great Space* upon a *Weight* which it causeth to be raised by a Lesser, are

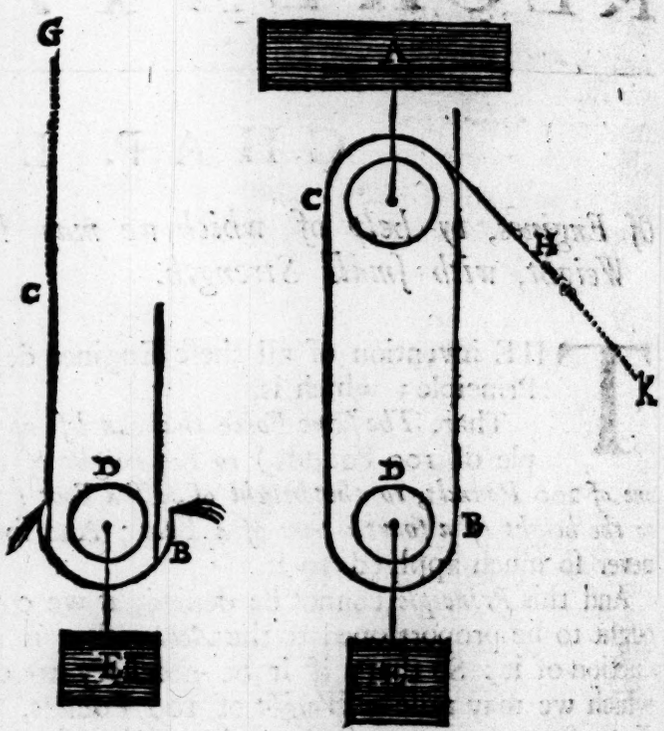
Trochela, the Pulley.
The inclined Plain.
Cuneus, the Wedge.
Axis in Peritrochio, the Crane, Capsten, or Wheel.
Cochlea, the Screw.
Vectis, the Leaver.

And some others: For, if we will not apply, or compare *them* one to another, we cannot well number *more*: And if we will apply *them*; we need not instance in so *many*.

CHAP. II.

Of the Pulley PROCLEA.

LET A, B, C, be a Cord put about the Pulley D, to which let the Weight E be fastned; and first, supposing that two Men sustain, or pull up equally each of them, one of the Ends of the said Cord: It is manifest, that if the Weight weigheth 200 Pounds, each of those shall employ but the half thereof, that is to say, the Force that is requisite for sustaining, or raising of 100 Pounds, for each of them shall bear but the half of it. Afterwards, let us suppose that A, one of the Ends of this Cord, being made fast to some Nail, the other C be again sustained by a Man; and it is manifest, that this Man in C needs not (no more than before) for the sustaining the Weight E, more Force than is requisite for the sustaining of 100 Pounds; because the Nail at A, doth the same Office as the Man which we supposed there before. In fine, let us suppose that this Man in C do put the Cord to make the Weight E to Rise; and it is manifest, that if he there employeth the Force which is requisite for the Raising of an hundred pound to the height of two Foot, he shall Raise this Weight E of 200 Pounds to the height of one Foot; for the Cord A B C being doubled, as it is, it must be pulled two Foot by the end C, to make the Weight E rise as much, as if two Men did draw it, the one by the end A, and the other by the end C, each of them the length of one Foot only.



There is always one thing that hinders the exactness of the Calculation, that is the ponderosity of the Cord or Pulley, and the difficulty that we meet with in making the Cord to slip, and in bearing it:

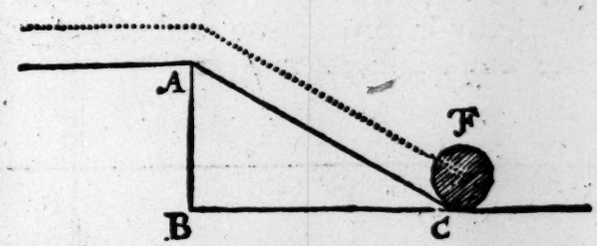
But

But this is very ſmall in compariſon of that which raiſeth it, and cannot be eſtimated ſave within a ſmall matter.

Moreover, it is neceſſary to obſerve, that it is nothing but the redoubling of the Cord, and not the Pulley, that cauſeth this Force; for if we faſten yet another Pulley towards A, about which we paſs the Cord A B C H, there will be required no leſs Force to draw H towards K, and ſo to lift up the Weight E, than there was before to draw C towards G. But if to theſe two Pulleys we add yet another towards D, to which we faſten the Weight, and in which we make the Cord to run or ſlip, juſt as we did in the firſt, then we ſhall need no more force to lift up this Weight of 200 Pounds, than to lift up 50 pounds without the Pulley; becauſe that in drawing four Feet of Cord we lift it up but one Foot. And ſo in multiplying of the Pulleys, one may raiſe the greateſt Weights with the leaſt Forces. It is requiſite alſo to obſerve, that a little more Force is always neceſſary for the raiſing of a Weight, than for the ſuſtaining of it; which is the Reaſon why I have ſpoken here diſtinctly of the one and of the other.

CHAP. III.

Of the Inclined PLANE.



IF not having more force than ſufficeth to raiſe an hundred Pounds, one would nevertheleſs raiſe this Body F, that weigheth 200 Pounds, to the height of the Line B A, there needs no more, but to draw or rowl it along the Inclined Plane C A, which I ſuppoſe to be twice as long as the Line A B, for by this means, for to make it arrive at the Point A, we muſt there employ the Force that is neceſſary for the raiſing 100 Pounds twice as high, and the more enclin'd this Plane ſhall be made, ſo much the leſs force ſhall need to raiſe the Weight F: But there is to be rebated from this Calculation, the difficulty there is in moving the Body F, along the Plane A C, if that Plane were laid down upon the Line B C, all the parts of which I ſuppoſe to be equidiſtant from the Center of the Earth.

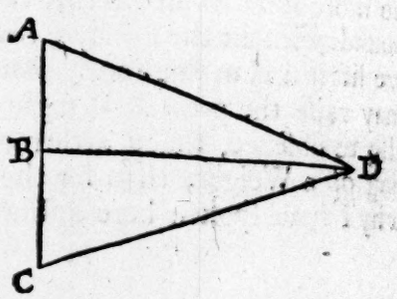
It is true, that this impediment being ſo much leſs as the Plane is more united, more hard, more even, and more polite, it cannot be likewiſe eſtimated but by gueſs, and it is not very conſiderable.

We need not neither much to regard that the Line B C, being a part of Circle that hath the ſame Center with the Earth, the Plane A C ought to be (though but very little) Curved, and to have the Figure of part of a Spiral, deſcribed between two Circles, which likewiſe

wiſe have for their Center that of the Earth; for that it is not any way ſenſible.

CHAP. IV.

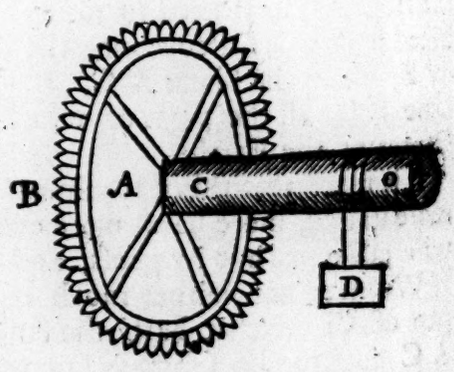
Of the WEDGE *Cuneus.*



THE force of the Wedge A B C D, is eaſily underſtood after that which hath been ſpoken above of the enclined Plane, for the force wherewith we ſtrike downwards, acts as if it were to make it move according to the Line B D; and the Wood, or other thing and body that it cleaveth, openeth not, or the Weight that it raiſeth doth not raiſe, ſave only according to the Line A C, inſomuch, that the force wherewith one driveth or ſtriketh this Wedge, ought to have the ſame proportion to the reſiſtance of this Wood or Weight, that A C hath to A B: Or elſe again, to be exact, it would be convenient that B D were a part of a Circle, and A D and C D two portions of Spirals that had the ſame Center with the Earth, and that the Wedge were of a matter ſo perfectly hard and polite, and of ſo ſmall Weight, as that any little force would ſuffice to move it.

CHAP. V.

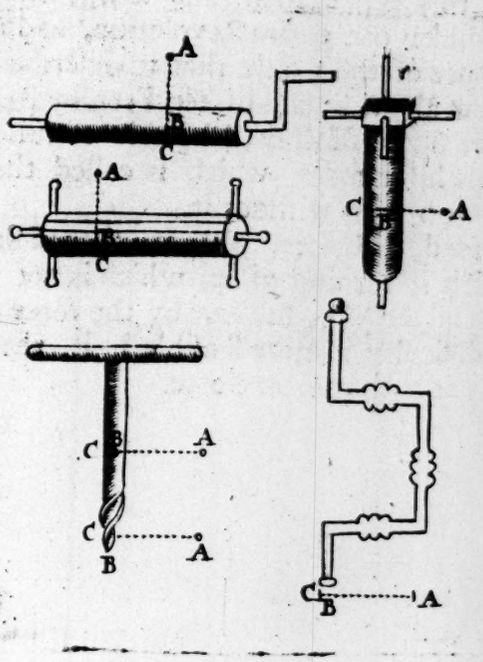
Of the CRANE, CAPSTEN, *or* WHEEL, *Axis in Peritrochio.*



WE ſee alſo very eaſily, that the force wherewith the Wheel A or Cogg B is turned, which make the Axis or Cylinder C to move, about which a Cord is Rolled, to which the Weight D, which we would raiſe is faſtned, ought to have the ſame proportion to the ſaid Weight, as the circumference of the Cylinder hath to the Circumference of a Circle which that force

force deſcribeth, or that the Diameter of the one hath to the Diameter of the other, for that the Circumferences have the ſame proportions as the Diameters: Inſomuch, as the Cylinder C having no more but one Foot in Diameter, if the Wheel A B be ſix Feet in its Diameter, and the Weight D do weigh 600 pounds, it ſhall ſuffice that the force in B ſhall be capable to raiſe an 100 Pounds, and ſo of others. One may alſo, inſtead of the Cord that rolleth about the Cylinder C, place there a ſmall Wheel with Teeth or Cogs, that may turn another greater, and by that means multiply the Power of the force as much as one ſhall pleaſe, without having any thing to deduct of the ſame, ſave only the difficulty of moving the Machine, as in the others.

Unto this Faculty of the *Wheel*, may be referred the Force of all thoſe *Engines* which conſiſt of *Wheels* with Teeth in them: And from hence alſo may be diſcerned the Reaſon why ſundry *Inſtruments* in common uſe are framed, as *Hand-Mills*, the *Piercer*, the *Gimlet*, &c. as in the Figure. All which are but ſeveral kinds, of this fourth *Faculty* the *Wheel*; in all which *Inſtruments*, the Points A B C do repreſent the Places of the *Power*, the *Fulſiment* and the *Weight*. The *Power* being in the ſame proportion unto the *Weight* as B C is unto B A.



CHAP VI.

Of the SCREW *Cochlea.*

WHen once the force of the Capſten, and of the inclined Plane is underſtood, that of the Screw is eaſie to be computed, for it is compoſed only of a Plane much inclined, which windeth about the Cylinder; and if this Plane be in ſuch manner inclined, as that the Cylinder ought to make *v. gr.* ten turns, to advance forwards the length of a Foot in a Screw, and that the

the bigneſs of the Circumference of the Circle which the force that turneth it about doth deſcribe be of ten Feet; foraſmuch, as ten times ten are one hundred, one Man alone ſhall be able to preſs as ſtrongly with this Inſtrument, or Screw, as one hundred without it, provided always, that we rebate the force that is required to the turning of it.

Now I ſpeak here of preſſing rather than of Raiſing, or removing, in regard that it is about this moſt commonly that the Screw is employed: but when we would make uſe of it for the raiſing of Weights, inſtead of making it to advance into a Female Screw, we joyn or apply unto it a Wheel of many Cogs, in ſuch ſort made, that if *v. gr.* this Wheel have thirty Cogs, whilſt the Screw maketh one entire turn, it ſhall not cauſe the Wheel to make more than the thirtieth part of a turn, and if the Weight be faſtned to a Cord, that Rolling about the Axis of this Wheel ſhall raiſe it but one Foot in the time that the Wheel makes one entire Revolution, and that the greatneſs of the Circumference of the Circle that is deſcribed by the force that turneth the Screw about be alſo of ten Feet, by Reaſon that 10 times 30 makes 300, one ſingle Man ſhall be able to raiſe a Weight of that bigneſs with this Inſtrument, which is called the *Perpetual Screw*, as would require 300 Men without it.

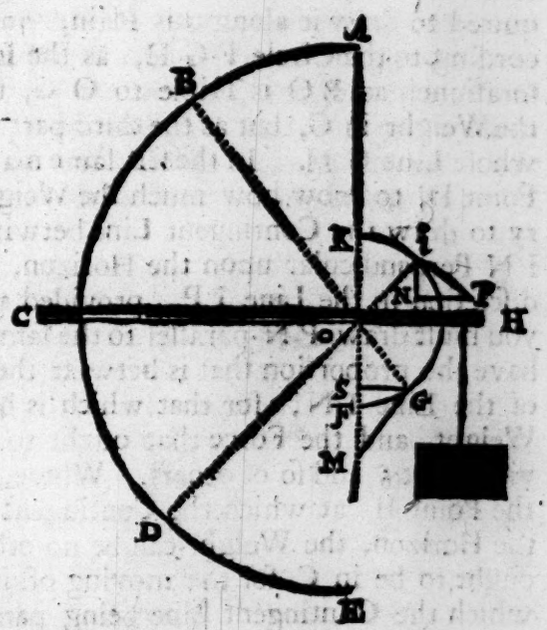
Provided, as before, that we thence deduct the difficulty that we meet with in turning of it, which is not properly cauſed by the Ponderoſity of the Weight, but by the force or matter of the Inſtrument, which difficulty is more ſenſible in it, then in thoſe aforegoing, foraſmuch as it hath greater Force.

CHAP. VII.

Of the LEAVER *Vectis.*

I Have deferred to ſpeak of the *Leaver* until the laſt, in regard, that it is of all Engines for raiſing of Weights, the moſt difficult to be explained.

Let us ſuppoſe that C H is a Leaver, in ſuch manner ſupported, at the Point O, (by means of an Iron Pin that paſſeth through it acroſs, or otherwiſe) that it may turn about upon this Point O, its Part C deſcribing the Semicircle A B G D E, and its Part H the Semicircle F G H I K, and that the Weight which we would raiſe by help of it were in H, and the force in C, the Line C O being ſuppoſed Triple of O H. Then let us conſider, that in the time whilſt the Force that moveth this Leaver, deſcribeth the whole Semicircle A B C D E, although that the Weight deſcribeth likewiſe the Semicircle F G H I K, yet it is not raiſed to the length of this Curved Line F G H I K, but only to that of the Line F O K; inſomuch, that the proportion that the Force which moveth this Weight ought to have to its Ponderoſity, ought not to be meaſured by that which is between the two Diameters of theſe Circles, or between the two Circumferences, as it hath been ſaid before of the Wheel, but rather by that which is between the Circumference of the Greater, and the Diameter of the Leſſer. Furthermore, let us conſider, that there is a neceſſity that this Force needeth not be ſo great, at ſuch time as it is near to A, or near to E, for the turning of the Leaver, as then when it is near to B or to D, nor ſo great when it is near to B or D, as then when it is near to C; of which the Reaſon is, that the Weights there do mount leſs; as it is eaſie to underſtand, if having ſuppoſed that the Line C O H is parallel to the Horizon, and that A O F cutteth it at Right Angles, we take the Point G equidiſtant from the Points F and H, and the Point B equidiſtant from A and C; and that having drawn G S Perpendicular to F O, we obſerve that the Line F S (which ſheweth



how

how much the Weight mounteth in the time that the Force operates along the Line A B) is much Leſſer than the Line S O, which ſheweth how much it mounteth in the time that the Force operates along the Line B C.

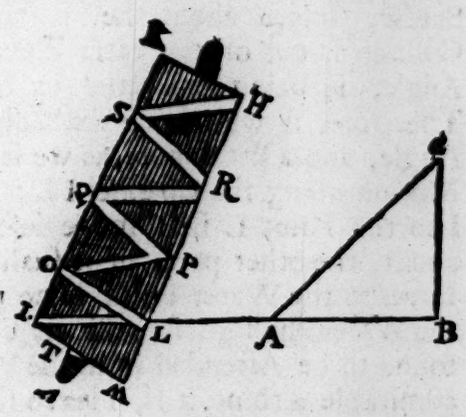
And to meaſure exactly what his Force ought to be in each Point of the Curved Line A B C D E, it is requiſite to know that it operates there, juſt in the ſame manner, as if it drew the Weight along a Plain Circularly Inclined; and that the Inclination of each of the Points of this Circular Plain were to be meaſured by that of the Right Line, that toucheth the Circle in this Point. As for Example; when the Force is at the Point B, for to find the proportion that it ought to have with the Ponderoſity of the Weight which is at that time at the Point G, it is neceſſary to draw the Contingent Line G M, and to account that the Ponderoſity of the Weight is to the Force which is required to draw it along this Plain, and conſequently to raiſe it, according to the Circle F G H, as the Line G M is to S M. Again, foraſmuch as B O is Triple to O G, the Force in B, needs to be to the Weight in G, but as the third part of the Line S M, is unto the whole Line G M. In the ſelf ſame manner, when the Force is at the Point D, to know how much the Weight Weigheth at I, it is neceſſary to draw the Contingent Line betwixt I and P, and the Right Line I N Perpendicular upon the Horizon, and from the Point P, taken at diſcretion in the Line I P, provided that it be below the Point I, you muſt draw P N parallel to the ſame Horizon, to the end you may have the proportion that is betwixt the Line I P, and the third part of the Line I N, for that which is betwixt the Ponderoſity of the Weight, and the Force that ought to be at the Point D for the moving of it; and ſo of others. Where, nevertheleſs, you muſt except the Point H, at which the Contingent Line being Perpendicular upon the Horizon, the Weight can be no other than Triple the Force which ought to be in C for the moving of it: In the Points F and K, at which the Contingent Line being parallel unto the Horizon it felt the leaſt Force that one can Aſſign, is ſufficient to move the Weight. Moreover, that you may be perfectly exact, you muſt obſerve that the Lines S M and P M ought to be parts of a Circle, that have for their Center that of the Earth; and G M and I P, parts of Spirals drawn between two ſuch Circles: And laſtly, that the Right Lines S M and I N, both tending towards the Center of the Earth, are not exactly Parallels: And furthermore, that the Point H, where I ſuppoſe the Contingent Line to be Perpendicular unto the Horizon, ought to be ſome ſmall matter nearer to the Point F than to K, at the which F and K the Contingent Lines are Parallels unto the ſaid Horizon.

CHAP.

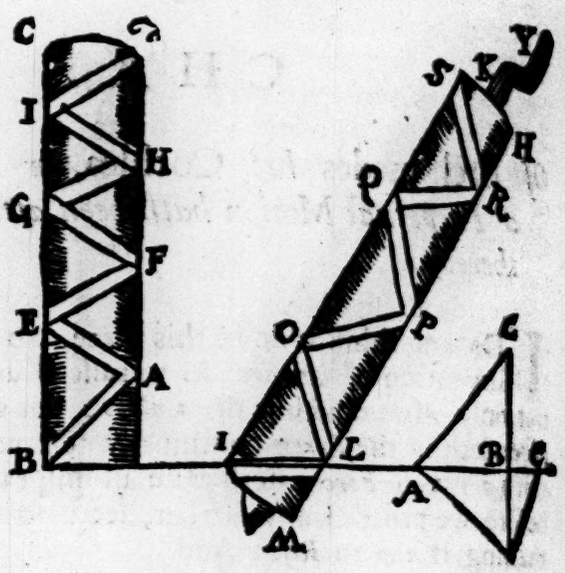
CHAP. VIII.

Of Archimedes *his* Cochlea, *or* Water Screw, *and how a Perpetual Motion hath been attempted to be performed thereby.*

I Do not think it fit in this place, to paſs over with ſilence, the Invention of *Archimedes* to raiſe *Water* with the *Screw*, which is not only *Marvelous* but *Miraculous*: For we ſhall find, that the *Water* aſcendeth in the *Screw* continually deſcending: And in a given *time*, with a given *Force*, doth raiſe an unſpeakable quantity thereof. But before we proceed any farther, let us declare the uſe of the *Screw* in making *Water* to *Riſe*: And in the Figure let us conſider the Line I L O P Q R S H, being an hollow Pipe of Metal, wrapped or turned about the Columb M I K H, or a hollow Channel, cut in the Columb of Wood, and covered with thin Plate of Braſs or Latten, through which the Water may run. If we ſhall put the end I into the Water, making the Screw to ſtand Leaning, ſo as the Point L may be lower than the firſt I, (as the Diagram ſheweth) and ſhall turn it round about on the two Axes T and V, the Water ſhall run thorough the Pipe or Channel, till, that in the end, it ſhall diſcharge forth at the Mouth H. Now I ſay, that the water, in its conveyance from the Point I, to the point H, doth go all the way deſcending, although the Point H be higher than the Point I: Which that it is ſo, we will declare in this manner: We will deſcribe the Triangle A C B, which is that of which the Screw H I is generated, in ſuch ſort, that the Channel of the Screw is repreſented by the Line A C, whoſe Aſcent, or Elevation is determined by the Angle B A C, that is to ſay, if ſo be the Angle B A C be one third, or one fourth part of a Right Angle, then the Elevation of the Channel A C ſhall be according to one third, or one fourth of a Right Angle alſo: And it is manifeſt, that the Riſe of that ſame Channel A C, will be taken away, debaſing the Point C as far as to B, for then the Channel A C ſhall have no Elevation: And debaſing the Point C a little below B, the water will naturally run along the Channel A C downwards from the Point A towards C. Let us therefore conclude, that the Angle A, being one third of a Right Angle, the Channel A C



ſhall no longer have any Riſe, debaſing it on the Point C, for one third of a Right Angle.

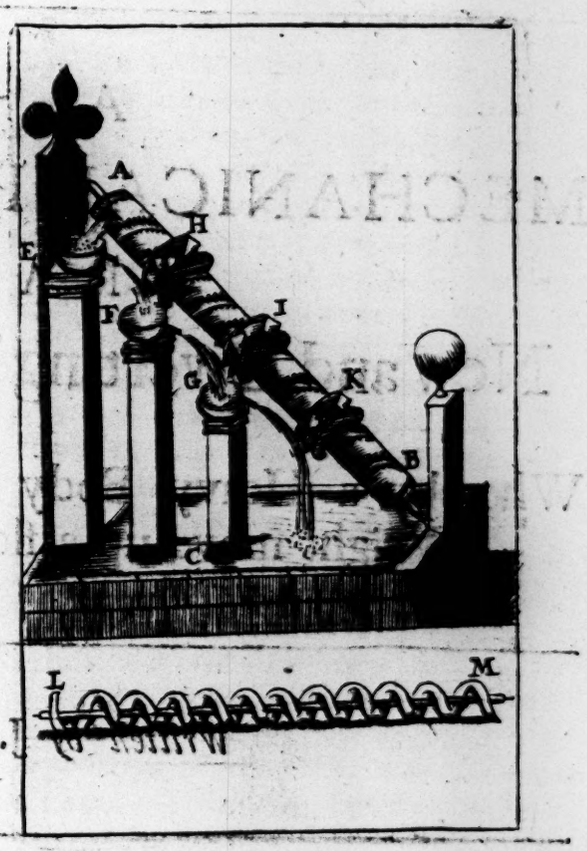


Theſe things underſtood, let us unfold the Triangle about the Columb, and let us make the the Screw B A E F G, *&c.* which if it ſhall be placed at Right Angles with the end B in the water, turning it about, it ſhall not this way draw up the water, the Channel about the Columb being Elevated, as may be ſeen by the Part B A, but altho' the Columb ſtand erect at Right Angles, yet for all that, the Riſe along the Screw, folded about the Columb is not of a greater Elevation, than of one third of a Right Angle, it being generated by the Elevation of the Channel A C: Therefore, if we incline the Columb but one third of the ſaid Right Angle, and a little more, as we ſee I K H M, there is a Tranſition and Motion along the Channel I L: Therefore, the Water from the Point I to the Point L ſhall move deſcending, and the Screw being turned about, the other parts of it ſhall ſucceſſively diſpoſe, or preſent themſelves to the Water in the ſame poſition as the part I L: Whereupon the Water ſhall go ſucceſſively deſcending, and in the end ſhall be found to be Aſcended from the Point I, to the Point H. Which, how admirable a thing it is, I leave ſuch to judge who ſhall perfectly have underſtood it.

How it hath been by ſome ſuppoſed, that from this Water Screw a Perpetual Motion may be contrived:

For, *ſay they,* if there were but ſuch a Water-Wheel made on this Inſtrument, upon which the Stream that is carried up, may fall, in its Deſcent it would turn the Screw Round, and by that means convey as much Water up, as is required to move it; ſo that the Motion muſt needs be continual, ſince the ſame Weight which in its fall does turn the Wheel, is by the turning of the Wheel carried up again.

Or, if water falling upon one Wheel would not be forcible enough for this effect; why, there might be two or three, or more, according as the Length and Elevation of the Inſtrument will admit; by which means the Weight of it may be ſo multiplyed in the fall, that it ſhall be equivalent to twice or thrice that quantity of water which Aſcends; as may be more plainly diſcerned by the Figure.



In this Figure L M at the bottom, repreſents a Wooden Cylinder with Helical Cavities cut in it; which at A B is ſuppoſed to be covered over with Tin Plates, and three water Wheels upon it, as H I K; the Lower Ciſtern which contains the water, being C D. Now this Cylinder being turned round, all the water which from the Ciſtern Aſcends through it, will fall into the Veſſel at E, and from that Veſſel being conveyed upon the Water Wheel H, ſhall conſequently give a circular Motion to the whole Screw: Or if this alone ſhould be too weak for the turning of it, then the ſame water which falls from the Wheel H, being received into the other Veſſel F, may from thence again deſcend on the Wheel I; by which means the force of it will be doubled. And if this be yet inſufficient, then may the water which falls on the ſecond Wheel I, be received into the other Veſſel G, and from thence again deſcend on the third Wheel at K; and ſo for as many other Wheels as the Inſtrument is capable of. So that beſides the greater diſtance of theſe three Streams from the Center or Axis, by which they are made ſo much heavier; and beſides, that the fall of this outward water is forcible and Violent; whereas the Aſcent of that within, is natural: Beſides all this, there is thrice as much water to turn the Screw, as is carried up by it.

But on the other ſide, if all the water falling upon one Wheel would be able to turn it round, then half of it would ſerve with two Wheels.

Although what is here ſaid concerning the probability of effecting ſuch a Motion, yet Trial and Experience have diſcovered the contrary, and found it wholly inſufficient: And that,

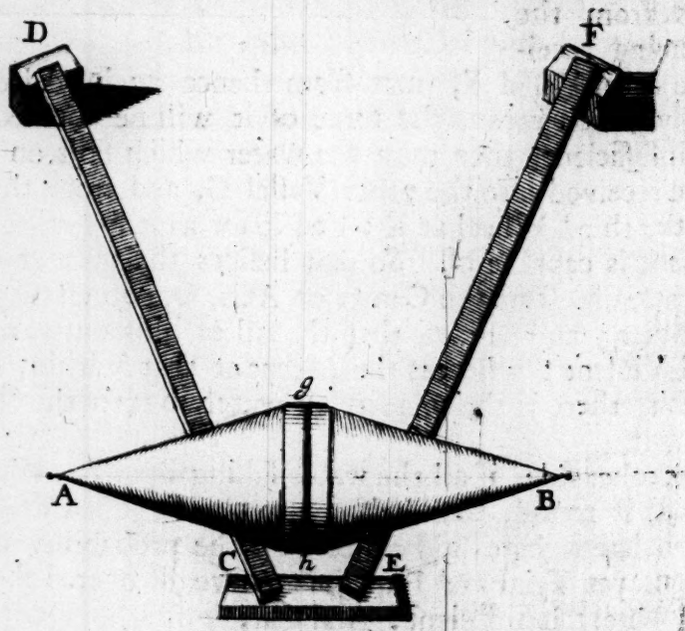
Firſt, For that the water that Aſcends will not make any conſiderable Stream in the fall. And

Secondly, This Stream (though multiplyed) will not be of force enough to turn about the Screw.

A MECHANICAL PARADOX: OR, A New and Diverting Experiment.

Whereby a Heavy Body ſhall by its own Weight move up a ſloping Aſcent.

Written by J. P.



THE Things neceſſary for this Experiment are, Firſt, A Roller of Wood, turned in a *Lathe-Ink*, a Figure like as here is repreſented at A B, (*viz.*) of two Cones (or Sugar Loaves) abutting one againſt the other. Let the thickneſs in the middle (g h) be about 5 or

5 or 6 Inches, the length AB about 3 times the thickneſs; at the end A & B may be left two little Pins turned. 2. You muſt provide two ſtraight ſmooth Rulars about a Yard in length, and ſtrong enough to bear the weight of the Roller. 3. Laſtly, You muſt have three pieces of Wood to ſupport the ends of the Rulars; the firſt about two or three Inches thick, the other two (to ſtand at D and F) muſt be thicker than this firſt by ſomewhat leſs than half the Diameter of the Roller; ſo that if the Roller be 6 Inches Diameter, the firſt piece of Wood 2 Inches, then let the other 2 pieces be about 4 ½ Inches apiece. Being thus provided when you would try the Experiment, 1. Place the two thicker pieces upon a level Table almoſt the length of the Roller off of one another, as at D F, ſet the other piece of Wood almoſt the length of the Rular off of the other two. 2. Place the two Rulars with their ends upon the pieces of Wood in the manner as is repreſented in the Figure, with their lower ends near together, and the upper ends ſtradling. 3. Place the middle of your Roller between the two lower ends of the Rulars, and you will ſee (if you have placed all right) what you deſire, *viz.* The Roller will of it ſelf climb to the upper ends of the Rulars.

When you would divert any perſon with this Experiment, you may firſt put the Rulars Parallel, (or with lower ends as wide as the upper) and let it be ſeen how faſt the Roller will run down the Deſcent; which will make it the more ſtrange to ſee it afterwards climb the ſame Aſcent, by only bringing the lower ends nearer.

The reaſon of this (ſeeming) Aſcent of the Rhomb or Roller, is a real *Deſcent* or *Lowering* of its Center of Gravity, for tho' the way or line of the motion on the *Rulars* be an *Aſcent*, yet the line which the Roller deſcribes on its own ſurface is ſuch, that every point of it approaches nearer to the Axis of the Rhomb, the opening of the Rulars cauſing the Contact to be nearer to the ſmall ends of the two Cones; and conſequently, nearer to their Axis: Whereupon the Axis of the Rhomb is ſo much lower at the top of the Rulars, as their Elevation comes ſhort of the Semi-diameter of the Rhomb.

CHAP.

CHAP. IX.

Of other Engines, deduced from the former, for the moving of Great *or* Heavy Bodies; *and of others for violent Motions.*

ALthough great and heavy *Works* may be effected by many Hands, according to the Verſe,

Multorum manibus grande levator opus:

And as may be inſtanced in the Wild *Indians*, who have performed with Hand Labour (they not having the knowledge of Engines) ſuch ſtrange matters, as the removing of Stones of an incredible greatneſs.

Arcoſta in his Hiſtory of *Italy* relates, That he himſelf meaſur'd one at *Tiaguanaco* which was 38 foot long, 18 foot broad, and 6 foot thick. And he there alſo affirms, That in their ſtatelieſt *Edifices*, there were many other of far vaſter Magnitude.

I. *Of* Pullies.

1. It is Reported of *Archimedes*, that with an Engine of *Pullies*, to which he applied only his *Left-hand*, he lifted up 5000 *Buſhels* of Corn at once; and drew a *Ship*, with all its *Lading*, upon the Land.

2. The ſame *Archimedes* with an *Inſtrument* called *Helix*, which *Rivoltus* ſuppoſes to conſiſt of many *Screws*, Launched into the Sea that great Ship which *Hero* had built, without any other help. And what is here ſaid may in ſome meaſure be verified by the Experiment following.

3. *How a Child, with a ſmall Thread, or Sewing Silk, ſhall draw up a Weight of* 3 *or* 400 *Pounds.*

Prodigious is the force of Engines made by the Multiplication of Wheels; but this feat may be performed by a Common Kitchen Jack for Roaſting of Meat, provided the Pullies be well fixed, and the Cord of ſufficient ſtrength: For let a *Man*, or a greater *Weight*, be faſtned where the Jack-Weight uſually is, then let a ſmall ſilk *String* or *Thread* (nay a *Horſes Hair*) be tied faſt to one end of the *Fly* of the *Jack*, which put into a *Childs* Hand, he can with it eaſily turn the *Fly* about, which he continuing ſo to do, ſhall draw the *Man* or *Weight* up to what height he pleaſeth.

But if the number of Wheels were any Invented, it would draw up a far greater Weight; nay, it might be made of ſufficient ſtrength to pull

pull a great *Oake*, or other *Tree*, out of the Ground by the Root. Nay, as *Archimedes* affirmed, That the whole *Globe of the Earth* might be moved were there any other place to fix his *Engine* in.

4. *How to contrive a* Ladder by Pullies; *whereby a* Man *may draw himself up to any assigned height.*

For the performance hereof, there is required only an *upper* and a *lower Rundle* or *Pullies*. To the uppermost of these at A, there should be fastned a sharp *Grapple*, or *Cramp* of Iron, which may be apt to take hold of any place where it lights: This part being first cast up and fastned, and the Staff DE, at the neither end, being put betwixt the Legs, so that a Man may sit upon the other BC, and take hold of the Cord at F; it is evident, that the weight of the Person at E, will be but equal to half so much strength at F, so that a man may easily pull himself up to the place required, by leaning but little more than half of his own weight on the string F: But if the Pullies be Multiplied, this Experiment might be performed with less Labour.

II. *Of the* Sling.

The *Sling* is an Instrument or *Engine* of incredible force, and of such *swiftness*, that it exceeds that of a *Stone* thrown by the Hand, by how much the end of the *Sling* is farther off from the *Shoulder joint*, which is the *Center* of *Motion*; the Victory by *David* over *Goliah* is a sufficient Evidence of the force of this Engine. And again, in Holy Writ, we read of 700 *Benjamites Left-handed that could Sling a Stone at a Hairs breadth and not miss. Vigetius* relates, that it was usual this way to strike a Man Dead, and beat the *Soul* out of his *Body*, without so much as breaking his *Armor*, or fetching *Blood*. And it is likewise *Storied*, that there is a whole *Nation* among the *Indians*, who for their Excellency in the use of this Engine are Stiled *Baleares*; and they are so strict in Teaching this Art to their young Children, that the Mother will not give Meat to her Child, till (it being set at some distance) he could hit it with Slinging.

CHAP. X.

Of Engines of War used among the Ancients.

THose among the Ancient *Romans* were principally of two sorts,

1. *Ballistæ*, Engines for Shooting or Casting of *Stones*.
2. *Catapultæ*, For the discharging of *Arrows*.

1. *Athenians*

1. *Athenus* mentions one of these *Baliſt*, that was proportioned into a Stone of three Talents Weight, each Talent being 120 pound (ſaith *Vitruvius*) and ſo the whole 360 pounds.

2. It is reported by *Plutarch*, that *Archimedes* caſt a Stone into one of *Marcellus* his Ships, which was found to weigh 10 Talents; that is, 1200 pound weight.

3. Among the *Turks* there have been uſed ſuch Inſtruments, for *Nab. Naudæus* tells us of one Bullet ſhot from them at the Siege of *Conſtantinople*, which was of above 1200 pound weight. This he affirms from the Relation of an Archbiſhop, who was then preſent and did ſee it.

4. Of the *Catapultæ*; ſome of them were proportioned to carry Spears of 12 Cubits long, which they would diſcharge with ſuch force (ſaith *Amianus*) that the Weapons diſcharged from them were ſometimes ſet on Fire by the ſwiftneſs of their motion.

5. It is related by Sir *Francis Bacon* in his 704 Experiment, that the *Bows* uſed now in *Turky* can ſtrike an Arrow through a piece of *Steel* or *Braſs* two Inches thick; being headed only with *Wood*, it will pierce Timber of 8 Inches thick, and hath pierced the ſides of Ships.

6. *Barelay*, in his *Icon animorum*, affirms that he was an Eye-Witneſs how one of theſe *Bows*, with a little *Arrow*, did pierce through a piece of *Steel* three Fingers thick. Theſe *Bows* (ſays the ſame Author) are ſomewhat like the *Long Bows* now in uſe with us, and bent by the immediate ſtrength of one Man.

7. Mr. *John Greaves*, in his *Pyramadographia* reports, that he hath ſeen Turkiſh Bows of that ſtrength, that they would pierce a Plank of 6 Inches thick.

CHAP. XI.

Of Automonata, *or Self-Movers.*

OF theſe there are principally two kinds, *viz.*

1. Such as are moved by ſomething which is *Extrinſical* unto their own *Frame*, As *Mills* by *Water* or *Wind*.

2. Such as receive their Motion from ſomething that does belong to the *Frame* it ſelf, As *Clocks*, *Watches*, &c. by *Weights*, *Springs*, or the like.

Mills by Wind and Water are commonly known, and to what uſes they ſerve; as for Grinding of Corn, Making of Paper, Grinding of Brazil, Logwood, *&c.* Forging of Iron, *&c.* But there are ſome other Motions by *Wind* and *Air*, though not ſo common, yet of Excellent Curioſity. As for Inſtance.

1. In

1. In *Ægypt* there is a Statue of *Memnon* which makes a ſtrange noiſe whenever the Sun begins to ſhine upon it; which Statue, *Strabo* affirms he hath both ſeen and heard it. Much like unto this was that *Muſical Inſtrument* made by *Cornelius Dreble*, which being ſet in the Sun-ſhine, would of it ſelf render a ſoft and pleaſant Harmony; but being removed into the ſhade, would preſently become ſilent.

2. *Cardane* makes mention of a Spit that may be turned (without the help of Weights) by the Motion of the *Air* that aſcends the Chimney, and may be uſed for the Roaſting of many a great Joint of Meat. In which contrivance there are theſe conveniencies: Firſt, It makes little or no noiſe. Secondly, It needs no Winding up. Thirdly, It is much Cheaper than an ordinary Jack, it conſiſting only of a pair of *Sails*, one *Wheel*, to the *Axis* whereof the Jack-Line muſt be faſtned. This may be an Engine of good uſe in many other Domeſtick Affairs, as in Reeling of Yarn, Rocking of a Cradle, *&c.*

Unto theſe kind of Motions may be added all thoſe repreſentations of *Living Creatures*, as *Birds* or *Beaſts*, Invented by *Cteſibius*, which were for the moſt part performed by the motion of the *Air*, being forced up by Rarefaction by *Fire*; or elſe by compreſſion through the fall of Water, which by poſſeſſing the place of the Air, did thereby drive it to ſeek for vent.

3. The late Invention of the *Wind Gun*, which is Charged by the forcible compreſſion of *Air*, being injected through a *Syrenge*, the ſtrife and diſtenſion of the impriſoned *Air* ſerving by help of little *Falls* within to ſtop and keep cloſe the Vents by which it was admitted; the force of which Gun is almoſt equal to our common Powder Guns: For *Merſennus* ſaith, that he hath often found by frequent Trials, that a *Leaden-Bullet* from one of theſe Guns being ſhot againſt a Stone-Wall at 24 Paces diſtance, hath been beaten into a thin Plate.

4. The ſame Author (in the 31th Prop. of the ſame Book *Phænomena Pneumatica*,) whereby he will make the ſame Charge of Air to ſerve for the diſcharge of ſeveral *Arrows* or *Bullets* one after another, by giving the Air only ſo much room as may immediately ſerve for to impreſs a violence in ſending away the Arrow or Bullet.

5. *Boterus* mentions that *Sailing Chariots* are commonly uſed in Champion Countries, as in *China*, by which a Man may Sail on the Land as well as by a Ship on the Water. Amongſt theſe *Sailing Chariots*, that at *Scheveling* in *Holland* is very remarkable; it was made by the Directions of *Stephinus*, and is Celebrated by divers Authors. *Walchius* affirms it to be of ſo great a ſwiftneſs for its Motion, and yet of ſo great Capacity for its Burthen, that it did far exceed the ſpeed of any Ship; and that in ſome few hours ſpace it would convey ſix or ten Perſons 20 or 30 *German* Miles, and all this with very little Labour to him that ſitteth at the Stern. One *Peireskius* an inquiſitive Man, Travelled to *Scheveling* for to ſee the Experience of this *Chariot*, and would often ſpeak of the Extraordinary ſwiftneſs of it: And ſaith (among other Excellencies of it) that in two hours time it would paſs from *Scheveling* to *Putten*, which are diſtant above 14 *Horaria Milliaria*; that is, more than 42 Miles.

6. Of this kind in *Holland* they have frequently other little Veſſels for one or two Perſons to go upon the Ice, having Sledges inſtead of Wheels, being driven by a Sail; the Bodies of them like little Boats, that if the Ice ſhould burſt they might yet ſafely carry a Man upon the Water, where the Sail would be ſtill uſeful.

III. *Of ſuch* Automata's *that have their Motions from ſomething within their own Frames.*

1. Of the ſame kind with our *Clocks* or *Watches*, was that Sphere of Glaſs Invented by *Archimedes*, which did repreſent the Heavenly Motions, the Diurnal and Annual Motions of the *Sun*; the *Changes*, and other Aſpects of the *Moon*, &c. This is frequently Celebrated in the Writings of the Ancients, particularly by the Epigramiſt *Claudius*, &c.

Jupiter in Parvo, &c.

Thus *Engliſhed*.

IN a ſmall Glaſs *when* Jove *beheld the* Skies,
He Smiles, and thus unto the Gods *Replies*:
Could Man ſo far extend his Studious Care,
To mock my Labours in a Brittle Sphere,
Heaven's *Laws*, Man's *Ways, and* Nature's *Sovereign Right;*
This Sage of Syracuſe *Tranſlates to ſight.*
A Sol *within on various* Stars *attends,*
And moves the quick Work unto certain ends.
A feigning Zodiack *runs his proper* Year,
And a falſe Cynthia *makes new* Months *appear.*
And now bold Art *takes on her to Command,*
And Rule the Heavenly Stars *with Humane Hand:*
Who can admire Salmonean *harmleſs* Thunder,
When a ſlight Hand ſtirs Nature *up to* Wonder.

This Spherical Engine I cannot think to be of *Glaſs*, but rather of *Braſs* ſet on Work by Springs as the Movements of our *Watches*, and that it was incloſed in a *Caſe* of *Glaſs*, through which the Motions might be viſible. In imitation whereof there was lately made one by *Cornelius Dreble*, and preſented to King *James* I. of *England*. And like unto theſe *P. Ramus* Reports to have ſeen two at *Paris*, not of *Glaſs* but of *Iron*; one of which was brought by *Ruellius* the Phyſician from the Spoils at *Sicily*, and the other was recover'd by *Orontius* from the *Grecian* Wars.

Of Walking or Flying Automata's.

1. Such were thoſe ſtrange Inventions attributed to *Dedalus*, Self-moving Statues, which (unleſs they were violently detained) would of themſelves run away. *Ariſtotle* affirms that *Dedalus* did this by putting

putting *Quickſilver* into them; it is more likely he did it with *Wheels* and *Springs* within.

2. Of this kind likewiſe were *Vulcan's Tripodes*, Celebrated by *Homer*, that were made to move up and down the Houſe, and fight with one another.

3. *Cardan* makes mention of an *Image* holding in its Hand a *Golden Apple*, beautified with many coſtly Jewels, which if any Man offered to take, the Statue preſently ſhot him to Death; the touching of which Apple ſerving to diſcharge ſeveral ſhort Bows, or other like Inſtruments couched in the Body of the Image.

4. *Walchinus* his *Iron-Spider* is very remarkable, which being but of an ordinary bigneſs, beſides the outward ſimilitude, which was very exact, had the ſame motions with a Living *Spider*, creeping up and down as if it had been alive.

5. I my ſelf remember, and have often ſeen it, a Figure of a *Dutch Froe* made; the Caſe of Silver about 5 or 6 Inches high, one Arm by her ſide, and in the other a Silver Cup, which Cup being filled with Wine, and her Face directed to the party Drank to, who would ſoftly Travel over the Table to the very Party; and when ſhe came to the edge of the Table, ſhe would there ſtand ſtill till the party took the Cup out of her Hand and Drank to another; and ſo to a 3d, 4th, 5th or 6th Party, or more, one after another. And thus would ſhe do for the ſpace of half an hour, beſides the times that ſhe ſtood ſtill between every Parties taking the Cup and Drinking.

6. There have been ſome Inventions alſo, which have been made, able to utter ſome certain Words; and ſuch are ſome of the *Ægyptian Idols* related to be: And ſuch was the *Brazen Head* made by Friar *Bacon*. And that Statue, in the framing of which *Albertus Magnus* beſtowed thirty years in the framing, who *Aquinas* coming to ſee, broke it, purpoſely, that he might boaſt, how that in one Minute he had ruin'd the Labour of 30 years.

Of Flying Automata's.

1. The Wooden *Dove* made by *Architas*, a Citizen of *Tarentem*, and one of *Platres* Acquaintance.

2. The Wooden *Eagle* framed by *Regiomontanus* of *Noremberg*, which by way of Triumph did fly out of the City to meet *Charles* the Fifth.

3. The Iron *Fly* made by the foreſaid *Regiomontanus*, which, when he Invited any of his Friends, would fly to each of them round the Table; and at length (as being weary) return unto his Maſter.

CHAP. XII.

Of the Magnificent Works *of the* Ancients, *and wherein they exceed these of our Age.*

I Shall here make a reictal of some of the largest *Monuments* of this Nature which are Recorded in the *Ægyptian*, *Jewish*, *Grecian* and *Roman* Histories. And

1. Amongst the *Ægyptians*, we read of divers *Pyramids*. *Herodotus* mentions one of them Erected by *Cleopes* an *Ægyptian King*, wherein there was not one Stone less than 30 foot long.

2. *Amasis*, another *Ægyptian*, made himself a *House* of one entire *Stone*, which was 21 Cubits long, 14 Broad, and 8 High.

3. The forementioned *Amasis* is Recorded to have made the *Statue* of a *Finx*, or *Ægyptian Cat*, all of one Stone, whose *Length* was 143 foot, its *Height* 62 foot, and the *Compass* about it 102 foot.

4. *Pliny*, in his 37 Book, and 5th Chapter, Reports, that in one of the *Ægyptian Temples* Dedicated to *Jupiter*; there is an *Obelisque*, consisting of four *Emeralds*; which is 40 *Cubits High*, 4 *Cubits Broad* at the *Bottom*, and 2 at the *Top*.

5. *Diodorus Siculus*, Reports, that *Sesostris* an *Ægyptian* King, Erected in a Temple at *Memphis* Dedicated to *Vulcan*, two *Statues*, one for himself, the other for his Wife; both consisting of two several Stones; each of which *Statues* were 30 *Cubits* High.

6. Amongst the Jews, *Solomons Temple*, which for its *State* and *Magnificence*, (besides the great *Riches* of the *Materials*,) there were *Pillars* of *Brass* 18 Cubits high, and 12 Cubits about; and great and costly Stones for the Foundation of it; some of which *Josephus* tells us, were some 40, others 45 Cubits long.

7. The same *Josephus*, in the 6th Chap. of his 6th Book, tells us, that *Herod* built three Famous *Towers*, each 30 Cubits high, of White Marble, each Stone whereof was 20 Cubits Long; 10 Broad, and 5 Thick. These *Temples* were scituate upon a steep rising Ground, and *Hills* upon that; upon the Tops of which *Hills* these *Towers* were Erected.

8. *Pliny*, in the 14th Chapter of his 36 Book Relates, that in the *Ephesian Temple* Dedicated to *Diana*, there were 127 *Columbs* made of so many several Stones, each 60 foot high; which Stones were digged out of the Quarries in *Asia*. This Temple was fired seven times, the last time by *Erestratus*, only to get himself a Name to Posterity.

9. Amongst the rest, the *Brazen Colossus* at *Roads* must not be forgotten, which *Pliny* in the 3d Chapter of his 34th Book, Reports to be Erected over a *River*, the two Feet whereof were fixed on either side of the *River*, and of such a *Magnitude*, that it was 70 Cubits high, the *Thumbs* of it being so big, that no Man could grasp one of them

them about with both his Arms: And it ſtanding upright, a Ship might have paſſed betwixt the Legs of it, with all its Sails fully diſplayed: This *Statue* was thrown down by an Earth Quake, and the *Braſs* of it did Load 900 Camels. Now, ſuppoſing a *Camels* Burthen to be 1200 Weight, the Braſs in this Statue did contain 1080000 Pound Weight.

10. But above all theſe (had it been effected) that would have taken place, which was by a *Grecian* Architect propounded unto *Alexander*, for to have cut the Mountain *Athos* into the Form of a *Statue*, which in his *Right Hand* ſhould hold a *Town*, capable to receive ten thouſand Men, and in his Left, a *Veſſel* to receive all the *Water* that flowed from the ſeveral *Springs* in the *Mountain*: But *Alexander* (for what Reaſon I know not) embraced not the propoſal, and ſo the World muſt reſt ſatisfied without this *Wonder*. But had this *Statue* been Erected, according to the Propoſal, let us a little enquire into the *Magnitude* of ſome of the *Members* of this *Statue*. (1.) For the *Statue* it ſelf, that could not be higher than the *Mountain*, and the *Mountain* was not above *one Mile* in *Perpendicular height*. Therefore (2) The *Hand* muſt be the 10th part of the height, that is 500 foot, and conſequently, (3.) The *Breadth* thereof 250 foot. And, (4) The *Superficies* of the Hand would be 125000 *Square Foot*, in which ten thouſand Men were to be received, and that it would afford, (5.) Each *Man* 12 Foot and a half of *Ground*. And, (6.) For the other *Members* you may compare their *Magnitudes* by what hath been ſaid concerning the Proportions of the *Members* of Mans *Body*.

11. Amongſt the *Romans*, we Read of a *Coloſſus* of *Braſs*, made by the command, and at the Charges of *Nero*, which was 100 Foot high, which *Martial* calls *Syderius*.

Hic ubi Syderus *proprius videt Aſtra Coloſſus.*

12. It is ſtoried of *M. Curio*, that he Erected two *Theatres* ſufficiently capable of People, contrived moveable upon ſeveral Hinges. Sometimes there were *Plays* or *Shews* in each of them, neither being any diſturbance to the other; and ſometimes they were both turned about with the People in them, and the ends meeting together, did make a perfect *Amphitheatre*. So, that the *Spectators* which were in either of them might joyntly behold the ſame Spectacles.

13. At *Rome* were Erected ſeveral *Obeliskes*, each conſiſting but of one entire Stone; of which ſome were 40, ſome 80, and ſome 90 Cubits high. The choice of theſe were brought out of *Ægypt*, where they were dug out of ſeveral *Quarries* there, and being there wrought into Form, were afterwards (with great Charge and Labour) conveyed to *Rome*.

14. In the year 1586, there was Erected an *Old Obeliske*, which had been formerly Dedicated to the Memory of *Julius Cæſar*, it was one ſolid Stone, an *Ophite*, or *ſpotted Marble*; the *Height* of it was 117 foot, the *Breadth* at the bottom 12 foot, and at the Top 8 foot. Its whole Weight is Recorded to be 956148 Pounds, beſides the heavineſs of all thoſe *Inſtruments* and *Utenſils* that were employed about it, which (as it is thought) could not amount to leſs than 1040824 Pounds.

Pounds. This *Obeliske* was Transplaced at the Charges of *Pope Sixtus* the V. from the Left side of the *Vatican*, unto a more Eminent place, about a hundred foot off, where now it stands upon a Pedistal 12 foot and a half high: And the height of the Gilded Cross of Brass, at the Top of the *Obeliske* is 19 foot and a half high. This great Work was done in a few days; by five hundred Men.

15. *Plammiticus* King of *Ægypt*, built a Labyrinth on the South side of the Pyramids, which contained within one continued Wall 3500 Persons, (as *Herodotus* saith) and 12 Royal Palaces, all covered with Marble. The Labyrinth had only one Enterance, but innumerable turnings and windings, sometimes one over another: The Building whereof was more under ground than above, the Chambers so disposed, that the Doors upon their opening, did give a Report like a crack of Thunder. The main Entrance was all of white Marble, Adorned with stately Columbs and most curious Images. Being at the end, a pair of Stairs of 90 steps, conducted into a stately *Portico* supported by Pillars of *Theban* Marble, which was the Entrance into a Far and Stately Hall (the place of their General Convention) all of Polished Marble set out with the Statues of their Gods.

16. The House of *Nero*, which he called *Domum Aurum*, by *Suetonius* is thus described: In the Porch was placed a *Colossus*, shaped like himself of 120 foot high. The spaciousness of the House was such, that it had in it three Galleries, each of them a Mile long: A standing Pool like a Sea, beset with Buildings in manner of a City; Fields in which were Arable and Pasture Grounds, Vineyards and Woods, with a Multitude of Tame and Wild Beasts of all kinds: In the other parts thereof all things were covered with Gold, and distinguished with Precious Stones or Mother of Pearl: The Supping Rooms were Roofed with Ivory, moveable for the casting down of Flowers, and Pipes in them for the sprinkling of Oyntments: The Roof of the principal Supping Room was Round, which (like the Heavens) Day and Night moved round about. This House when he had thus finished and Dedicated it, he said then, that he began to live like a Man.

17. *Pancirollus* Reports, that *Ptolomæus Philopater* built a Ship, or Galley which was 280 Cubits in length, 52 Cubits from the Keele to the upper Deck: It had 400 Banks or Seats for Rowers; 400 Mariners, and 4000 Rowers, and on the Deck would contain 3000 Souldiers. *Plutarch* in the Life of *Demetrius* saith, there were Gardens and Orchards on the Top of it.

Of the Time, and Number of Men *employed in the* Building *of some of these* Magnificent Works.

1. In the making of one of the *Ægyptian Pyramids*, we Read, that three hundred and sixty thousand Men were employed for *twenty years*. And *Herodotus* tells us, that one *Million* (or ten hundred thousand) of Men were as long in building another of them.

2. About the carriage of one Stone from *Amasis*, the distance of 20 days Journey, there was for three years together employed 2000 chosen Men *Governours*, besides many other under *Labourers*.

3. In

3. In the building of *Solomons* Temple, there were threescore and ten thousand Men that bore burthens, and 80 thousand *Hewers* in the *Mountains*.

4. The *Ægyptian* Temple was built by all *Asia* joyning together: The 127 *Pillars* thereof were made by so many Kings, according to their several Successions; the whole Work being not finished under the space of 215 years.

5. In the building of one of the *Pyramids*, there was expended for the maintenance of the *Labourers* with *Radish* and *Onions*, no less than eighteen hundred Talents, which is reckoned to be about 470000 *i. e.* four hundred and seventy thousand pound Sterling.

6. It is Reported, that the making of the *Rhodean Colossus* (before spoken of) did cost 300 Talents.

7. *Cleopes*, an *Ægyptian King*, is reported to have been so desirous to finish one of the *Pyramids*, that having spent all about it he was worth, he was forced at last, to prostrate his Daughter, for necessary maintenance.

8. We Read of *Ramises*, another *Ægyptian* King, how that he was so careful to Erect an *Obeliske*; about which he had employed twenty thousand Men; that when he feared, least through the negligence of the *Artificers*, or weakness of the *Engine*, the Stone might fall down and break, he tied his own Son to the Top of it, that so the care of his safety might make them more circumspect in their business.

An Account of the Heights *of several* Obeliskes, Steeples, Pyramids *and* Pillars *in the* World, *according to our* English Measure, *most of which are now in being.*

1. St. *Pauls* Steeple in *London*, when the *Spire* was on it, (which was Fired by Lightning *Anno* 1555) the *Stone Work* was 260 foot high, and the *Spire* as much, which was 520 foot in all.
2. The Steeple at *Cremona* in *Italy*, is 528 foot high.
3. The *Ball* upon St. *Peters* in *Rome*, is 466 Feet.
4. The *Steeple* at *Roan* in *Normandy*, is 399 Feet.
5. The *Steeple* at *Strasburg* in *Germany*, is 431 Feet.
6. The *Steeple* at *Landhoven* in *Bavaria*, is 451 Feet.
7. The *Steeple* at *Medera* in *Italy*, is 279 Feet.
8. The *Tower Asinel* in *Banonia* in *Italy*, is 316 Feet.
9. The *Cupulo*, or *Lanthorn* at *Genua*, 324 Feet.
10. The *Highest* of the *Pyramids*, 1350 Feet.
11. The *Lowest* of the *Pyramids*, 883 Feet.
12. *Boston Steeple* in *Lincolnshire* all of Stone, and without any *Spire*, is 264 Feet.
13. The height of the *Obeliske* in *Rome*, removed by *Sixtus* the V. is 78 Foot high.
14. The *Columb* or *Monument*, Erected in Memorial of the great Conflagration in *London*, *Anno* 1666, wherein 13200 Houses, 89 Parish Churches were burned, all which stood upon 437 Acres of Ground. The *Monument* is of Stone; the *Pedistal* whereof is 27 foot broad, and the height of the *Columb*, from the Ground (besides the Foundation) to the Top of the *Flame*, is 202 Foot, the Circumference

rence of the *Shaft* is 47½ Foot, its Diameter 15 Foot, and of the hollow Cylinder 9 Foot, with *Steps* up to the Top, and an hollow *Newel* from the Top to Bottom.

CHAP. XIII.

An Account of some such admirable Pieces of Work of several kinds, made by several eminent Artists, both Ancient and Modern; which have been by divers Historians and others related; and of some in this our present Age.

1. REgiomontanus, a Famous *Geometrician* of *Norimberg* made an *Eagle* of Wood, and a *Fly* of Iron —— This Eagle he put to Flight out of the City, which raised it self aloft into the Air, and met the Emperour *Maximilian* a good way off, as he was coming towards it; and having saluted him, returned again to the City Gates. The Fly, at a Feast, flew forth of his Hand, and taking a Round about the Table, returned to his hand again. Of these two admirable Pieces of Wormanship, (performed by Geometrical Proportion,) *Du Bartus* thus Writeth.

Why should not I that Wooden Eagle *mention,*
A Learned Germans *late admir'd Invention;*
Which Mounting from the Fist that framed her,
Flew far to meet an Almain *Emperour;*
And having met him with her Nimble Train,
And wearied Wings; turning about again;
Follow'd him close, unto the City Gate
Of Norimberg, *whom all their Shews of State,*
(Streets hang'd with Arras, Arches Rarely Built:
Gray-headed Senate, and Youths Gallantries;)
Grac'd not so much, as did this one Device.

Then he Describes the FLY as followeth,

Once as the Artist, more with Mirth than Meat,
Feasted some Friends, whom he Esteemed Great:
From under's Hand an Iron Fly flew out,
Which having flown a perfect Round about,
With weary Wings Returned to her Master,
And (as Judicious) on his Arm he plac'd Her.
O! Divine Wit, that in the Narrow Womb
Of a small Fly, could find sufficient Room,
For all those Springs, Wheels, Counterpoise and Chains,
Which stood instead of Life, and Spur and Reins.

2. *Doctor*

2. *Doctor Hackwell* in his Third Book of his Apology of the Power and Providence of God, in the Government of the VVorld——*Verstegan* in his Antiquities, *Chap.* 2.—— And *Knowls* in his Turkish History, *Pag.* 513. Make mention of a Silver Sphere, which was sent by the Emperour *Ferdinand*, to *Solyman* the Great Turk: It was carried (Unframed) by Twelve Men, and Re-framed in the Grand Signior's Presence by the Maker of it; who also delivered him a Book containing the Mystery of using it. Of which Sphere *Du Bartus* thus VVriteth.

Nor may we smother, or forget ungrately
The Heaven of Silver that was sent, but lately
From Fardinando, *as a Famous Work*
Unto Bizantiam, *to the* Greateſt *Turk*;
Wherein a Spirit, still moving to and fro,
Made all the Engine orderly to go,
And tho' th'one Sphere did always slowly Glide:
And contrary, the other swiftly Slide:
Yet still the Stars, kept all their Courses even
With the True Courses of the Stars in Heaven.
The Sun there shifting in the Zodiack,
His shining Houses never did forsake
His pointing Path. There in a Month his Sister
Fulfill'd her Course, and changing oft her Lustre,
And Form of Face, (now Larger) Lesser soon;
Follow'd the Changes of the other Moon.

III. *Archimedes* the *Syracusan* Mathematician, who flourished *Anno Mundi* 3739, and before the Nativity of Christ 263, who is said to have composed a Sphere of Transparent Glass; representing to the Life the whole Frame of the Heavens; wherein the Sun, Moon and Stars, with their true Motions, Periods and Limits were shewed to the Sight, in such sort as if it were Natural: Of which Sphere *Claudius*, an Eminent Poet in the time of *Theodosius* the Emperour, who was born at *Alexandria*, and flourished *Anno Christi* 390, Elegantly VVrote in his Epigrams; which is thus excellently Translated into English, by our Late English Poet, Mr. *T. Randolph.*

Jove saw the Heavens, Fram'd in a little Glass,
And Laughing to the Gods, these Words let pass:
Comes then the Power of Mortal Cares so far;
In Brittle Orbs, my Labours Acted are.
The Statutes of the Poles, the Faith of things,
The Laws of Gods, this Syracusian *brings*
Hither by Art; Spirits inclos'd attend
Their several Spheres; and with set Motions bend
The Living Work; each year the feigned Sun,
Each Month returns the Counterfeited Moon;
And viewing now her World, bold Industry
Grows Proud, to know the Heavens his Subjects be,
Believe, Salmonius *hath false Thunder thrown,*
For a poor Hand is Natures Rival grown.

IV. One *Mark Scalios* a Black-Smith, in the year of Christ 1573, made a Lock, which consisted of eleven pieces of Iron, Steel and Brass, all which, together with a Pipe Key to it, weighed but one Grain of Gold—— The same party also, made a *Chain* of *Gold*, consisting of 43 Links, to which he fastned the forementioned Lock and Key, and putting the Chain about the Neck of a Flea, which drew them all about with ease; and all these particulars weighed together but one Grain and a half.

V. One *Colicrates* would make Ants, Pismires, and other such like Creatures of Ivory, so small, that others could not discern the parts thereof one from another by the bare Eye, without the help of Glasses.

VI. One *Mermicides* (another excellent worker in Ivory,) made a Coach with four Wheels, and as many Horses; which a Fly might cover with her Wings,—— He also made a Ship, with all her Tackling to it, which a small Bee would cover with her Wings.

VII. One *Cornelius van Drebble*, a Dutch-man (in imitation of *Memnons* Statue) made a kind of Organ that would make an excellent Symphony of it self, being placed in the open Air and clear Sun, without the Fingers of an Organist; but in a shady place it would yield no Musick, but only where the Sun Beams had the Liberty to play upon it.

VIII. One *Janellus Tarrianns*, a Great Mathematician, for to Recreate the Emperour *Charles* the V. would oftentimes delight him, by sending sometimes, Wooden Sparrows into the Emperours Dining Room, which would Fly about that, and return———Sometimes he would cause little Armed Men to Muster themselves upon his Table; and Artificially to move according to the Discipline of War.

IX. There was an Artificer in *Rome*, who made Vessels of Glass of such a Temper, that they were as little liable to be broken, as these of Gold or Silver. This Artificer made a Vessel of this pure sort of Glass, and presented it to *Tiberius Cæsar*, the then Emperour: The Gift was received, and the Artist applauded; and that he might gain more Applause, he desires the Vessel again, and threw it with great Violence against the ground, whereby it received no more damage than the like Vessel made of the solidest Metal would have sustained thereby; at which *Cæsar* was astonished: But the Artificer drew an Instrument out of his Bosome, wherewith he suddenly reduced it to its former shape: At the sight whereof, the Emperour was much moved, and enquired of the Artist, whither there were any other besides himself were privy to the like tempering of Glass? When he had told him no, he commanded his Head immediately to be stricken off: Saying, should this Artifice come once to be known, Gold and Silver would be of as little Value as Stones in the Street.

X. *Knolls* in his Turkish History, *Pag.* 1253 Relates, (long after the forementioned) *Viz.* in *Anno* 1610, that amongst other rare Presents then sent from the *Sophy* of *Persia* to the King of *Spain*, were Six Glasses of Malleable Glass, so exquisitely tempered, that they could not be broken.

XI. At *Dantzick*, a City of *Prussia*, one Mr. *Harrison* sent a Mill, which without help of Hands did saw boards, having an Iron Wheel which did not only drive the Saw, but did also hook in the boards, and

and drive them out, to and from the Saw——Dr. *John Dee*, in his Mathematical Preface before *Euclides Elements*, mentions the like seen by him in *Prague*; but whither the Mill before mentioned, or that which he see at *Prague*, moved by Wind or VVater, is not set down by either of them.

XII. At the Mint of *Segovia* in *Spain*, there is an Engine that moves by water, so made, that one part of it distendeth an Ingot of Gold into that breadth and thickness, as is required to make Coyn of. It delivereth the Plate that it hath wrought, into another that Printeth the Figure of the Coyn upon it; and from thence it is turned over to another, that cutteth it according to the Print in due shape and weight. And lastly, the several Pieces fall into a Receiver in another Room; where the Officer, whose charge it is, finds it ready Coyned.

XIII. *Oswaldus Morhingerus*, made 1600 Dishes (or Platters rather) of turned Ivory, so small and little, so thin and slender, that all of them were included at once in a Cup turned out of a Pepper Corn of the ordinary size. This Piece was carried to *Rome*, and shewed to Pope *Paul* the V. who (by help of Spectacles) told them all distinctly.

XIV. *Johannes Baptista Ferrarius*, a Jesuit, had in his Possession, not long since, Cannons of VVood with their Carriages, VVheels, and other Military Furniture; of which 25 of these, and 30 Cups turned out of wood, were all at once contained in one Pepper Corn, which exceeded not the common bigness.

XV. *George Whitehead*, an English Man, made a Ship with all her Tackling, to move it self on a Table, with Rowers plying the Oars; a VVoman playing on the Lute, and a little VVhelp creeping on the Deck. *Schottus* relates this in his History of Man, *Chap.* 12.

XVI. About the year 1649 or 50, one *Fromanteel* an English Man (though of Dutch Parentage) made here in *London* for Sr. *Parston*, a little Figure in Silver, of a VVoman in Dutch Habit, holding upright in her two Hands a Silver Cup, which would hold about the tenth part of a Pint of VVine. This Figure Sir *Parston* called his Froe (in respect of her Habit) and after Dinner, (for the Entertainment of his Friends) while his Servants were at their Dinner, would call for a Bottle of VVine, and his Froe to wait at his Table in his Servants Absence; which being brought, he would take the Cup out of his Froes Hands, and fill it with VVine; then drinking to one of his Friends on the other side of the Table, he would fill the Cup, and put it into the Froes Hands again, and directing her Face to the Person he drank to, the Froe would move of her own accord, cross the Table, to the party drank to; and when she came to the very edge of the Table, (till some part of the Skirt of her Garment hang over) she would there stand still; till the party drank to, took the Cup out of her Hands, and he drinking to another, filled the Cup again, and directed the Froe to whom he drank; to whom she would directly go as before; and would thus continue going throughout the Table. This Figure have I often seen, and been at the Table when the Experiment was put in practice.

XVII. There was one in Queen *Elizabeth's* days, that wrote the Ten Commandments, the Creed, the Pater-Noster, the Queens Name, and the Year of our Lord, within the Compass of a Penny; and gave with

it a pair of Spectacles, by help whereof the Queen did distinctly discern every Letter.

XVIII. Since which time, Namely in the Year of our Lord 1656, one *Abraham Svitzer*, born in the Parish of St. *Andrews* in *Holborn*, *London*, (by profession an Ingraver, or Cutter of Figures, Pictures or Schemes to print in Books, in *Box* or other VVood.) Did write in a Roman Hand in words at Length, these following particulars; *Viz.* 1. *Every good and perfect Gift comes from above.* 2. *Lord teach us to Pray.* 3. *The Lords Prayer.* 4. *The Creed.* 5. *The* 134 *Psalm.* 6. *Lord Preserve the Church.* And Lastly, in the Center, or middle; (for it is written in a Spiral or Serpentine Line, with a Red Line, between every Line, for the better guiding of the Eye in Reading.) 7. *All the Figure of a Deaths Head*, which, one of the Farthings (then in use) would easily cover. This writing was written in the 61 year of his Age, and my own Father when I first shew'd it to him, being then 63 years of Age, could Read without Spectacles (for he never used any) and this piece of small writing I have this present time, *Febr.* 16. 1690 by me, to be seen of any that shall desire it.

XIX. One *Francis Alumnus* wrote the Apostles Creed, and the 14 first Chapters of St. *Johns* Gospel, in the Compass of a Peny, and this was done in full words, in the presence of the Emperour *Charles* the V.

XX. One *J. Rich*, an English Man, and an excellent Steniographer, did in the year 1671, take in Short-Hand, a Sermon, word for word, Preached before King *Charles* the II. at *Whitehall*, by a Bishop (whose Name I do not remember) in less than half a quarter of a sheet of Paper; and afterwards shewed it to the Bishop, and did distinctly Read it before him, who acknowledged it to be his, and that he did think there was not one word added or diminished, from what he then Preached: VVhereupon he wished he would present some piece of his Art to the King; upon which, he wrote the same Sermon in a little Book of Six Leaves of Fine Paper, and had it bound in Crimson Sarcenet, with Silver Clasps and Corners upon the Cover; all which Book and Cover was less than the nail of his little Finger; which Book he afterwards presented to the King.

XXI. *Wind Guns*, or *Muskets* are a Rare and Late Invention, which will shoot Bullets without Powder, or any thing else but *Wind*, or *Air* compressed in the bore of it, or injected by a Spring: These will discharge with as much force as others with Powder.

XXII. In the Duke of *Florentines* Garden at *Pratoline*, is a Statue of *Pan* sitting upon a Stool, with a wreathed Pipe in his Hand, and Syrinx beckoning him to play upon it. *Pan* putting away his Stool, and standing up, plays upon his Pipe; which done, he looks upon his Mistress, as expecting thanks from her, but his expectation being frustrate, he sits down with a sad Countenance.

XXIII. There is also in the same Garden another Statue, of a *Laundress* beating a Buck, and turning the Cloaths up and down with her Hand and Battledore, with which she beats them in the water.

XXIV. There is also the Statue of Fame loudly sounding her Trumpet. An Artificial Toad creeping up and down. A Dragon bowing down his Head to drink water, and then vomiting up again: VVith divers other pieces of Art.

XXV. In

XXV. In Cardinal *Farraras* Garden at *Tivoli* near *Rome*, are the Repreſentations of ſundry Birds ſitting on the Tops of Trees; which by ſecret conveyances of water through the Trunks and Branches of the Trees, are made to Sing and clap their Wings; but at the ſudden appearance of an Owl out of a Buſh, they immediately all become Mute and Silent: Theſe were made by *Claudius Gallus*.

XXVI. In *Dantzick* in *Poland*, there was ſet up a rare Engine for weaving of four or five Webs at the ſame time, without any humane help, and would work Night and Day. This Invention was ſoon ſuppreſſed, and the Inventor ſecretly made away.

XXVII. *Hadrianus Junius* ſaw at *Mickland* in *Brabant*, a Cherry-Stone cut in the form of a Basket; wherein were fourteen pair of Dice, each with their ſeveral Spots, eaſily to be diſcerned by a good Eye.

XXVIII. In the year 1524, the City of *Colonia Agrippina*, was Painted with much exactneſs, and yet ſo little, that a Fly might cover it.

XXIX. In the Reign of *Anaſtacius Dicorus*, the Famous *Geometrician* and *Aſtronomer Proclus*, made burning Glaſſes of that admirable force, that therewith he burnt (at a great diſtance) the Ships of the *Myſians* and *Thraſians* that had blocked up the City of *Conſtantinople*.

XXX. One *Peter de Peene*, by Profeſſion a Printer, made a Printing Preſs, of VVood and Iron, with all the Members belonging thereunto, which was no bigger than a common Hour Glaſs, and in it was a form of Letters, conſiſting of twelve Lines of Verſes, written in the Praiſe of that Famous Art or Myſtery of Printing, which this Preſs (as ſmall as it was) would Print off; which Preſs my ſelf have often ſeen in *London*: And this *Peter de Peene* was adviſed (he being but a poor Man) to put this Preſs into a Box, with ſome Ink and Letters, and go about the Countries to ſhew the Art of Printing to the Country People, and to Print their Names either in Red, Black or Gold upon Paper, Parchment, or Satten Ribband.

XXXI. The Duke of *Holſted* had made in the City of *Gottorp*, a double Globe of Copper, which was ten foot and a half Diameter; ſo that within it ten Perſons might ſit at a Table, which (with the Seats about it) hangeth at one of its Poles: In which Globe, a Man may ſee (by means of an Horizontal Circle within the Globe) how the Sun and Stars move of themſelves through the Degrees of the Ecliptick, and their Riſing and Setting regularly. The motion of this Globe exactly followeth that of the Heavens, and deriveth that motion from certain Wheels and Pinnions, driven by water, which is drawn from a Mountain not far off, and let in as it requireth More or Leſs.

XXXII. One *Linus*, an Engliſh man, of the Society of *Jeſus*, had a Vial Glaſs of water, wherein a little Globe did float, with the 24 Letters of the Alphabet deſcribed upon it: In the inſide of the Vial was a Style or Index, to which the Globe did turn and move it ſelf at the Period of every Hour, with the Letter which denoted the Hour of the Day or Night ſucceſſively.

XXXIII. The ſame *Linus*, in *Anno* 1668, came over into *England*, (by way of Diſguiſe, in Habit of a Country Gentleman) and in *Anno* 1669, did make in the Kings Privy Garden at *Whitehall*, a Pyramidical Dial. This Dial did ſtand upon a Pillar, or Pediſtal of Stone,

and

and consists chiefly of Six Parts or Pieces, one less than another, and placed one above another, in form of a Pyramis.——— The whole Fabrick contained 113 Dials, some Plain, some Concave, some Convex, some Upright, or Vertical; some Inclining, some Reclining; some Globular and Hollow; some covered with Glass, *&c.* Some of these shew the common Hour of the Day, others the *Jewish*, others the *Babylonish*, others the *Italian* Hour. Some shew the Suns Declination, others the Suns Altitude, others the Suns Azirnath, others the Suns Rising, others the Suns Setting, others the Amplitude, some the Day of the Month, *&c.* Some by Fire, some by Water, some by Earth, and some by Air. In some of these Dials you shall see the Hour Lines, but no Stile, in some the Stile but no Hour Lines; in some neither Stile nor Hour Lines; and in some both Stile and Hour Lines. Infinite Varieties there were upon this Pyromis, too many here to recite; but for such as would know more concerning this Piece of Art, may have recourse to my Book of Dialling in *Folio*, *Pag.* 323, *&c.* where there is a Figure of the whole Pyramis, and a Description of every particular Dial upon it. This Pyramis stood in the Garden aboveſaid about 18 Months, and then in a Frolick much Battered and Defaced, so that there is now nothing of it to be seen.

XXXIV. I saw, *saith my Author*, at *Legorn*, a Clock brought thither by a *German*, to be sold, which had so many Rarities in it, as I should never have believed, if my own Eyes had not seen it: For, (besides an infinite Number of strange Motions which appeared not at all to the Eye) you had there a company of Shepherds, some of which played on the Bag-Pipes, with such Harmony and Excellent Motion of the Fingers, as that one would have thought they had been alive; others Danced by Couples, keeping exact time and Measure; whilst others Leap'd and Capred up and down, with so much of Nimbleness, that my Spirits were wholly Ravished with the Sight.

XXXV. *Copernicus* made a Clock, in which there was not only to be heard a Number of different Noises, occasioned by its various Motions; but also (most exactly to be discovered) the circutions of all the Cœlestial Orbs; the distinction of Days, Months, and Years: There the Zodiack did explicate its Signs; so performing the Circle of the Year; there the playful *Ram* began the Spring; *Cancer* produces the Summer; *Libra* inriches it with Autumn; and *Capricorn* makes the Winter. Here also the Moon changes in the *Nones*, shines out more Bright in the *Ides*; and shamefully conceals her Conjunction with the Sun in the *Calends*. But, besides all these, there were produced into the Scene, upon the Entrance of every hour made shew of some Mystery of our Faith. The first Creation of Light, the powerful separation of the Elements, and all other intermediate Mysteries he had traced upon this Engine, even to the Great Eclipse that was when our Saviour suffered upon Mount *Calvary*, *&c.* for to insist upon the particulars were the Work of an Age.

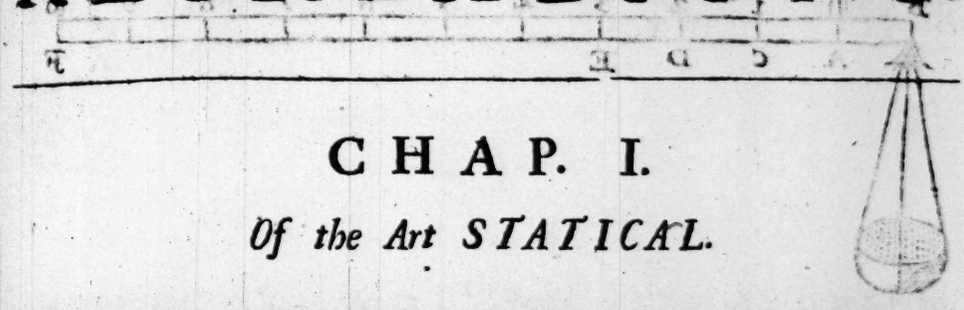
XXXVI. There is a Clock at *Strasburg*, invented by *Cornelius Dasipodius* in the year 1571. Before this Clock stands a Globe on the Ground, shewing the Motions of the Heavens; Namely, of the Heaven carried about by the first Mover in 24 Hours: of *Saturn* in 30 years, of *Jupiter* in 12, of *Mars* in 2, of the *Sun*, *Venus* and *Mercury* in

in one, and of the *Moon* in a Month. In the Clock it ſelf there be two Tables, ſhewing the *Eclipſes of the Sun* and *Moon*, from *Anno* 1573 to 1624. There is alſo a third Table, which conſiſteth of three Parts: In the firſt Part, the Statues of *Apollo* and *Diana* ſhew the courſe of the year, and the Day thereof: The ſecond ſhews the Year of our Lord, and of the World, the Hour and Minute of the Day. The third Part hath the Geographical Deſcription of all *Germany*, and particularly of *Strasburg*, and the Names of the Inventor, and all the Work-Men. In the middle Frame of the Clock is an Aſtrolabe, ſhewing the Sign in which each Planet is every day; and there be the Statues of the Seven Planets, upon a round Piece of Iron lying flat, ſo that every Day that Planet which Denominates the Day, comes forth, the others being all hid; as upon Sunday the *Sun*, *Monday* the *Moon*, *&c.* And there is a Terreſtrial Globe upon which the Hour, half Quarter and Minutes are ſhewed. There is alſo a Skull of a Dead Man, and the Statues of two Boys, whereof one turns the Hour-Glaſs every Hour when the Clock hath done ſtriking; and the other puts forth the Rod in his Hand at each ſtroke of the Clock. There are alſo the Statues of the *Spring*, *Summer*, *Autumn* and *Winter*, and many Obſervations of the *Moon*. In the upper part of the Clock, are the Statues of four Old Men, which ſtrike the Quarters of Hours; the Statue of Death coming out at each Quarter to ſtrike, but being driven back by the Statue of *Chriſt*, with a Sphere in his Hand for three of the Quarters; but at the fourth Quarter, that of *Chriſt* goeth back, and that of Death ſtriketh with a Bone in his Hand, and then the Chimes begin. On the Top of the Clock is an Image of a Cock; which twice in a day Croweth aloud, and chappeth his Wings. Beſides, this Clock is Beautified with many Rare Pictures, and being in the inſide of the Church carrieth another Frame to the outſide of the Wall; wherein the Hours of the Sun, the Courſe of the Moon, the Length of the Day, and ſuch other things, are ſet out with great Art.

XXXVII. The forementioned *John Fromanteele*, in the year 1649, did make a Table Clock with Springs, for Mr. *Dudley Palmer* of *Grays Inn*, which beſides the Hour and Minute of the Day, there was ſhewed upon the Dial Plate, the time of the Suns *Riſing* and *Setting*, with the *Length* of the Day and Night; the *Suns Declination*, his *Amplitude*, his *Declination*, and other things relating to the Courſe of the Sun. This Clock gives one ſtroke upon a ſmall Bell every *Minute*, and at every Quarter 2, 4, 6 and 8, ſtrokes upon a bigger Bell, and upon a third Bell, at every *Quarter* is repeated the preceding *Hour*, and at every third Hour the Chimes went; which were ſo contrived, that they Rang 12 ſeveral times, by the ſhifting of three ſeveral Barrels, 4 Tunes upon each Barrel. This Clock I have often ſeen, and had ſome Hand in the making of it.

The End of the Mechanical Part.

Statical RECREATIONS.



CHAP. I.

Of the Art STATICAL.

THE *STATICKS* is defined to be an *Art Mathematical*, which demonſtrateth the Cauſes of *Heavineſs* and *Lightneſs* of all Things: And of *Motions* and *Properties* belonging to *Heavineſs* and *Lightneſs*.

So that by this Art, *Spheres* or other *Regular Bodies* may be *meaſured*, by having their *Quantity Ponderal*, according to any *Metal* and *Weight* aſſigned;) or, by having their *Quantity Dimentional*, (according to any aſſigned *Meaſure*;) there being the ſame Mathematical Reaſon of theſe two *Quantities*: And ſo they are by Mathematicians compared together in ſeveral kinds of *Bodies*; or divers kinds of *Bodies* are compared together among themſelves in this twofold *Reaſon* of *Quantity*: Viz. *Magnitude*, or *Dimenſion*; and *Gravity*, or *Ponderoſity*. So that theſe two do proportionally anſwer each other, inſomuch that the one may be deduced from the other; as *Gravity* from *Magnitude*, and *Magnitude* from *Gravity*: Or *Weight* from *Solid Meaſure*, and *Solid Meaſure* from *Weight*.

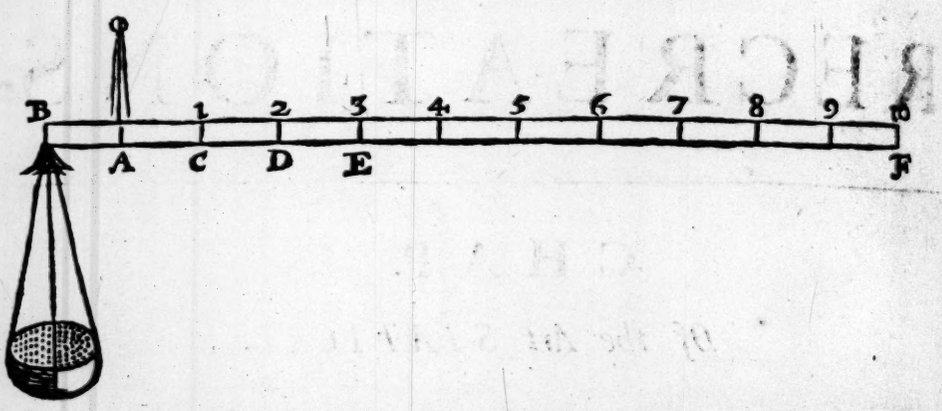
And from hence are deduced theſe following *Theorems* of *Marinus Ghetaldi*, in his Tractate called *Archimedes Promotus*.

CHAP. II.

Concerning the Ballance.

THE firſt Inventor of this *Statical Faculty*, commonly called *Libra*, or the *Ballance*, is attributed to *Aſtrea*, who is therefore Deified for the Goddeſs of *Juſtice*; and that Inſtrument is advanced among the Twelve *Celeſtial Signs*, and in that part of the *Zodiack*, to which, when the Sun cometh, it makes the *Days* and *Nights* of *equal Length* throughout the whole World.

The chief End and Uſe of this Inſtrument, is for the diſtinction of ſeveral *Ponderoſities*: For the underſtanding whereof it is to be noted, That, if the Length of the *Sides* in the *Ballance*, and the *Weights* at the *Ends* of them, be both mutably *equal*, then the *Beam* will be in an *Horizontal* Situation: But, if either the *Weights* alone be *Equal*, and not their *Diſtance*; or the *Diſtance* alone, and not the *Weights*; then the *Beam* will accordingly *decline*. As in the Figure.



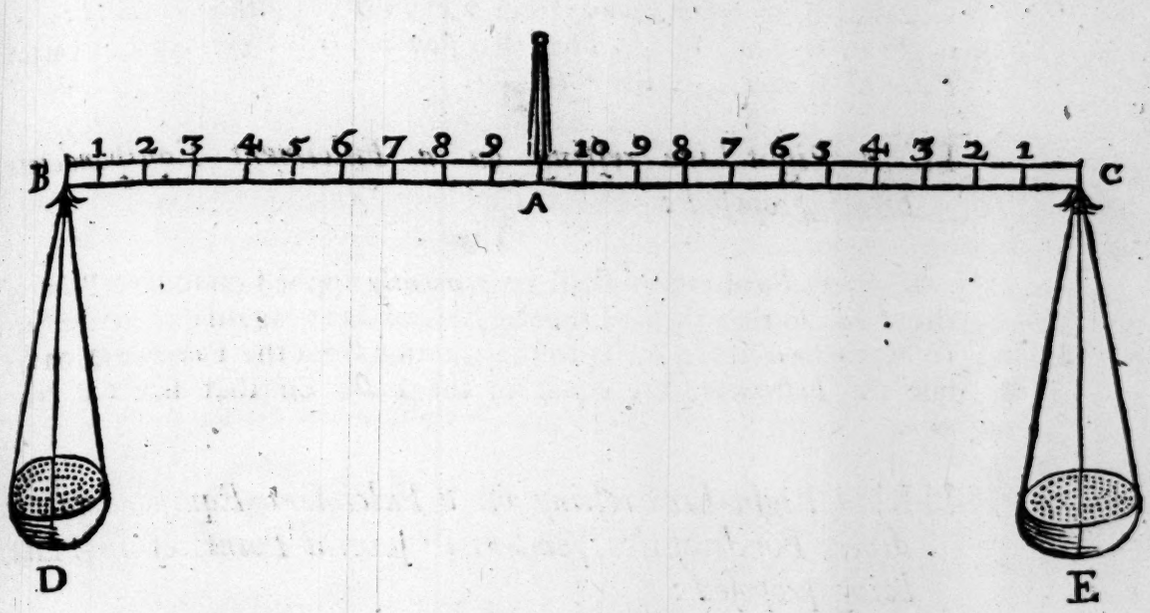
In which *Ballance*, ſuppoſe an *equal Weight* at C, to that at B, (both which Points are equidiſtant from the *Center* A) it is evident, that then the *Beam* BF will hang *Horizontally*: But if the *Weight* at C be unequal to that at *B*; or, if there be an equal *Weight* at D, or E, or any other of the unequal Diſtances noted upon the *Beam*; the *Beam* muſt then neceſſarily *decline*.

And with ſuch a kind of *Ballance* as this, it is uſual with *one Weight* only, to *weigh*, or *meaſure*, ſundry different *Gravities*, whether *more* or *leſs* than that by which they are *meaſured*. And thus by the Figure here deſcribed, a Man may (with *one Pound* alone) weigh any other Body, or Subſtance, within *ten Pounds*; becauſe the *heavineſs* of any *Weight* doth increaſe proportionably to its *Diſtance* from the *Center*. And thus, *one Pound* at D, will equiponderate *two* at B, becauſe the *Diſtance* AD, is *double* to that at AB. And for the ſame Reaſon, *one Pound* at E, will equiponderate *three Pound* at B; and *one Pound* at F, unto *ten* at B; there being the ſame diſproportion between their ſeveral *Diſtances*.

This kind of *Ballance* is of great Antiquity; *Ariſtotle* giving it the Name of φάλαγξ. It is alſo called *Romana Statera*; and now commonly, the *Stillyard*.

From what hath been ſaid, it is eaſie to apprehend, how *falſe Ballances* may be compoſed; too often uſed, and ſo much condemned by *Salomon*, Prov. XI. 1. XVI. 11. XX. 10 & 23. as being *an Abomination unto the Lord*. For, if the *Sides* of the *Beam* be not *equally* divided, As ſuppoſe one have 10 *Parts*, and the other 11 of the ſame *Parts*, then any two *Weights* which differ according to this Proportion; as *Pappus* in his Third Book of *Mathematical Collections* obſerves: The *Heavier* being placed on the *Shorter* Side, and the *Lighter* on the *Longer*, will *Equiponderate*: And yet both the *Scales* being *empty*, ſhall be in *Æquilibrio*, as in the Figure.

Sup-



Suppoſe AC to have *eleven* ſuch Parts, whereof AB hath but *ten*; and yet both of them (in themſelves) of equal Weight: It is certain, that whether the *Scales* be empty, or whether in the *Scale* D we put 11 Pounds, and in E 10 Pounds, yet both of them ſhall *Equiponderate*, becauſe there is juſt ſuch a diſproportion in the *length* of the *Sides*; AC being unto AB, as 11 is to 10.

How ſuch deceitful *Ballances* may be diſcovered, is by changing the *Weights* into each other *Scale*, and then the Inequality will be obvious.

From theſe *Principles* here delivered, it is eaſie to conceive how a Man may find out the juſt Proportion of a *Weight*, which in any Point given, ſhall *equiponderate* to ſeveral *Weights* given, hanging in ſeveral places of the *Beam*.

Of theſe *Ballances*, ſome of them are ſo exactly made, (eſpecially thoſe that are in the *Say-Office* in the *Tower*, and at the *Goldſmiths Hall*,) as to be ſenſibly turned with the Eightieth part of a Grain. And *Capellus*, in his Book *De Ponderibus & Nummis*, mentions one at *Sedan*, that would turn with the Four hundredth part of a Grain.

Certain THEOREMS of Mr. *Michael Dary*'s, concerning the *Conſtruction* and *Uſe* of the SCALE of PONDEROSITY, (commonly called) the STILLYARD.

I. *A Right-Line reſting upon a Fulciment Equiponderate being propoſed:*

Then,

If any *Ponderoſity* ſhall be applied to a *Point* of *Pondency* in that *Line*, it ought to be underſtood, That that *Ponderoſity* is tranſplanted from the *Fulciment*, to that Point of *Pendency*: But if any *Ponderoſity* ſhall be

withdrawn, or taken away, from a *Point* of *Pendency* in that *Line*, it ought to be understood, That that *Ponderosity* is transplanted from the *Point* of *Pendency*, to the *Fulciment*.

II. *A Right-Line resting on a Fulciment Equiponderate being proposed:*

Then,

If divers *Ponderosities* shall be *pendently* applied on sundry *Points* of that *Line*, so that the said *Line* be *Equiponderate* again; then the *Facts* (of each *Ponderosity*, by its transplantation from the *Fulciment*) on this side the *Fulciment*, are equal to the *Facts* on that side the *Fulciment*.

III. *A Right-Line resting on a Fulciment Equiponderate by divers Ponderosities, pendent in several Points of that Line, being proposed:*

Then,

If *two* of the *Ponderosities Pendent* shall be transplanted, so that the said *Line* shall be *Equiponderate* again; then the *Facts* of each *Ponderosity* by his *distance* run in transplantation, are *equal*.

IV. *If a* Stillyard, *or Scale of Ponderosity, or* (*as the* Dutch *call it*) The Roman Beam, *be true* (i. e. *doth give the Truth*) *in two Points,* (*the farther distant, the better.*)

Then,

It is *true* in all the *Points* of *Pendency* throughout, the *Divisions* being *equal*.

Unto what Uses these Ballances may be applied.

THere are several Contrivances to make use of these Ballances: as,

In measuring of the
- Weight of *Blows*.
- Force of *Powder*.
- Strength of *Strings*.
- Condenced *Air*.
- Distinct Proportion of several *Metals* mix'd together.
- Different Gravity of divers *Bodies* in the *Water*, from what they have in the open *Air*.

With divers the like Ingenious *Enquiries*.

CHAP.

CHAP. III.

Of the BALLANCE *of Signeur* Galileo Galilei. *In which, in Imitation of* Archimedes *in the Problem of the* Crown, *he ſheweth how to find the Proportion of the* Alloy *of* mix'd Metals; *with the manner how the ſame Inſtrument is to be made. With ſome Annotations of* Dominico Montovani *upon the ſaid* Ballance.

AS it is well known, by ſuch as take the pains to Read Old Authors. That *Archimides* detected the Cheat of the Goldſmith in the Crown of *Heron*, King of *Sicily*, and Kinſman to that great Mathematician *Archimedes*; ſo I think it hitherto unknown what Method this great Philoſopher obſerved in that Diſcovery.

For the Opinion that he did perform it, by putting the Crown into the Water, having firſt put into it ſuch another Maſs of *Pure Gold*, and another of *Silver* ſeverally, and that from the *differences* in their making the Water more, or leſs riſe and run over, he came to know the *mixture*, or *Alloy* of the *Gold* with the *Silver*, of which that *Crown* was compounded; ſeems a thing (if I may ſpeak it) very *Groſs*, and far from Exactneſs. And it will ſeem ſo much the more dull, to ſuch who have read and underſtood the exquiſite Inventions of ſo Divine a Man, amongſt the Memorials that are extant of him; by which it is very manifeſt, that all other Wits are inferior to that of *Archimedes*. Indeed I believe, that Fame divulging it abroad, that *Archimedes* had diſcovered that ſame Fraud by means of the Water, ſome Writer of thoſe Times committed the Memory thereof to Poſterity; and that this Perſon, that he might add ſomething to that little which he had heard by common Fame, did relate, that *Archimedes* had made uſe of the Water in that manner, as ſince hath been, by the generality of Men, believed.

But in regard I know that that Method is altogether Fallacious, and falls ſhort of that Exactneſs which is required in *Mathematical Matters*, I have often thought in what manner, by help of the Water, one might exactly find the *mixture* of two *Metals*; and in the end, after I had diligently peruſed that which *Archimedes* demonſtrates in his Book *De inſidentibus Aquæ*, and thoſe others *De Æquiponderantium*; there came into my Thoughts a *Rule* which exquiſitely reſolveth our *Queſtion*; which *Rule* I believe to be the ſame that *Archimedes* made uſe of; ſeeing, that beſides the uſe that is to be made of Water, the exactneſs of the Work dependeth alſo upon certain *Demonſtrations* found by the ſame *Archimedes*.

The way is by help of a *Ballance*, whoſe *Conſtruction* and *Uſe* ſhall be ſhewn by and by, after we ſhall have declared what is neceſſary for the knowledge thereof. You muſt know therefore, that the Solid Bodies that ſink in the Water, weigh ſo much Leſs in the Water

than

than in the Air, as a Maſs of Water equal to the ſaid Solid, doth weigh in the Air; which hath been by *Archimedes* Demonſtrated. But, in regard his Demonſtration is very mediate, becauſe I would not be over long; laying it aſide, I ſhall declare the ſame another way. Let us conſider therefore, that putting into the Water, *v. g.* a *Maſs of Gold*, if that Maſs were of Water, it would have no weight at all; for the Water moveth neither upwards nor downwards in the Water: It remains therefore, that the Maſs of Gold weigheth in the Water only ſo much, as the *Gravity* of the Gold exceeds the *Gravity* of the Water. And the like is to be underſtood of other Metals. And becauſe that Metals are different from each other in Gravity, their Gravity in the Water ſhall diminiſh according to ſeveral Proportions. As for *Example*: Let us ſuppoſe that *Gold* weigheth twenty times more than *Water*, it is manifeſt by that which hath been ſpoken, that the *Gold* will weigh leſs in the *Water* than in the *Air*, by a *Twentieth* part of its whole Weight. Now, let us ſuppoſe that *Silver*, as being leſs grave than *Gold*, weigheth *Twelve* times more than *Water*; this then weighed in the Water, ſhall diminiſh in Gravity the *Twelfth* part of the whole Weight: Therefore, the *Gravity* of *Gold* in the Water, decreaſeth leſs than that of *Silver*; for that diminiſheth a *Twentieth* part, and that a *Twelfth*. If therefore, in an exquiſite *Ballance*, we ſhall hang a *Metal* at the one *Arm*, and at the other end a *Counter-poiſe*, that weigheth equally with the ſaid Metal in the Water, leaving the Counter-poiſe in the *Air*, to the end it may equivalate and compenſate the *Metal*, it will be neceſſary to hang it nearer the Perpendicular or Cock.

Example. Let the Ballance be A B, its Perpendicular C, and let a Maſs of ſome Metal be ſuſpended at B, counter-poiſed by the Weight D: Putting the Weight B into the Water, the Weight D in A would weigh more; therefore, that they may weigh equally, it would be neceſſary to hang it nearer to the Perpendicular C; as *v. g.* in E; and look how many times the Diſtance C A ſhall contain A E, ſo many times ſhall the Metal weigh more than the Water. Let us therefore ſuppoſe, that the Weight in B be *Gold*, and that weighed in the Water, it withdraws the Counterpoiſe D into E: And then doing the ſame with pure *Silver*, let us ſuppoſe that its Counterpoiſe, when afterwards it is weighed in the Water, returneth to F, which Point ſhall be nearer to the Point C, as Experience ſheweth, becauſe the *Silver* is leſs *Grave* than the *Gold*: And the Diſtance that is between A and F, ſhall have the ſame difference with the Diſtance A F, that the Gravity of the Gold hath with that of the Silver.

But

But if we have a mixture of *Gold* and *Silver*, it is clear, that by reaſon it participates of *Silver*, it ſhall weigh Leſs than the *Pure Gold*; and by reaſon it participates of *Gold*, it ſhall weigh more than the pure *Silver*; and therefore being weighed in the *Air*, and deſiring that the ſame Counterpoiſe ſhould Counterpoiſe it, when that Mixture ſhall be put into the Water, it will be neceſſary to draw that Counter-poiſe more towards the Perpendicular C, than the point E is; which is the Term of the *Gold*; and more from C than F is, which is the Term of the pure *Silver*: Therefore it ſhall fall between the points E and F; and the Proportion into which the diſtance E F ſhall be divided, ſhall exactly give the Proportion of the two *Metals* which compound that *Mixture*.

As for *Example*; Let us ſuppoſe the Mixture of *Gold* and *Silver* to be in B, Counterpoiſed in the Air by D, in the undermoſt Figure which Counterpoiſe, when the Compound Metal is put into the Water, returneth unto G: I ſay now, that the *Gold* and the *Silver* which compound this *Mixture*, are to one another in the ſame Proportion, as the diſtance F G is to the diſtance G E. But you muſt know, that the diſtance G F terminated in the Mark of the *Silver*, ſhall denote unto us the quantity of the *Gold*; and the diſtance G E, terminated in the Mark of the *Gold*, ſhall ſhew us the quantity of the *Silver*: Inſomuch, that if F G ſhall prove double to G E, then that Mixture ſhall be *two* parts *Gold*, and *one* part *Silver*: And in the ſame Method proceeding in the Examination of other *Mixtures*, one ſhall exactly find the quantity of the ſimple *Metals*.

The Compoſition of the Ballance.

To Compoſe the *Ballance*, therefore, Take a *Rod*, at leaſt a Yard long (and the longer it is, the exacter the *Inſtrument* ſhall be) and divide it in the midſt, where place the *Perpendicular*; then adjuſt the *Arms*, that they may ſtand in *Equilibrium*, by Filing, or Shaving that leſs, which weigheth moſt; and upon one of the *Arms*, Note the *Terms* to which the *Counterpoiſes* of ſimple *Metals* return, when they ſhall be weighed in the *Water*; taking care to weigh the pureſt *Metals* that can be found, this being done, it remaineth that we find out a way, how we may with facility diſcover the proportion, according to which the diſtances between the *Terms* of the *ſimple* and *pure Metals* are divided by the *marks* of the *mixt Metals*; which ſhall be effected in this manner.

We are to have two very ſmall *Wiers* drawn thorough the ſame *drawing Iron*, one of *Steel*, the other of *Braſs*, and above the Terms of the *ſimple Metals* we muſt wind the *Steel Wire*; as for Example; Above the point E, the Term of the *pure Gold*, we are to wind the *Steel Wire*, and under it the other *Braſs Wire*; and having made *Ten* folds of the *Steel Wire*, we muſt make *Ten* more with that of *Braſs*; and thus we are to continue to do with *Ten* of *Steel*, and *Ten* of *Braſs*, until that the whole ſpace between the points E and F, the *Terms* of the *Pure Metals*, be full; cauſing thoſe two *Terms* to be always viſible and perſpicuous: And thus the diſtance E F ſhall be divided into many

many equal parts, and numbered by *Ten* and *Ten*. And if at any time we would know the *Proportion* that is between F G and G E, we muſt count the *Wires* F G, and the *Wires* G E; and finding the *Wires* F G to be, for *Example*, 40, and the *Wires* G E 21: We will ſay, That there is in the *mixt Metal* 40 parts of *Gold*, and 21 of *Silver*. But here you muſt note, that there is ſome difficulty in the counting; for thoſe *Wires* being very ſmall, as it is requiſite for exactneſs ſake, it is not poſſible with the Eye to tell them, becauſe the ſmallneſs of the ſpaces dazeleth and confoundeth the Sight: Therefore, to number them with facility; take a *Bodkin* as ſharp as a *Needle*, and ſet it into a *Handle*, or a very fine pointed *Pen-knife*, with which we may eaſily run over all the ſaid *Wires*; and this way, partly by help of *hearing*, partly by the *impediments* the *Hand* ſhall feel at every *Wire*, thoſe *Wires* ſhall be counted; the Number of which, as I ſaid before, ſhall give us the exact quantity of the *ſimple Metals* of which the *mixt Metal* is Compounded; taking Notice that the *ſimple* anſwer alternately to the *Diſtances*. As for Example, in a *mixture* of *Gold* and *Silver*; the *Wires* that ſhall be towards the *Term* of *Gold*, ſhall ſhew us the quantity of the *Silver*; and the ſame is to be underſtood of other *Metals*.

CHAP. IV.

Six Theorems, *extracted out of* Archimedes *his Tract*; *Entituled*, De Incidentibus Aqua: *Very neceſſary for the better underſtanding of ſeveral Statical Experiments and Concluſions herein contained.*

THEOREM I.

THE *Superficies* of every *Liquid*, that is conſiſtant and [ſettled,] ſhall be of a *Spherical* Figure, which *Figure* ſhall have the ſame *Center* with that of the *Globe* of the whole *Earth* and *Waters*.

THEOREM II.

SOlid *Magnitudes* that being of equal *Maſs* with the *Liquid*, are alſo *Equal* to it in *Gravity*, being demitted into the [ſettled] *Liquid*, do ſo ſubmerge in the ſame, as that they lie, or appear not at all above the *Surface* of the *Liquid*, nor yet do they *Sink* to the Bottom.

THEOREM

THEOREM III.

SOlid *Magnitudes* that are *Lighter* than the *Liquid*, being demitted into the [ſettled] *Liquid*, will not totally *Submerge* in the ſame; but ſome part thereof will lie, or ſtay, above the *Surface* of the *Liquid*.

THEOREM IV.

SOlid *Magnitudes* that are *Lighter* than the *Liquid*, being demitted into the [ſettled] *Liquid*, will ſo far Submerge, till that a *Maſs* of *Liquor*, equal to the *Part* Submerged, do in *Gravity* equalize the whole *Magnitude*.

THEOREM V.

SOlid *Magnitudes Lighter* than the *Liquid*, being thruſt into the *Liquid*, are repulſed upwards with a *Force*, as great as is the exceſs of the *Gravity* of a *Maſs* of *Liquor* equal to the *Magnitude*, above the *Gravity* of the ſaid *Magnitude*.

THEOREM VI.

SOlid *Magnitudes*, *Heavier* than the *Liquid*, being demitted into the [ſettled] *Liquid*, are born downwards as far as they can deſcend; and ſhall be *Lighter* in the *Liquid*, by the *Gravity* of a *Liquid Maſs* of the ſame *Bigneſs* with the *Solid Magnitude*.

Here followeth ſome other Theorems *of Mr.* William Oughtred *concerning this matter.*

THEOREM I.

IF four Pieces of *Metals*, whereof the *Third* is of the ſame kind with the *Firſt*; and the *Fourth* of the ſame kind with the *Second*, are proportional: Their *Gravities* [or *Weights*] ſhall be Proportional.

THEOREM II.

IF there be four Pieces of *Metal*, whereof the *Third* is of the ſame kind with the *Firſt*; and the *Fourth* of the ſame kind with the *Second*; and the *Firſt* and *Second* be of *equal Greatneſs*; and the *Third* and *Fourth* of equal *Weight*: The *Weight* of the *Firſt* and *Second* ſhall be reciprocally Proportional to the *Magnitudes* of the *Third* and *Fourth*.

THEOREM III.

IF *Spheres* of the ſame *Matter*, are in *Gravity*, or *Weight*, as the *Cubes* of their *Diameters* are in *Magnitude*. *Et contra.*

THEOREM IV.

PIeces of *Metal*, if they be of equal *Magnitude*, have their *Weights* in *Direct Proportion*, as is ſet down in the following Table: But if they be of equal *Weight*; they have their *Magnitudes* in Reciprocal Proportion.

CHAP V.

Concerning the Compariſon of ſeveral Metals *in* Quantity *and* Weight.

COncerning this matter, I will give you the Proportions or Compariſons of the moſt Principal kinds of *Metals* in common Uſe; Namely of *Gold* ☉; *Quick-Silver* ☿; *Lead* ♄; *Silver* ☾; *Braſs* ♀; *Iron* ♂; *Tin* ♃; according to the Experiments and Obſervations of *Marinus Ghetaldus*, in his foremention'd Book; Entituled, *Archimedes Promotus*; as followeth, *Viz.*

☉	Gold	3990	♀	Braſs	1890
☿	Quick-Silver	2850	♂	Iron	1680
♄	Lead	2415	♃	Tin	1554
☾	Silver	2170			

Wherefore,

☉ Gold, hath ſuch Proportion to ☿ Quick Silver, ♄ Lead, ☾ Silver, ♀ Braſs, ♂ Iron, ♃ Tin — As 5, 38, 57 — Hath to 5, 23, 31 — Or in Decimals, as 19, 19, 95 — Hath to 9, 8, 37 — Or in Decimals, as 2850, 2415, 2170, 1890, 1680, 1554 to 3990.

Which Numbers Mr. *Gunter* in the 5th Chap. of his 3d Book of the *Sector* hath expreſſed in the ſame Terms, and that in whole Numbers by changing the *Vulgar Fractions* of thoſe Numbers into *Decimals*, putting the firſt Number 10000, whereas *Ghetaldus* puts in but 100; which Fractions Dr. *Wyband* in his *Tactometria* Pag. 201. hath put into a Table, both in *Direct* and *Reciprocal* Proportion; in reſpect of the *Equal Magnitudes* and *Gravities* of like Bodies of different Metals.

Pro-

Proportion Direct.			Reciprocal Proportion.		
In like Bodies of ſeveral Metals and equal Magnitude: Having the weight of the one, to find the weight of the other.	☉ Gold	10000	3895	Gold ☉	In like Bodies of ſeveral Metals and equal weights, having the Magnitude of the one, to find the Magnitude of the reſt. *The Converſe of the former.*
	☿ Quick Sil.	7143	5453	Quick Sil. ☿	
	♄ Lead	6053	6435	Lead ♄	
	☾ Silver	5439	7161	Silver ☾	
	♀ Braſs	4737	8222	Braſs ♀	
	♂ Iron	4210	9250	Iron ♂	
	♃ Tin	3895	10000	Tin ♃	

Mr. *William Oughtred* in the 10th *Chapter* of his *Circles of Proportion*, ſays, that *Ghetaldus* uſeth the *Ancient Roman Foot*, which by the Meaſure ſet down in his Book ſeemeth to be very little leſs than our uſual *Engliſh Foot*, if not exactly the ſame. But, concerning this matter, let us make farther ſcrutiny.

CHAP. VI.

The Roman and Engliſh Foot Compared.

FOR farther Enquiry into this Matter, Mr. *John Graves*, in his Treatiſe of the *Roman Foot*, hath deduced from divers Obſervations and Experiments, by him made, with great Pains and Induſtry in his Travels, (eſpecially in *Italy* and there at *Rome*,) more clearly expreſſes the *Dimenſion* or *Magnitude* of the *True Roman Foot*, not only *Linearly*, but alſo *Numerally*; in comparing it with the *Standard Meaſures* of *England*, and divers other *Nations*.

Of the ſeveral ſorts of Roman Feet.

There are (as the above named Mr. *Graves* relates) ſeveral ſorts of *Ran Feet*, uſed by divers Authors; amongſt which, that which *Ghetaldus* made uſe of was this next following.

1. *Pes Colotianus*, to be found *in Hortis Colotianis* in *Rome*: Which Foot, he comparing with our Engliſh Foot, findeth, that it containeth 967 ſuch Parts, as our Engliſh Foot doth 1000: And ſo our Engliſh Foot to contain 1034. 13 ſuch Feet, as the ſaid *Pes Colotianus* (or Roman Foot) contain'd 1000: Whereby this Roman Foot ſhould be exactly 11. 604 ſuch Parts, as our Engliſh Foot is 12, *Viz.* 11. 604 Inches, wanting of our Engliſh Foot only 0. 396 Inches.

2. There is another *Roman Foot* mentioned by the ſame Mr. *Graves*, which is that on the Monument of *Statilius*, In *Hortis Vaticanis* in *Rome*, which comes ſomewhat nearer to our Engliſh Foot than the former, for he obſerved it to be 972 ſuch parts, as the Engliſh Foot is 1000 (and to be 1005. 17 of the *Pes Colotianus*, being 1000) whereby this Foot will be 11. 7 *ferè* of the Engliſh Foot, being 12, *Viz.* 11. 66 Inches; which wanteth only 0. 34 Inches of the Engliſh Foot.

3. A Third Roman Foot mentioned by him, is that of *Villalpandus*, deduced from the *Congius* of *Veſpaſian* in *Rome*, which he ſaith to be 986 parts of our Engliſh Foot, containing 1000. (and 1019. 65 of *Pes Colotianus*, being 1000) and ſo is 11. 8 of the Engliſh Foot being 12, *viz.* 11.8 Inches; which wants of the whole Engliſh Foot, only 0.2 Inches.

4. But the Ancient *Greek Foot*, doth (by his Obſervation) more nearly agree with our Engliſh Foot, than this laſt Roman Foot, being (by his Collation) 1007.29, of which the Engliſh Foot is 1000, which is hardly .09 parts of an Inch above a Foot Engliſh, it being 12.087 Inches Engliſh.

CHAP. VII.

Of Weights: And the Ancient Roman, and our Engliſh Weights compared.

AS for the *Weights* uſed by *Ghetaldus*, they are ſurely the *Roman Weights*, for he ſaith, they were the *Weights* uſed in his time, and thoſe *Weights* have continued the ſame for many Ages. And Mr. *Graves* in his Diſcourſe of the *Denarius*, puts this as an undeniable *Principle* and *Foundation*, from whence the *Weights* of the *Ancients* may be deduced [as the *Roman Foot* for the *Principle* of their *Meaſures*] having Collated with the *Troy Weights* from our *Engliſh Standard* for *Gold* and *Silver*, by *Grains* thereof ſaith; That the *Roman Pound*, both *Ancient* and *Modern*, containeth 5256 ſuch *Grains*: And ſo the *Roman Ounce* both *Ancient* and *Modern*, 438 of the ſame *Grains*; the *Troy Pound* containing 5760 *Grains*; and ſo the *Troy Ounce* 480: Whereupon the *Roman Pound* and *Ounce* ſhould be (in the leaſt Terms) but $\frac{73}{80}$ of our *Engliſh Pound* and *Ounce Troy*. And ſo the Proportion of the *Roman Pound* and *Ounce*, to our *Troy Pound* and *Ounce*, as 80 to 73: Or in *Decimals*, as 10000 to 9125. And this is the commonly received proportion of the *Ounce Averdupois* to the *Ounce Troy*; and the contrary.

Moreover,

The *Pound* and *Ounce* uſed by *Mar. Ghetaldus* in his Book, Entituled, *Archimedes Promotus*, being compared with our *Engliſh Pound* and *Ounce Troy*, have ſuch proportion as followeth; *Viz.*

One { Ounce / Pound } Of *Ghetaldus* Weight contains { 5760 / 6912 } Grains.

And

And

One {Ounce / Pound} Of our *Troy Weight* contains. {480 / 5760} Grains

Which Reduced to the leaſt Terms, is, As 10 *To* 12.

And then,

As 10,
Is to 12 :
So is 480 Grains, or one Ounce or Pound of Engliſh *Troy Weight*,
To 576 Grains, or one Ounce or Pound of *Ghetaldus* Weight.

Other Obſervations.

There is at this time in the Tower of *London*, (which hath been there ever ſince the Reign of K. *Henry* the VII.) in a Velvet Caſe ; an exact Cube of fine Iron, which by the Standard Ballance, there was found to Weigh of Troy Weight 4 Ounces, 3 Peny-Weight, 9.12 Grains.

And from hence by Experiment there made ; one Cubick Inch of Caſt Iron will weigh 3.968 Ounces Troy, or 0.33068 Pound Troy.

And from hence,

A Sphere or Bullet of one Inch Diameter, of fine Forged Iron will weigh 2.18288 Ounces Troy, or 0.1819 Parts of a Pound Troy Weight.

And,

A Sphere or Bullet of 4 Inches Diameter, made of the ſame fine Forged Iron, will weigh 11.642 Pound, or 11 Pound 6 Ounces 2 Peny Weight, and 20 Grains Troy.

Alſo,

A Sphere of 1 Inch Diameter of {Gold / Silver / Tin} Will weigh of Troy Weight {2625.6 / 1427.95 / 1022.6} Grains.

CHAP. VIII.

The Weight of Spheres *of ſeveral* Metals *compared together.*

AS For the Weights of *Metals* compared in *Spheres* of one and the ſame *Magnitude* in *Troy Weight* (wherein alſo are Obſerved the foreſaid common proportions between *Troy* and *Averdupois* Weights) they are according to the Experiments of *Ghetaldus*, being dedueed from his Comparitive Numbers, into the Parts of *Troy*

Troy Weight, and by him inſerted into a *Table*; though not very preciſely: Which Table Dr. *Wybard* in his *Tactometria* Pag. 237, hath put moſt Correct and Exact, and in *Grains* and Milleſimal parts of a *Grain*: He ſuppoſing firſt (with *Ghetaldus*) a *Sphere* of *Gold* to weigh juſt one Pound *Troy*: And from thence the other *Metalline Spheres* of the ſame *Magnitude* to Weigh accordingly.

The TABLE.

	Ounc	Pen. W.	Grain	Min.	Grains. Parts.
☉ Gold	12	00	00	00	5760.00
☿ Quick Sil.	8	11	10	6 *fere*	4114.285
♄ Lead	7	5	6	6	3486.316 *fere*
☾ Silver	6	10	12	12	3132.631
♀ Braſs	5	13	16	8	2728.421
♂ Iron	5	1	1	7	2425.294
♃ Tin	4	13	11	5	2243.368

CHAP. IX.

Some Obſervations concerning the Worth, Weight, Magnitude, &c. of ſeveral Metals, and other Liquids.

I. Of GOLD.

GOLD ſurpaſſeth all other Metals in theſe Reſpects, (1.) Greatneſs of Weight. (2.) Cloſeneſs of Parts. (3.) Fixation. (4.) Pliantneſs or Softneſs. (5.) Immunity from Ruſt. (6.) Colour or Tincture of yellow.

The Worth of Gold.

			l.	s.	d.	q.
One	Pound Weight	*Troy* is worth	40	0	0	0
	Ounce		3	6	8	0
	Peny Weight		0	3	4	0
	Grain		0	0	1	2

This is the Price of ordinary Gold: Angel Gold is worth ſomewhat more; And Sovereign Gold ſomewhat leſs.

The

The Worth of Silver.

			l.	s.	d.	q.
One Pound Weight	*Troy* is worth		3	0	0	0
One Ounce			0	5	0	0
One Peny Weight			0	0	3	0
One Grain			0	0	0	0½

But of *Engliſh Coin*,

			l.	s.	d.	q.
Of Gold	One Pound of *Troy* Weight is worth		40	18	4	3
Of Silver			3	2	0	0

And by the foregoing Proportions.

			l.	s.	d.	q.
Of Gold	One Pound Weight *Averdupois* is worth		49	13	8	1
Of Silver			3	15	3	2

CHAP. X.

Of the Weight of Water, and other things, in Weight and Magnitude.

One Ounce { *Troy* / *Averdupois* } of Water contains { 1.8949 / 1.72556 } Inches.

One Ounce { *Troy* / *Averdupois* } of Water contains { 0.001096 / 0.00099859 } Feet.

One Pound { *Troy* / *Averdupois* } of Water is of ſolid Meaſure { 22.7368 / 27.609 } Inches.

One Pound { *Troy* / *Averdupois* } of Water is of ſolid Meaſure { 0.013158 / 0.115917 } Foot.

A Cubical Foot of Water weigheth of *Troy*-Weight 912 Ounces; which is 76 *l. Troy.* And,

A Cubical Foot of Water weigheth of *Averdupois*-Weight 999.463 Ounces; which is 62.588 *l.* That is, 62 *l.* 9 Ounces, 6 Drams and a half.

From thoſe Proportions, divers *Statical* Experiments may be made by weighing ſeveral Metals, and other things, in the Water, and in the Air; The Meaſuring of Irregular Bodies, either in whole or in part, *&c.*—— And although there may be ſome difference in the Weight of ſeveral Waters, as Rain-Water, River-Water, Fountain and Spring Water,

Water, *&c.* yet the greateſt difference that Dr. *Wybard* could find, was, but as 1.000000 to 1.002104, which is little above 2 in 1000.

Yet the difference which *Snellius* makes between the Gravity of Rain-water and others, is 1.000000 to 1.007522; but Dr. *John Wybard* makes it but as 1.000000 to 1.002104, which is little above $\frac{2}{1000}$, or 2 in a thouſand. Which latter Proportion I do rather adhere unto, for that (to my own knowledge) the aboveſaid Dr. *Wybard* made this following Experiment, *viz.* He cauſed a Glaſs to be blown on purpoſe which held about five and a half Wine-Pints, which he filled with clean Rain-water, which had ſettled ſome days, and filling the Glaſs therewith, he weighed it in a Standard Ballance in the Tower of *London* (or at Goldſmiths-Hall, I remember not well which) which would turn at the weight of one Grain, and he found the differences of the Weights of theſe ſeveral Waters from Rain-water to be as followeth, *viz.* in the aboveſaid Quantity of five Pints and a half.

1. Snow-water he found to be lighter than Rain-water by 8 Grains.
2. *Thames*-water, and *Middleton's* New-River-water, to be Equal in Weight with Rain-water.
3. *Cheapſide*-Conduit-water to be heavier than Rain-water by 14 Grains.
4. *Lambs*-Conduit-water in *Grays-Inn-Fields* to be heavier than Rain-water by one Peny-weight, or 24 Grains.
5. The Water at the Poſtern-Spring upon the *Tower-hill* heavier than Rain-water by 57 Grains. And,
6. *Crowders*-Well-water, by *Cripplegate*-Church, he found to weigh in the ſame Quantity 84 Grains.

Unto the foregoing Proportions, I ſhall here inſert a Table not long ſince Publiſhed by Sir *Jonas Moor*.

A TA-

A TABLE comparing the Weight of Metals, Stones, Grains, Liquors, &c.

The Names of the Metals, &c.	*The Weight of a Cubick Inch in Magnitude in* Troy *Weight.*	*The Magnitude in Inches and D. parts, answ. to one Ounce of* Troy *W.*	*The Weight of a Cubick Inch in the Water in* Troy Ounc. *Dec. parts.*
	Oun. Parts.	Inch. Parts.	Oun. Parts.
☉ Gold	9.91735	0.10083	9.33962
☿ Quick-silver	7.93388	0.12604	7.35615
♄ Lead	6.16198	0.16229	5.58425
☾ Silver	5.50083	0.18179	4.92310
♀ Copper	4.81342	0.20776	4.23569
♂ Hammer'd-Iron	4.27715	0.23380	3.69942
Cast-Iron	3.96821	0.25253	3.29048
♃ Tin	3.96694	0.25208	3.38921
Marble	1.59631	0.62644	1.01858
Common Stone	1.09835	0.91045	0.51062
Honey	0.79339	1.26042	0.21566
Salt Waters	0.57773	1.79490	0.00000
Fresh Water and Wine	0.52773	1.77190	
Oil	0.47603	2.10069	
Wheat	0.37628	2.65757	
Dried Oak	0.40745	2.45609	

A TABLE of Foreign Weights and Measures compared with the *Engliſh*. By Sir *Jonas Moore*.

Names of Places.	*The* Engliſh *Foot divided into* 1000 *equal parts.*	*The* Engliſh *Foot divided into Inches, and* 10th *parts of an Inch.*	*The Pound* Averdupois *divided into* 100 *parts.*
ENGLAND.			
London Foot	1000	0.12.0	100
FRANCE.			
Paris Royal Foot	1.068	1.00.8	0.93
Lyons Ell	3.976	3.11.7	1.09
Boloyn Ell	2.076	2.00.8	0.89
XVII. *PROVINCES.*			
Amſterdam Foot	.942	0.11.3	0.93
Amſterdam Ell	2.269	2.03.2	
Antwerp Foot	.946	0.11.3	0.98
Antwerp Ell	2.278	2.03.3	
Brill Foot	1.130	1.01.2	
Dort Foot	1.184	1.02.2	
Rynland, *or* Leyden Foot	1.033	1.00.4	0.96
Rynland, *or* Leyden Ell	2.260	2.03.1	
Lorrain Foot	.958	0.11.4	0.98
Mechalin Foot	.919	0.11.0	0.98
Middleborough Foot	.991	0.11.9	0.98
GERMANY.			
Strasborough Foot	.920	0.11.0	0.93
Bremes Foot	.964	0.11.6	0.94
Cologn Foot	.954	0.11.4	0.97
Frankford ad Menam Foot	.948	0.11.4	0.93
Frankford ad Menam Ell	1.826	1.09.9	
Hamborough Ell	1.905	1.10.8	0.95
Leipſig Ell	2.260	2.03.1	1.17
Lubick Ell	1.903	1.09 8	
Noremberg Foot	1.006	1.00.1	0.94
Noremberg Ell	2.227	2.03.3	
Bavaria Foot	.954	0.11.4	
Vienna Foot	1.053	1.00.6	0.83
SPAIN and *PORTUGAL.*			
Spaniſh Palm, *or* Palm *of* Caſtile	.751	0.09.0	0.99
Spaniſh Vare, *or* Rod, 4 Palms	3.004	3.00.0	

Names

Names of Places.	The Engliſh Foot divided into 1000 equal parts.	The Engliſh Foot divided into Inches, and 10th parts of an Inch.	The Pound Averdupois divided into 100 parts.
Their Foot is one Third of the Vare	1.001	1.00.0	
Lisbon Vare	2.750	2.09.0	1.06
Gibraltar Vare	2.760	2.09.1	1.03
Toledo Foot	.899	0.10.7	1.00
Toledo Vare	2.685	2.08.2	
ITALY.			
Roman Foot *on the Monum. of* Colſutius	.967	0.11.6	1.23
Roman Foot *on the Monum. of* Statillius	.972	0.11.7	
Roman Palm for Building, whereof Ten make the Cauna	.732	0.08.8	
Bononia Foot	1.204	1.02.4	1.27
Bononia Ell	2.147	2.01.7	
Bononia Perch, whereof 500 make a Mile	12.940	12.09.5	
Florence Brace *or* Ell	1.913	1.11.0	1.23
Naples Palm	.861	0.09.6	1.43
Naples Brace	2.100	2.01.2	
Naples Cauna	6.880	6.10.5	
Genoa Palm	.830	0.09.6	1.42
Mantoua Foot	1.569	1.06.8	1.43
Milan Calamus	6.544	6.06.5	1.40
Parma Cubit	1.866	1.10.4	1.43
Venice Foot	1.162	1.01.9	1.53
Other Places.			
Dantzick Foot	.944	0.11.3	1.19
Dantzick Ell	1.903	1.10.8	
Copenhagen Foot	.965	0.11.6	0.94
Prague *in* Bohemia Foot	1.026	1.00.3	1.06
Riga Foot	1.831	1.09.9	
China Cubit	1.016	1.00.2	
Turin Foot	1.062	1.00.7	
Cairo Cubit	1.824	1.09.9	1.62
Perſian Araſh	3.197	3.02.3	
Turkiſh Pike, at *Conſtantinople*, the greater	2.206	2.02.4	0.86
The Greek Foot	1.007	1.00.1	
The Univerſal Meaſure	3.267	3.03.2	

A Pendulum of the juſt length whereof will Vibrate 60 times in one Minute of Time.

CHAP. XI.

Experiments STATICAL.

REgard being had to what hath been before delivered about the *Ballance*; and alſo concerning the *Weights* and *Meaſures* of *England*, and thoſe of the *Romans* Ancient and Modern; as alſo the Proportions of the *Weights* of *Metals* and *Waters*; ſeveral *Experiments* may be deduced; ſome whereof follow.

PROBL. I.

If a Sphere *of* Tin *of one Inch Diameter, do Weigh* 0.17753 *l.* Troy *Weight* (*or* 0.146285 *l. of* Averdupois *Weight*) as it hath been found ſo to do, by Dr. *Wybares* exact Experiment; *What ſhall a Sphere of* Tin *weigh, whoſe Diameter is* 7 *Inches.*

SPheres (and other *Regular Bodies*) are in Proportion one to another, as are the *Cubes* of their *Diameters*; VVherefore, Cube the Diameter given 7, it makes 343, which multiplyed by 0.17753 l. the Product will be 60.89279 for the VVeight of the Sphere of Tin, whoſe Diameter is 7 Inches.

Or, If the VVeight of the Bullet were required in *Averdupois* VVeight, then multiply 0.146285 (the proportional Number for *Averdupois* VVeight) by 343 (the Cube of 7, the Diameter,) the Product will be 50.175755 l. *Averdupois.* So that a Sphere of Tin of 7 Inches Diameter will weigh

			lb.	Oz.	P.wt.	Gr.
Of *Troy*	VVeight		60.	10.	14.	16.
Of *Averdupois*	VVeight		lb.	Oz.	Dr.	
			50.	2.	12.	1/4

PROBL. II.

If a Sphere of Tin do Weigh 60.89279 *Pound, or* 1022.6 *Grains* Troy, *what ſhall the Diameter of that Sphere be?*

THE VVeight of a Sphere of Tin, whoſe Diameter is one Inch, was found to weigh 1022.6 Grains (as in *Chap.* VII.) which is 0.17753 Parts of a Pound *Troy.* Wherefore, divide 60.89279 lb. by 0.17753 lb. and the Quotient will be 343, whoſe Cube Root is 7, for the Diameter of a Sphere of Tin, which weigheth 60.89279 Pound.

PROBL.

PROBL. III.

Having the Weight and Diameter of a Sphere of one ſort of Metal given, to find the Length of the Diameter of a Sphere of the ſame Weight, of any other Metal; and alſo the Weight of a Sphere of the other Metal, whoſe Diameter ſhall be equal to the given Spheres Diameter.

THere is a Sphere of Tin, whoſe Diameter is 7 Inches, and it weigheth 60.89279 lb. *Troy*; what muſt the Diameter of a Sphere of Silver be that ſhall have the ſame VVeight.

I. *For the Diameter of the Silver Sphere, equal in Weight to the given Sphere.*

As 2170 (the Proportional Number for Silver,)
Is to 343 (the Cube of 7, the Diameter of the Sphere give)
So is 1554 (the Proportional Number for Tin)
To the Cube of the Diameter of the Silver Sphere;

Wherefore,

Multiply 1554, by 343, the Product will be 5320223 which divided by 2170, gives in the Quotient 245.171; whoſe Cube Root is 6.228 Inches, and ſo much muſt the Diameter of a Silver Sphere be, that will Weigh 60.89279 Pounds *Troy*.

II. *For the Weight of the Silver Sphere, equal in Diameter to the Sphere given.*

As 1554 (the Proportional Number for Tin)
Is to 2170 (the Proportional Number for Silver)
So is the Weight of the Sphere of Tin 60.89279.
To the VVeight of the Sphere of Silver, 85.03054.

Multiply the given VVeight in Tin 60.89279, by 2170, the Product will be 132137.3543, which divided by 1554, the Quotient will be 85.03054, for the VVeight of a Sphere of Silver, whoſe Diameter is 7 Inches.

PROBL. IV.

Being an Experiment of a Marble Bullet VVeighed in the Air, and in the VVater, by Dr. Wybard *in* Goldſmiths-Hall, *and Mr.* Jackſon *the Sea Maſter there.*

THey took a Marble Bullet, whoſe Diameter taken by a very fine pair of Calopers, they found to be 4.95 Inches; and ſo found the Solid Content to be 63.5 Inches, which Bullet VVeighed in the Air,

Air, they found to Weigh 89.10 Ounces *Troy*: And afterwards weighed in the Water, they found it to Weigh 55.75 Ounces, the Difference is 33.35 Ounces.

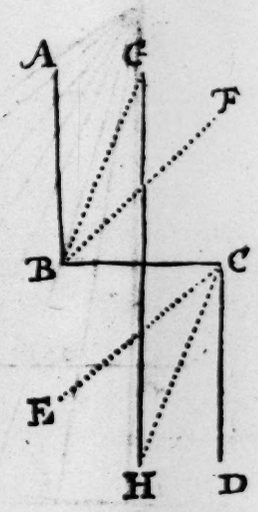
Now, one Ounce *Troy* of VVater is equal to 1.8949 Inches *Troy*; wherefore, divide 63.5 the solid Content in Inches, by 1.8949, the VVater equal to one Solid Inch, and the Quotient will be 33.51 equal to the Solidity.

In like manner, if an irregular Figure, or Statue of the like matter should VVeigh in the Air of *Averdupois* VVeight 231 lb. And when VVeighed in the VVater, the Water overflowing, weighed 66 lb. Now (by what hath been said before) one Pound *Averdupois* of VVater is equal to 27.609 Inches. VVherefore, divide 66.00000 lb, by 27.609, the Quotient will be 2.390.

The End of the Statical Part.

Poſtſcript.

How is it, that a Man ſitting in a Chair, his Back to the back of the Chair, and his Legs by the fore frame of the Chair upon the Ground, cannot riſe off the Chair except he bend his Body forward, or put his Legs backwards.



IN the Poſture of Sitting, our Legs are ſuppoſed to make a Right Angle with our Thighs, and they with our Backs: As in the Figure.

Wherein, Let A B repreſent the Back; B C the Thighs; and C D the Legs. It is now evident, That a Man cannot riſe from this Poſture, unleſs, either the Back A B, do firſt incline unto F, to make an acute Angle with the Thighs B C; or elſe that the Legs C D, do encline towards E, which may alſo make an acute Angle with the Thighs B C: Or, laſtly, unleſs both of them do decline to the Points G and H, where they may be included in the ſame Perpendicular.

For the reſolution of which, Philoſophers give theſe two Reaſons:

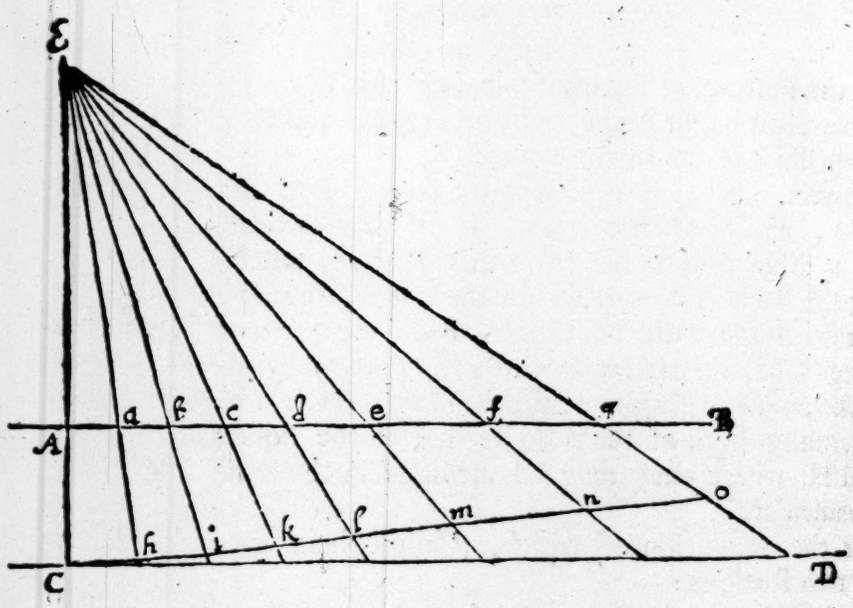
1. A Right Angle (ſay they) is a kind of Equality, and that being Naturally the cauſe of Reſt, muſt needs be an impediment to the motion of Riſing.

2. Becauſe, when either of the Parts are brought into an acute Angle, the Head being removed over the Feet, or they under the Head, in ſuch a Poſture the whole Man is much nearer diſpoſed to the form of ſtanding, wherein all theſe Parts are in one ſtreight Perpendicular Line, then he is by the other of Right Angles, in which Back and Legs are two Parallels; or that of turning theſe Right Angles into Obtuſe, which would not make an erect Poſture, but declining rather.

Theſe are the Philoſophical Reaſons; the Geometrical may be theſe. Suppoſe B C to be a Leaver, towards the middle of which is the place of the Fulciment, Prop or Center, A B as the Weight, and C D, the Power which is to raiſe it.

Now the Body being ſituate in this Rectangular Form, the Weight A B muſt needs be augmented proportionably to its diſtance from the Center, which is about the middle of the Thighs; whereas, if we ſuppoſe either the Weight to be inclined unto F, or the Power C D to E, or both of them to G and H, then there is nothing to be lifted up but the bare Weight it ſelf, which in this ſituation is not at all increaſed with any Addition by diſtance.

How upon a Plain Superficies, a Right Line being drawn; to draw another Line, which shall incline more towards that Right Line, yet never concur or meet with it in any part.



THis seems to be against the Definition of Parallels; but you are to note the Line to be thus drawn, will not be a streight, but a Conoyd or mixt Line.

Let there be drawn a Right Line at pleasure, as A B, and another Parallel to it at any distance, as C D, at one end of the Line A B, as at A, Erect a Perpendicular at pleasure, as A E, and draw it down to C; then divide the Line A B, into what Number of Parts you please, equal or unequal, as at the Points a, b, c, d, e, f, g, *&c.* Then take in your Compasses the distance A C, and set it upon the respective Lines that are drawn from C, as from, a, to h, from b to i, from c to k, from d to l, from e to m, from f to n, and from g to o. Lastly, draw Lines (or rather one continued Line with an even Hand, through the Points h, i, k, l, m, n, and o, and you will find that it still inclines nearer and nearer to the Line A B, but will never concur or meet with it.

Astronomical RECREATIONS.

CHAP. I.

A Brief View of the Principles of ASTRONOMY.

ASTRONOMY was Anciently called ASTROLOGY; But in after times, this Synonomy ceased; the former tending only to *Divine* and *Prognosticate* of future *Events*, from the *Sight* and *Aspects* of the *Planets* and *Stars*. But the Later (*Viz.* ASTRONOMY) of which I shall only Treat; is a *Science* which teacheth the *Laws* and *Rules*, whereby the *Motions* of the *Stars* and *Planets* are *Regulated* and *Determined*. As to the *Original* of this *Science*, it cannot be referred to a better *Patron* then *Admiration*: And so says *Gassendus*; *Originem ipsi fecit Admiratio*: For the *Ancients* admiring the *Splendor*, *Variety*, *Multitude* and *Magnitude* of the *Stars*, together with their *Constant* and *Regular Motions*; Transferred their *Admiration* into *Observation*, and that (in Process of time) into *Tables*, for the Information of Posterity.

CHAP. II.

Of the WORLD, *and of such Places as in General it consisteth of.*

THere is nothing wherein the wonderful Power and Glory of God is more visibly expressed, than in the admirable Frame of this great Universe of Heaven and Earth. And the Reasons why most Men are no more affected with it, are; First, the Commonness of the things. Secondly, the ignorance of the Order and Causes thereof; whereas they that know any thing hereof, cannot but admire the Works of God herein; as we may see by the Prophet *David*, who so often speaks hereof in the *Psalms*. I shall therefore as briefly and plainly as I can shew you some part of these Wonders, and then I shall

ſhew you how theſe things are found out, and proved to be ſo, that you may not take them for vain Fancies, but in ſome meaſure ſee them to be Truths.

Firſt, you muſt know that in this viſible VVorld there are theſe Eminent parts which deſerve a particular conſideration.

1. The Globe of the *Earth.*
2. The Globe of the *Moon.*
3. The Globe of *Mercury.*
4. The Globe of *Venus.*
5. The Globe of the *Sun.*
6. The Globe of *Mars.*
7. The Globe of *Jupiter.*
8. The Globe of *Saturn.*
9. The Heaven of the fixed Stars.

Some add to theſe,

10. The Criſtalline Heaven.
11. The firſt moving Heaven.

Now concerning the Number and Order of theſe ſeveral Spheres, Globes or Orbs, there are ſeveral Syſtems; Namely, (1.) That of *Pythagoras.* (2.) The Platonick and Porphyrian Syſtems. (3.) The Egyptian Syſtem. (4.) The Copernican. (5.) The Tychonean. (6.) The Hypotheſis of *Ricciolus.* Of theſe ſeveral Syſtems, there are two which are generally received; Namely, the *Ptolomaick,* and the *Copernican*; concerning both which I ſhall briefly inſiſt, for concerning theſe two, there hath been great *Diſputes,* and therefore, I ſhall in this Place give you a brief Account of ſeveral *Objections* made againſt the one, (*Viz.* the *Copernican,*) with their *Anſwers,* and alſo *Arguments* for the Confirmation of the other, (*Viz.* The *Ptolomaick.*) ——And for the better underſtanding of the following Diſcourſes, it will be neceſſary to acquaint you with what a material Sphere or Globe is, and what Circles, Lines and Points are deſcribed thereon, as imagined really to be in the Heavens.

CHAP. III.

Of a Material Sphere *or* Globe.

THeodoſius, an Ancient and Learned *Mathematician*, who Writ much of the Sphericks, thus defineth a Sphere or Globe. *Sphæra eſt ſolidum, una ſuperficie contentum, in cujus medio punctum eſt, à quo omnes, rectæ lineæ ductæ ad ſuperficiem ambientem ſunt equales.* That is, a *Sphere* or *Globe* is a ſolid Body, containing one Superficies; in whoſe middle there is a point, from which all right Lines drawn to the Superficies are equal.

Upon

Upon the Superficies of this *Sphere* or *Globe* are deſcribed ſeveral *Circles* and *Points*, (which are only imagined in the Heavens) thereby the better to inform the Fancy; for by them (in the *Cæleſtial Globe*) the ſeveral Conſtellations of the *Fixed Stars* are limited, and their Situation denominated: And the ſame Circles deſcribed upon the *Terreſtrial Globe*, do confine and bound the Zones and Climates.

Theſe *Imaginary Circles* are in Number *Ten*; of which *Six* are called *Great Circles*, and *Four* or *Leſſer Circles*: Thoſe *Circles* are called *Great Circles*, which being drawn upon the Globe, do divide the Superficies thereof into two equal parts; and thoſe are called *Leſſer Circles*, which divide the Superficies of the Globe unequally.

CHAP. IV.

Of the Circles *of the* Sphere.

EVery of the forementioned Circles hath a peculiar Name Attributed unto it; and the Names of

The Six Great Circles, are theſe.

1. *The Horizon.*
2. *The Meridian.*
3. *The Equinoctial.*
4. *The Zodiack.*
5. } *The Colures.*
6. }

Of theſe two *Colures*, the one is called the *Equinoctial Colure*, dividing the Sphere into two equal parts in the Points of *Eaſt* and *Weſt*, and the other is called the *Solſticial Colure*, dividing the Sphere into two equal parts in the Points of *North* and *South*: So that theſe two Colures croſſing one another at Right Angles in the *Poles* of the World, do divide the Globe into four equal Quarters.

The Four Leſſer Circles *are,*

1. The *Tropick* of *Cancer*; Or the Northern Tropick.
2. The *Tropick* of *Capricorn*; Or the Southern Tropick.
3. The *Circle Artick*; Or the North-polar Circle.
4. The *Circle Antarick*; Or the South-polar Circle.

Beſides theſe Circles, there are upon the Superficies of the Globe, other Points and Lines of ſpecial Note and Uſe: As,

1. The two *Poles* of the World, North, and South,
2. The Zenith, Or Point in Over our Heads.
3. The Nadir, the Heaven. Under our Feet.
4. The *Poles* of the *Ecliptique*.
5. The *Ecliptique Line*, or *Via Solis*. And
6. There is another imaginary Line ſuppoſed to paſs from *Pole* to *Pole*, through the Body of the *Globe*; and this Line is called, *the Axis*

of the World, for that the Motion of the Heavens is wholly upon the Poles of the World.

Having thus given you the Names of the ſeveral *Circles*, *Lines* and *Points*, which are in the Heavens imagined, I will give you a brief Account of each particular, by which you may eaſily conceive how they are drawn upon the *Globe*.

CHAP. V.

Of the Uſe and Office of theſe Circles, Points *and* Lines.

1. *Of the* HORIZON.

THE *Horizon* is a great Circle of the Sphere, dividing the viſible part of the Heavens from the not viſible, that is, the lower Hemiſphere from the Higher: As ſuppoſe you were at Sea, or upon a high Hill upon the Land, and looking round about you, you imagine that the *Skie* and *Water*, or the *Skie* and the *Earth* did meet and touch each other. Now this Line of Separation of the *Skie* and *Water*, or *Earth*, is the *viſible Horizon*, and is that which is imagined to be in the Heavens.

Unto this Circle, when either the *Sun*, *Moon*, *Stars*, or any of the *Planets* do come on the Eaſt-part of the Heavens, they are then ſaid to riſe; and when any of them come to this Line on the Weſt-part of the Heavens, then the *Sun*, or ſuch a *Star* or *Planet* is ſaid to ſet: So that the *Riſing* and *Setting* of the *Sun*, *Moon* and *Stars*, are limited and bounded by this Circle.

2. *Of the* MERIDIAN.

The *Meridian* is a great Circle of the Sphere, which paſſeth through the *Zenith* and *Nadir* Points, and alſo thro' both the *Poles*, and divideth the *Horizon* into two equal parts in the Points of *North* and *South*, which paſſeth through the *Zenith* Point in the Heavens, which is directly over your Head, in what part of the World ſoever you are, and the *Nadir* (which is the point directly under your Feet, and oppoſite to the *Zenith*) and it alſo croſſeth or divideth the *Horizon* in the *North* and *South* Point.

Unto this Circle, when the *Sun*, *Moon*, or any other *Star* or *Planet* cometh, it is ſaid to be upon the *Meridian*, or at the higheſt it can be that Day or Night; ſo that when the *Sun* riſeth towards the *Eaſt* in the Morning, it aſcendeth higher and higher, till it cometh to this Circle, and then it is Noon; after which it deſcendeth lower and lower, till it ſet in the Evening in the Weſt-part of the *Horizon*.

3. *Of the* ÆQUINOCTIAL.

The *Æquinoctial* is a great Circle, dividing the Sphere into two equal parts, and passeth through the *East* and *West* Points of the *Horizon*, and also through the *Meridian* in two Points, which are so far distant from the *Zenith* and *Nadir* Points, as the *Poles* are distant from the *Horizon*: So that the *Axis of the World*, which passeth from *Pole* to *Pole*, cutteth this Circle at Right Angles.

Unto this Circle when the *Sun* cometh (which is twice every year, Namely, about the tenth of *March*, and the twelfth of *September*) it maketh the Days and Nights of equal length through the whole World; for that at that time the *Sun* riseth due *East*, and sets due *West*; which at no other times of the year it doth, but rises and sets Northward of the *East* or *West* Points, from the tenth of *March* to the twelfth of *September*, and Southward of the *East* or *West*, from the twelfth of *September* to the tenth of *March* again.

4. *Of the* ZODIACK.

The *Zodiack* is a great Circle, dividing the Sphere or Globe into two equal parts, in the *East* and *West* Points of the *Horizon*, and cutteth both the *Horizon* and *Equinoctial* at oblique Angles; for it cuts the *Horizon* at an Angle equal to the *Suns* greatest *Meridian Altitude*, in any *Latitude*, and the *Æquinoctial* at an Angle equal to the *Suns greatest Declination*. This Circle is called by some, *The Girdle of the World*, it having imbroidered (as it were) upon it, the Figures of the 12 *Cœlestial Signs*. But through the middle thereof, there is a Line, which is called the *Ecliptick Line*, or *Via Solis*, for that the *Sun* in his motion never deviates from this Line (though the other Planets do more or less.) The Poles of this Circle are upon the *Meridian*, so far distant from the Poles of the World, as is the *Suns* greatest Declination.

5. *And* 6. *Of the two* COLURES.

The two *Colures* are two great Circles, passing through the *Poles* of the World, and there cutting each other at right Angles: The one of these Colures cutteth the *Ecliptick Line* in the Points ♈ and ♎, and is called the *Equinoctial Colure*: The other cutteth the *Ecliptick* in the Points ♋ and ♑, and is called the *Solsticial Colure*.

When the *Sun* is in that Sign or Point of the *Ecliptick*, which meeteth with the *Equinoctial Colure*, which is in the first scruples of ♈ or ♎, then are the Days and Nights equal: But when the *Sun* cometh to those Points of the *Ecliptick*, which are cut by the *Solsticial Colure*, which are in the beginning of ♋ and ♑; then the *Sun* being in ♋, it maketh the longest day to all the Northern, and the shortest day to all the Southern Inhabitants of the World: And being in *Capricorn*, it maketh the shortest day to the Northern, and the longest to the Southern In-

7. *Of the two* TROPICKS.

The *Tropicks* of *Cancer* and *Capricorn*, are two Circles drawn upon the Superficies of the Globe, parallel or equidiſtant from the *Æquinoctial*, on the North and South-ſide thereof at 23 Degrees, 31 Minutes diſtance therefrom; the one is called the *Northern Tropick*, the other the *Southern*. And between theſe two *Tropicks*, is ſeated that part of the World which is called the two Torrid *Zones*.

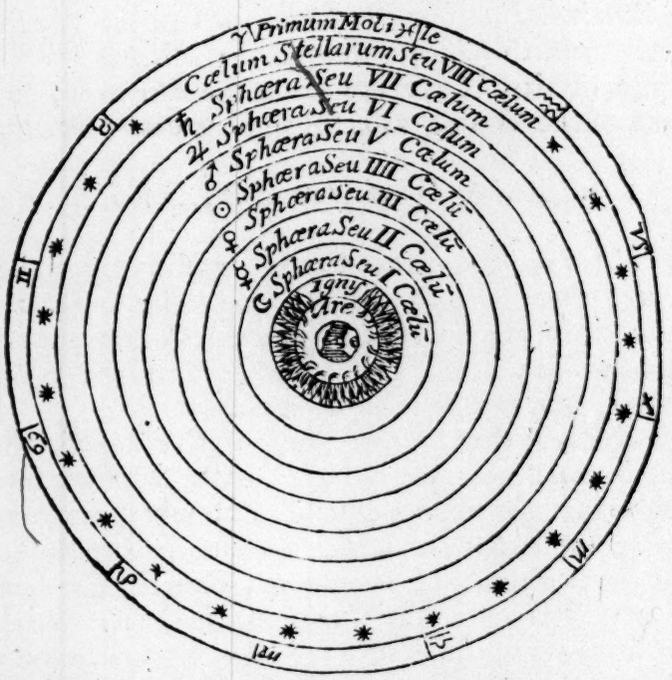
8 *And* 9. *Of the* ARTICK *and* ANT-ARTICK *Circles.*

Theſe are two ſmall *Circles* drawn about the *Poles* of the *World*, and parallel to the *Equinoctial* and *Tropicks*: And the *Circle Artick* is ſo far diſtant from the *North Pole*, as the *Tropick* of *Cancer* is from the *Equinoctial*: And the *Antartick* is at the ſame diſtance from the *South Pole*, *Viz.* 23 Degrees, 30 Minutes, and in theſe two *Circles* and ſeated the two *Poles* of the *Ecliptick*: And between them and either *Pole*, are the two *Frigid Zones*; the two *Temperate Zones*, lying between theſe two *Circles*, and the *Tropicks* of *Cancer* and *Capricorn*.

CHAP. VI.

Of the SYSTEM *of the WORLD.*

1. Of the *Ptolomaick* Syſtem.

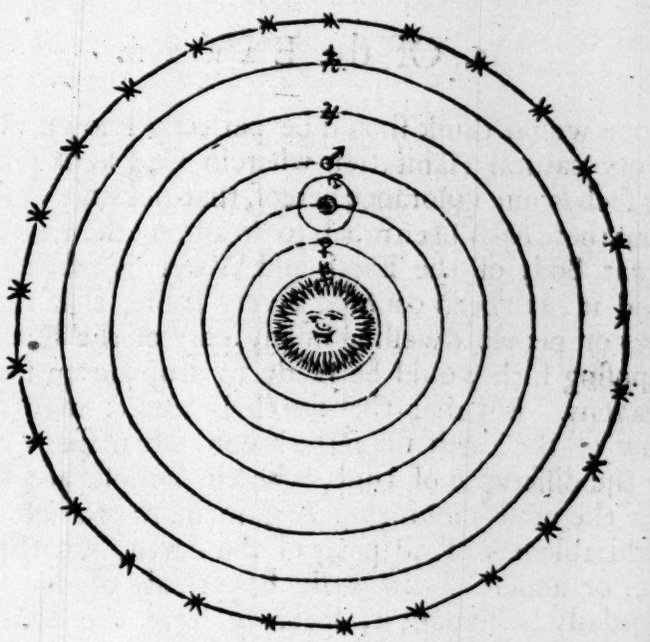


THIS *Syſtem* ſuppoſeth the *Earth* to be in the *Center* of the *Univerſe*; above it the *Elementary Region*; next above that the *Moon*, then *Mercury*; next above him *Venus*, the *Sun* as Chief Moderator

rator of all the reſt, in the midſt of the *Planets*, environed not only by the three foregoing, called *Inferior*; but likewiſe by *Mars*, *Jupiter* and *Saturn*, called the *Superior Planets*; above *Saturn* is the *Sphere* of the *fixed Stars*; by ſome called the *Firmament*.

This was the the firſt *Pythagorean Syſtem*, and was embraced by *Archimides*, the *Chaldeans*, *Ariſtotle*, *Cicero*, *Livy*, *Ptolomy*, *Alphonſus*, *Purbaccus*; and the greateſt part of *Aſtronomers*, until the time of *Magi-nus* and *Clarius*.

2. Of the *Copernican* Syſtem.



THIS *Syſtem* giveth to the *Earth*, not only a *Diurnal* Motion about its *Axis*, but alſo an *Annual* Motion about the *Sun*, as the Center of the *Univerſe*: This *Syſtem*, about two Ages ſince was reſuſcitated (but imperfectly) by Cardinal *Caſanus*, until *Copernicus* came and gave it a perfect conſummation, followed by the greateſt Wits of *this* and the foregoing *Ages*.

In this *Syſtem* we may perceive the *Sun* placed in the *Center* of the *World*, next above him *Mercury*. finiſhing his Courſe about 80 Days: Then *Venus*, making her Revolution in *Nine Months*: Above her the *Earth*, with the *Elementary Sphere*, in the *Annual Orb*, which it runs through in 365 days and a half, by a *Motion* from *Weſt* to *Eaſt*; that is, in the ſame Circle wherein the other *Syſtem* places the *Sun*; beſides which Annnal Motion, *Copernicus* Aſſigns to the Earth a *diurnal Revolution*, in which it turns about its own Center in the ſpace of 24 Hours from *Weſt* to *Eaſt*: The *Moon* by a Menſtrual Revolution being carried about the *Earth* as in an *Epicicle*: *Mars* running about the *Sun*, (as the Center of the Univerſe,) in two years time; *Jupiter* about him in *Twelve years*; and *Saturn* in *Thirty*: The *Sphere* of the *fixed Stars*, being diſtant by ſo vaſt an Interval from the Sphere of *Saturn*, that the *Annual Orb* in which the *Earth* moves, appears in reſpect to it, no other then a point.

CHAP.

CHAP. VII.

The Ptolomean Syſtem *ſtrongly Argued for and Maintained, and many of the* Copernican Arguments *Anſwered by Mr.* Henry Phillippes.

1. Of the EARTH.

THis one would think ſhould be perfectly known of us, ſince it is our natural Habitation wherein God hath placed us; but yet ſuch is our ignorance hereof, that not only among the common ſort, but there hath been much to do among the Learned, to prove that this great Body of the Earth and Water is exactly round like a Globe. And it hath been counted a very Fable, that there ſhould be any *Antipodes* or people dwelling in any part of the Earth directly under us, ſuppoſing ſuch would be ready to drop down from the Earth into the Heavens. But that the Earth is round, may appear by the round ſhadow of the Earth upon the Moon, when ſhe is Eclipſed; and likewiſe by the difference of time, wherein Eclipſes are ſeen in ſeveral places. But the moſt convincing Argument to prove both the roundneſs and habitableneſs of all parts of the Earth, whether we account them above, or under, is the daily Experience of our Seamen, who have and do daily Sail about it, ſpecially from the Eaſt to the Weſt. And though the Earth cannot be ſurrounded by the North and South Poles, in regard of the Cold and Ice, yet by the agreement which is found between the diſtance and the Latitude of places lying North and South, it is evident that the Earth is neither Flat, nor Square, nor Oval (as ſome have imagined) but perfectly round.

The ſecond thing worthy conſideration herein, is the Compaſs of the Earth. And herein there is much difference between Ancient and Modern Writers, which ariſeth partly from the difference of their Meaſures, unknown to each other. The beſt Account hereof I ſuppoſe is given by Mr *Norwood*, who for this purpoſe *Anno* 1635. meaſured the diſtance between *York* and *London*, and ſo exactly taking the Latitudes of thoſe two places, found that one Degree in the Compaſs of the Earth did contain 367200 of our Engliſh Feet; ſo that the whole Compaſs of the Earth being 360 Degrees, will be 132192000 Feet, which reduced into Miles, according to the Statute, each Mile containing 5280 Feet, it yields 25036 Miles. By this you may ſee this Globe of Earth and Water, though it ſeem ſo great to ſome, that it cannot be meaſured, or incompaſſed, yet is but a ſmall thing, conſidered by the Rules of Art; inſomuch, that a Ship ſailing one hundred Miles every day (which it may well do) it will ſurround this Globe in 250 days; which is leſs than three Quarters of a Year.

Thirdly,

Thirdly, It may be enquired, whether the Earth be the Center of the Univerſe or not; and whether it ſtands ſtill, or hath any Motion?

For anſwer hereunto, I ſay, it hath been the moſt received, and in my Judgment the beſt grounded Opinion, That this Globe of Earth is the Center of the Univerſe, being without any Motion, and that the Sun, Moon and Stars are moved round about it: Yet it was the Opinion of *Pythagoras* and *Ariſtarthus* (very Ancient Philoſophers) that the Sun is the Center of the Univerſe, and that this Globe of the Earth is turned round about the Sun, together with the Moon, and the reſt of the Stars and Planets. This Theory was left off by *Ptolomy*, whom moſt of our following Aſtronomers have followed until *Copernicus*, who revived the foreſaid Hypotheſis, and ſince that he hath had many followers. And though this may ſeem to be a very ſtrange Conceit, yet they bring very probable Arguments (in point of Art) to confirm it. In anſwer to which, I ſhall propound theſe Six Conſiderations.

Firſt, That this Opinion is at the beſt Fortified, but with probable Arguments; whereas for the ſtanding ſtill of the Earth we have daily a ſenſible, if not a real Experience. And though our Senſes may be deceived herein; yet it will be hard to prove that they are deceived.

Secondly, Though this Theory, in ſome Reſpects, doth yield much Harmony in the places and motions of the ſeven Planets, giving a good Account of their direct and retrograde Motions, without any Epicicles; yet it fails much in the fixed Stars; ſuppoſing them to be ſo far diſtant from the Center of the Univerſe, as is beyond all probability. For it is certain by the moſt accurate Obſervations of *Tycho Brahe*, and his Succeſſor *Longomontanus*, that there is not the leaſt obſervable difference in the Latitude of places obſerved by the fixed Stars, at any one time of the year more than other; no not to the quantity of half a Minute. And therefore if this Theory be allowed, it muſt be Granted, that there is ſuch a diſtance between the fixed Stars, and the Center of the World, that not only the Globe of the Earth, but that vaſt diſtance of the Sun from the Earth (which is the ſemidiameter of the Suns Annual motion) muſt in reſpect thereof be but a little Point, not cauſing half a Minutes Parallax, or difference of appearance. Now if ſuch a diſtance be allowed as is requiſite for this purpoſe, obſerve how it will exceed all proportion. For whereas *Saturn*, the fartheſt of the Planets, by the conſent of all Aſtronomers, is diſtant from the Earth but about 12000 ſemidiameters of the Earth, the fixed Stars according to this Account muſt be diſtant 7904818 ſemidiameters: So that the place between *Saturn* and the Stars is 7892818 ſemidiameters, which is 658 times more than *Saturn* is diſtant from the Center. Thus you ſee there is no proportion between this vaſt diſtance of the Stars above *Saturn*, and the Planets under him. Beſides, all this vaſt diſtance is but Waſte and Uſeleſs, there being no Stars, nor any viſible Furniture therein, and God and Nature makes nothing in vain.

Again, if according to this foreſaid diſtance you Calculate the quantity of the fixed Stars by their viſible Diameters, a Star of the firſt Magnitude will be greater than the Sun it ſelf 16907143 times, and their true ſemidiameter, or half the breadth of ſuch a Star, will be more than 8 times the diſtance which is between the Sun and the Earth.

But theſe things are ſo unlikely, that it makes as much againſt this Hypotheſis, as the other probabilities plead for it.

Thirdly, The Followers of this Opinion hold, that the Earth hath a daily motion, whereby it is turned round about upon its own Axis every day. And for this motion they bring a very plauſible Argument; for by this (ſay they) the incredible ſwiftneſs of the Heavens and Stars is very well ſalved. For if according to *Tycho*, you allow the fixed Stars to be but 14000 ſemidiameters diſtant from the Earth; whereas *Ptolomy* accounts them 20000, yet the Stars near the Equator muſt move every Minute of an Hour 240000 Miles to perform its daily motion round about the Earth. This motion they count too ſwift for any natural body to perform; and therefore think it more likely that the Earth ſhould turn about once every day, and thereby cauſe the riſing and ſetting of the Sun and Stars.

This conjecture is ſo probable, that many who are againſt the Annual motion of the Sun, yet yield to this daily motion: But yet I conceive that the Earth is not ſubject to this motion neither: For if the Earth ſhould according to this Opinion move every day round upon its own Axis, then we which live upon the Surface of the Earth muſt be moved with the Earth above 1000 Miles every Hour. Which motion, though it be very ſlow in reſpect of the foreſaid ſwiftneſs of the Stars, yet conſidered by it ſelf, and compared with the motion of things here below, it is exceeding ſwift, far ſurpaſſing the Flight of the ſwifteſt Bird, or the Bullet from the Cannons mouth. And therefore ſurely if any ſuch motion were, we ſhould be more ſenſible of it, than we are.

But Grant that this motion ſhould be performed by the Earth, and we not ſenſible of it, in regard of conſtant uſe unto it; as men that are uſed to the Sea are not troubled ſo much with the motion of the Ship as others are: Yet judge whether it be not more likely that the Heavens, Sun, Moon and Stars, being all Light and pure Bodies, ſhould perform the foreſaid ſwift motion (though to us almoſt incredible) than that the Earth (a dull and heavy Body) ſhould perform this later motion. As we ſee by experience, that a Horſe being a Creature of a nimble fiery temper, is better able to run 20 or 30 Miles in one Hour, than a Snail which is a dull ſluggiſh Creature to creep one Poles Length, which is ſcarce the ten thouſandth part thereof.

Conſider alſo, that this ſwift motion in the Heavens is no more to be wondred at, than the vaſtneſs of their Circumference; and it is but reaſonable to think, that God who hath made them of ſo great Compaſs, hath likewiſe fitted them for ſo ſwift a motion. And the Truth is, ſwiftneſs of motion depends neither upon the Greatneſs or Smallneſs of the Creature, but upon the appointment of the Creator. Thus we ſee by experience, the little Hare exceeds for ſwiftneſs both the Sheep, a Midling, and the Oxe a Creature of the largeſt ſize; and yet is out-ſtripped by the Dog and Horſe, Creatures likewiſe of the middle and larger ſize. So that it is neither the ſmaller Compaſs of the Earth, nor the larger Compaſs of the Heavens, which can either prove or diſprove the daily motion of either: But herein the wonderful Power of God is the more magnified in making the Heavens, not only by their vaſtneſs to incompaſs the Earth, but alſo by their ſwiftneſs daily to ſurround the ſame.

Fourthly,

Fourthly, Though this Theory in ſome things may be more plainly demonſtrated by Geometry than the Theory of *Ptolomy*, being freed from thoſe Epicicles; yet this may alſo be performed by the Hypotheſis of *Tycho*; and the places and Aſpects of the Planets may be as truly and readily Calculated thereby, as by this. Experience we had hereof in that great Eclipſe of the Sun the 29 of *March* 1652, about which moſt of our Writers following new Tables, according to this Hypotheſis, were greatly miſtaken in the time of the Eclipſe, reckoning it half an hour too ſoon: Whereas I calculating it by the Tables of *Tycho*, and *Longomontanus*, it fell out very exactly. And you ſhall find, that in all the Eclipſes obſerved by *Tycho* and *Longomontanus*, their Tables never failed them ſo much. And for the motions of the Sun and Moon, there are none like them: So that you ſee there is no neceſſity of following this ſtrange Hypotheſis.

Fifthly, This Opinion is againſt the conſtant Tenor and ordinary Phraſe of the Holy Scriptures, which ought to be had in ſpecial Reverence; and from which we ought to be very careful how we vary in any thing, pretending the Scripture ſpeaks herein, according to the ſenſe and apprehenſion of common people, eſpecially when the thing pleaded for is ſo unlikely and unneceſſary.

Laſtly, This Opinion is of dangerous conſequence, the followers thereof being ſubject to fall into many vain curioſities, and Fruitleſs Errors: As that the Moon, and the reſt of the Planets and Stars, may be each of them habitable places, as well as the Earth; and ſome have Written largely to prove, that there is a World in the Moon, and that our Earth gives light unto them, as the Moon doth unto us; and ſo Account the Earth to be as one and the ſame kind with the Stars and Planets; whereas it is plain in *Gen.* 1. that God made the Earth on the third day of the Creation, and furniſhed it with all ſorts of Herbs and Trees; but the Sun, Moon, and Stars were not made until the fourth day: Which is enough to prove them Creatures of a different kind and nature; and we cannot find the leſt hint that God made the Moon, *&c.* for an habitable place, or the Earth to give Light to the Heavens; but *Ver.* 14, 15. it is plainly expreſſed, that *God ſaid, Let there be Light in the Firmament of Heaven, to ſeparate the Day from the Night. And let them be for Lights in the Firmament of Heaven, to give Light upon the Earth; and it was ſo.* There is not a ſyllable of the Earths mutual and reciprocal Light.

To conclude this particular, moſt of you I ſuppoſe are ready enough to think, that theſe Men err very much in making the Heavenly Bodies to be Earthly Habitations; yet give me leave to tell you, that there is a more common and dangerous Errour, which we had all need to take heed of; and that is, that we do not account this Earthly Dunghil to be our Heavenly Habitation, by laying out all our Care and Pains upon it: But whilſt we live upon the Earth, let our Converſations be in Heaven, from whence we look for a Saviour, the Lord Jeſus Chriſt. And now leaving the *Earth*, it is time to aſcend to the other Orbs, *Viz.* That of the *Moon*, *Mercury*, *Venus*, the *Sun*, *Mars*, *Jupiter*, *Saturn*, the *fixed Stars*, the *Chriſtaline*, and firſt moving Heavens.

2. *Of the* Moon.

It is time for me now to leave the Earth at reſt in the Center, and to ſpeak of the Heavenly Bodies and their Motions. And the Moon in the firſt place preſents her ſelf, being by the conſent of all Aſtronomers placed next the Earth.

Firſt, for the Form or faſhion of the Moon, it is round every way like a Globe. And this may be proved by the ſucceſſive and proportional Light which it receives from the Sun; for if it were round and flat like a Trencher; then the Light of the Sun coming to the flat ſide would preſently ſpread all over it, as it doth upon the flat ſide of an Houſe or upright Wall.

Secondly, As for the matter of the Moon, or what ſubſtance it is made of. Thoſe that hold the motion of the Earth, they think the Moon to be very little different from the Earth, having Sea and Land, Mountains and Valleys: And this they gather from the ſpots which are ſeen in the Moon, accounting thoſe darker places to be Sea, and the brighter places to be Land. But I conceive the Moon to be of ſome like ſubſtance with the Clouds: For if you look upon the Moon in the day time, you ſhall ſee many times in a fair day, when the Clouds are High and White, that the Moon and they are all of one Colour. Again look upon the Clouds in ſuch an Evening about Sun-ſetting, and you ſhall perceive the Clouds to reflect the Light of the Sun, and to ſhine as the Moon doth: And this Light thus reflected from the Clouds and the Vapours of theſe lower Regions, is the Cauſe of the Twilight, which continueth till the Sun is 18 Deg. under the *Horizon*, and then the ſhadow of the Earth falling upon the Clouds, doth take away their ſhining, juſt as the Moon loſeth her Light in her Eclipſe, which proceeds from the ſame Cauſe. Add to this the many Reſemblances of many Suns and Moons, which are divers times ſeen in the Air, which are nothing elſe but Clouds fitly formed and placed to Reflect the Light of the Sun unto us. All this may ſhew, that the Moons ſubſtance doth in ſome ſort reſemble that of the Clouds. And as for the duration of the Moon, longer than the Clouds, this it hath by Gods appointment, who alſo cauſed that Cloud to Guide the *Iſraelites* 40 years in the Wilderneſs, which was a Pillar of Cloud by Day, and a Pillar of Fire by Night.

Thirdly, It is a common queſtion, whether the Moon have any Light of her own? or whether ſhe receive all her Light from the Sun?

For anſwer hereunto, conſider that there are ſeveral ſorts of Lights; firſt there are ſome things that have not only Light in themſelves but give Light to other things, as the Fire and the Sun. Secondly, there are ſome things that have no great Light in themſelves, yet they ſhine in the Dark; as Glow-worms, Rotten-wood, and Fiſh-Skins. Thirdly, there are ſome things which may be ſaid to have Light, but this Light appears beſt in a greater Light; as white things, poliſhed Metals, Glaſſes and the Clouds. And ſuch a Light I conceive the Moon hath of her own, whereby ſhe is fitted to Reflect the Light of the Sun to our Sight. Yet I do not think, as ſome have ſuppoſed, that the Moon is ſo perfect a mirror as that the ſpots which we ſee in the

Moon

Moon are nothing elſe but the Image of our Earth and Sea which we ſee therein; as when we look upon a Glaſs we behold our own Image therein: Much leſs do I believe what ſome have ſaid, that things done, or a Writing ſet before the ſhining Moon, may be ſeen and read in the Moon at an other place of the World, very far diſtant from it; as by the Reflection of Glaſſes it may be performed for a ſmall diſtance.

Fourthly, If you demand what may be the Reaſon of thoſe ſpots which are ſeen in the Moon? I anſwer, they proceed from the roughneſs and inequality of the Moons Body. For though the Moon hath not Hills and Dales, Mountains and Valleys; yet as you ſee the Clouds are many of them ragged things, and not ſmooth and plain, ſo it may very well be in the Moon, though in regard of her diſtance they are not ſo diſcernable. And this ſuppoſition will very well agree with all the Obſervations of theſe ſpots, both in the New and Full Moon. For firſt, in the New Moon, by Reaſon of this inequality of the Moons body, ſome little ſpots of Light will appear before the main body of Light, on the tops of theſe rough places, and then again towards the Full Moon, this inequality will cauſe much difference in the Light of the Moon: Juſt as you ſee when the Sun ſhines full upon a Glaſs Window, though the Window be in a manner flat, yet if the panes of Glaſs lie never ſo little ſlanting, it will cauſe a difference in the Reflection, and one place will ſeem darker than another.

Fifthly, As for the quantity of the Moons body, and her diſtance from the Earth: How theſe things come to be known, I ſhall ſhew hereafter: In the mean time, it ſhall ſuffice to tell you, that according to *Tycho*, the Moon is diſtant from the Earth 56 ſemidiameters of the Earth, which reduced into Engliſh Miles, is 223048 Miles. And though the Moon appears ſo little to us in regard of this diſtance, yet the true Diameter or Breadth of the Moon is 2060 Miles, and the Compaſs of it 6474 Miles; ſo that the Breadth and Compaſs of the Moon is a little more than the one quarter of the Breadth and Compaſs of the Earth; and if you regard their ſolid Proportions, the Earth is 50 times bigger than the Moon.

Laſtly, The Moon hath a four-fold motion: The firſt is its daily motion, wherewith it encompaſſeth the Earth in 24 Hours 48 Minutes. The ſecond is her motion in the Zodiack, wherein ſhe is the ſwifteſt of all the Planets, running through all the Twelve Signs in 27 Days, 7 Hours, 43 Minutes. Thirdly, ſhe hath her motions from the Sun, whereby is cauſed the diverſity of her appearance; this motion is finiſhed in 29 Days, 12 Hours, 44 Minutes. Fourthly, ſhe hath a motion of Latitude from the Ecliptick Line, or way of the Sun, which is the Reaſon that there are not Eclipſes every New Moon and Full Moon; and this motion is finiſhed in 18 Years, 7 Months, and 12 Days. Many good Uſes might be made of theſe motions; but of theſe I have, and ſhall ſpeak more in its time.

3. *Of* Mercury *and* Venus.

I put these two Planets both together; for though they have some difference in their motions, yet in some respect they are both alike: These as two diligent Pages, continually wait upon the Sun their Lord, being never far distant from him.

As for their order in the Heavens, *Ptolomy* placeth *Mercury* next above the Moon, and *Venus* next above him, both under the Sun. But herein *Tycho* in some sort agrees with *Copernicus*, and the later Astronomers all consent therein, making the Sun the Center of the Planets, and so they place *Mercury* next the Sun, and *Venus* a little further off the Sun; supposing them to turn round about the Sun, as it were in two little Circles; so that either of these Planets, are sometimes above, and sometimes under the Sun: And sometimes *Venus* may be nearer to the Earth than *Mercury*.

Hereby also you may perceive the cause why these two Planets are never very far distant from the Sun, for they can go no farther than their Circles will give them scope: which for *Mercury* is about 29 degrees; and for *Venus* about 48 degrees.

Mercury, by reason of his continual nearness to the Sun, and the smallness of his body, is seldom seen. But *Venus* by her Lustre is known unto all men; she sometimes runs before the Sun, and then she is called the Morning Star, sometimes she follows the Sun, and then she is called the Evening Star. And so great is her brightness (by reason of her nearness to the Earth) that she is not only seen in the Night, but many times in the Day, which many will gaze at; but the Learned and Diligent *Tycho* made a very good use of it: For by observing the distance between the Sun and *Venus* in the Day time, and then the Night following observing the distance from *Venus*, to some of the fixed Stars, and from them to others; he thereby found out their places more exactly than by any way else he could attain to. But if such an Eclipse had happened in his time as that of the 29th of *March* 1652, or that *August* 9. 1654. both which happened about High-noon, wherein some of the fixed Stars were seen, he might have performed this conclusion with more ease and certainty. And I hope some lover of Astronomy or other, hath made such use of this occasion, which would be of great concernment, either for the rectifying or confirming of former Observations, in this so necessary a part of Astronomy. And I hope if all have neglected it this time, that some will be diligent in it, at the next great Eclipse of the Sun.

The distance of these two Planets from the Earth, is somewhat different, but according to their middle distance they are accounted to be removed from the Earth, as the Sun is, of which you shall have more in the next Section. The breadth and compass of their bodies is thus:

Breadth of { Venus 4397 / Mercury 2931 } Miles.

Compass

Compaſs of { Venus 13800 / Mercury 9354 } Miles.

4. *Of the Sun.*

The Sun is the principal and chiefeſt of all the Planets and Stars, being the Fountain of all the Light we receive; and therefore is moſt fitly placed in the midſt of the Heavens. As for the matter of it, it cannot be no other than pure Fire, as is manifeſt by the effects thereof, Light and Heat: For though Heat may be without Luſtre in ſome things, and Luſtre without Heat in many other things, yet Heat and Luſtre both cannot be without Fire: But this Fire I ſpeak of, you muſt not imagine to be any groſs Fire, but pure Elementary Fire, which needs not the continual ſupply of Fuel, but of it ſelf is laſting and durable. For though the Pſalmiſt ſaith, *Pſal.* 102. 26. that *the Heavens ſhall wax old as a Garment*, this is rather by the Sin of Man than by their own Nature.

This Sun, as it is the moſt glorious, ſo it is the moſt uſeful Creature of all others, being not only the Fountain of Light, but the Fountain of Life unto all Creatures; this made the Heathen Worſhip him for a God; and the Jews, though better Inſtructed, and expreſly forbidden, could ſcarce refrain from this Idolatry, but Worſhipped the Sun and Moon. Let us take heed of this, but let us praiſe God for them, and with them, who made both us and them, as we are exhorted, *Pſal.* 148. *Praiſe him Sun and Moon, praiſe him all ye Stars of Light; for he commanded, and they were created.*

As for the motion of the Sun; Firſt, It hath a daily motion round about the Earth, by which it makes the Day and Night. Secondly, It hath a Yearly motion round about the Heavens, whereby it cauſeth Summer and Winter, Spring time and Harveſt: This courſe it finiſheth in 365 Days 5 Hours, 49 Minutes, which odd 5 Hours 49 Minutes, in 4 Years makes up almoſt one whole Day, and is therefore added every fourth Year or Leap-year; for otherwiſe, in a little time the Month of *June* would fall in the Winter, and *December* in the Summer, as it did in the Egyptian year. But this was thus rectified by *Julius Cæſar*, which account, though it was far more exact than any of the former, yet by reaſon that thoſe odd hours and minutes did lack 44 minutes of one day in thoſe 4 years, it hath cauſed ſome alteration in the year ſince his time. For whereas the Sun in his time entred into *Aries* on the 25th of *March*, it now enters into *Aries* about the 10th of *March*; and hereupon Pope *Gregory Anno* 1582, cauſed ten days to be cut off from the old Account, and this is the reaſon the Account beyond Sea is ten days before ours, which makes their *Eaſter*, and all their Movable Feaſts, fall moſt times before ours.

The diſtance of the Sun from the Earth, according to *Tycho*, is 1150 Semidiameters of the Earth, which reduced into Miles, is 4580450 Miles. The diameter or breadth of the Suns body is 42600 Miles; and the compaſs thereof is 133886 Miles; ſo that the compaſs of the Sun is above 5 times the compaſs of the Earth: But if you have regard

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to the ſolid contents thereof, then the Sun by this Account is 153 times bigger than the Earth.

As for the daily compaſs which the Sun runs in the Heavens, it is 28789124 miles; ſo that every hour it runs 1199547 miles, which is every minute near upon 20000 miles. And yet to perform ſo ſwift and continual a motion, the Sun needs neither the Poetical Fiction of Horſes to draw it, nor the more probable Opinion of the Earths daily motion to eaſe it of its labour; but by the appointment of God, *cometh forth as a Bridegroom out of his chamber, and rejoyceth as a mighty man to run his race: his going forth is from the ends of the Heavens, and his compaſs to the ends of the ſame*, Pſal. 19. 5,6.

5. *Of the Three Superiour Planets*, Saturn, Jupiter *and* Mars.

Theſe three Planets are accounted by all Aſtronomers to be farther from the Earth than the Sun. As for the motions of theſe Planets, tho their daily motions are ſwifter than the other Planets, being farther diſtant from the Earth, yet their periodical motions are more ſlow; and according to their diſtance from the Earth, ſo is their ſlowneſs. For *Mars*, the neareſt of theſe three, finiſheth his courſe in one year 322 days. *Jupiter* the next, in 11 years 318 days. *Saturn*, the fartheſt, in 29 years 174 days; in theſe times they run thorow the 12 Signs in their Ecliptick.

Beſides theſe motions, there is alſo a retrograde motion, which not only thoſe three, but *Venus* and *Mercury* are ſubject to. Indeed, the Sun is exempted from it (as it were by extraordinary priviledge) and the Moon avoids it by her ſwift motion; but *Mercury* and *Venus*, tho in their mean motion equal to the Sun, yet are often driven backward.

Mercury is the moſt often ſubject hereunto of all the Planets, being very inconſtant in his motion; running over this motion every 116 days, being direct 92 days, and then retrograde 24 days. *Saturn* is more conſtant and ſerious in his walk, renewing this motion once in 378 days, being direct 238 days, and retrograde 140 days. And *Jupiter*, his next Neighbor, as it it were to keep him company, walks this round in 399 days, going forward 279 days, and then backward 120 days. But Warlike *Mars*, as ſcorning to turn his back, marches on furiouſly for 700 days together, yet then he is forced to countermarch as faſt for 80 days. Laſtly, Dame *Venus*, proud of her Beauty, which ſhe is loth to have too long hidden, or fears will be too much burned by the Sun, runs as faſt as ſhe can before him for 542 days; but when ſhe ſees the Sun will over-take her, ſhe turns about and runs the contrary way for 42 days, that ſo the Sun may the ſooner overſhoot her, and ſhe may again ſhew to the Earth her glorious Luſtre. Theſe motions you may evidently ſee in theſe Planets, if you obſerve their ſituation Nightly in reſpect of the fixed Stars; but the farther ground and demonſtration hereof, I have not time to ſpeak of.

Laſtly, For the diſtance of theſe three Planets from the Earth, *Mars* is diſtant 1745 Semidiameters, *Jupiter* 3990, *Saturn* 10550, which being granted, their diſtance, breadth, and compaſs in Miles, is thus:

Diſtance.

	Distance.	*Breadth.*	*Compass.*
Mars	6950535	3336	10484
Jupiter	15892170	12800	40228
Saturn	42020650	21920	68891

6. *Of the Fixed Stars.*

I call these *Stars* fixed, not because they are void of all motion, but because they keep their places very exactly one in respect of another. Insomuch, that those Stars which were to be observed in a right line by *Hpparchus* 1700 years ago, were found so by *Ptolomy* in his time 1300 years ago, and continue in the same posture still, as you may see by many Examples in *Tycho*, *lib.* 1. *p.* 134. Therefore it is very probable that they all are fixed in one Sphere, and are all equally distant from the Earth, since they are so uniform in their motions.

The motions of these Stars are two: The one daily from East to West, the other yearly according to the order of the Signs. In their daily motion they are exceeding swift; for the compass of their *Heaven* is above 350000000 of Miles; so that those Stars which are towards the Equator, must move 240000 Miles every minute of an hour. But in their other motion they are exceeding slow, not moving one whole minute in a year, but only one degree in 70 years; so that they will be 25000 years in making their progress through the 12 *Signs*. This motion by the Ancient Astronomers was thought to be finished in 36000 years, and is called by some the *Platonick year*: At which time, according to their Opinion, all the Stars having finished their Revolutions, shall return unto their first places; and that all other things shall return with them again to the same order wherein they were. But this Opinion is very vain, for neither will the motions of the Planets and Stars, calculated together, agree to such a Revolution. And the Apostle *Peter* affirms, *That both the heavens and the earth which now are, shall be destroyed with fire at the last day*, 2 Pet. 3. 7. which will certainly be long before this Period.

Concerning the number of the Stars, though it be great, yet it is not infinite. Astronomers take notice only of about 1000, which are the most conspicuous, and though sometimes in a clear Winter Night many more little ones may be perceived, yet they cannot certainly be observed; and those aforesaid seem many more than they are.

The Ancient Astronomers, the better to distinguish and describe these Stars, have drawn them upon their Globes and Maps in 48 Images; so that every Star comes to have a Name from that part of the Image they are placed in: As the *Bulls Eye*, the *Lions Heart*, and such like. Some Modern Astronomers have added 12 Constellations more towards the South Pole, which were not known to the Ancients.

Another way whereby these Stars are distinguished, is by their several bignesses, amd to this purpose they rank them into 6 Magnitudes. This distinction is far more apparent than the other, the Constellations being only Poetical and Imaginary, this difference real and visible. But though the biggest of these Stars seem little in regard of their great distance from us, yet are they of wonderful greatness. For if you reckon only their distance according to *Tycho*, to be 14000 semidiameters of the Earth (whereas *Ptolomy* allows 20000) this reduced into Miles, yields 55762000. Which being supposed, their Breadth and Compass, according to their visible Diameters, will be as followeth.

Magnitude.	Breadth	Compass	Number
1	32342	101643	15
2	24256	76232	45
3	16843	52935	208
4	12128	38116	474
5	08085	25410	217
6	05090	16941	49

Hereby you see, that the smallest Stars are almost as big as the Earth, the Compass of the Earth being 25036 Miles, as we shewed before. The next sort are full as big, or bigger, and all the rest in Order, far exceed the Earth. So that a Star of the first Order is four times the Breadth and Compass of the Earth; which Cubically considered, renders the Stars 65 times greater than the Earth. Thus the least things in Heaven are great, though they seem small, and the greatest things on Earth are small, though they seem great. Let this teach us to esteem of things, not as they seem, but as they are.

7. *Of the Christalline Heaven.*

This is fitly called the Watery Heaven, whose place is above the Stars: For though no such thing is apparent to our Eyes, yet the Scriptures make frequent mention of it. This was the Work of the second Day in the Creation, Gen. 1.6,7. *Again, God said, Let there be a Firmament in the midst of the Waters, and let it divide the Waters from the Waters. Then God made the Firmament, and separated the Waters which were under the Firmament, from the Waters which were above the Firmament.* This, though some interpret of the Waters in the Clouds, yet it cannot be so meant, for the Clouds are far below the Firmament, those Waters are above it; Besides, the Clouds are but Vapors arising daily out of the Earth. And though this may seem strange to Sense or Reason, yet it is the best way in this Point to Resolve with *Du Bartas*,

I'll rather give a thousand times the Lye
To mine own Reason, than but once deny
The Sacred Voice of that Non-Erring Spirit,
Which doth so plainly and so oft aver it,

That

That God above the Higheſt Firmament,
I wot not, I, what kind of Waters Pent.
Pſal. 104. 3. Pſal. 148. 4.

And the better to perſwade you to this, conſider what he ſaith afterwards;

I ſee not why Mans Reaſon ſhould withſtand,
Or not believe, that God, whoſe Powerful Hand
Bay'd up the Red Sea *with a double Wall,*
That Iſrael's *Hoſt might 'ſcape* Ægyptian *Thrall:*
Could prop as ſure theſe Waters thus on High,
Above the Heavens Starry Canopy.

And here, by the way, let thoſe who plead for the motion of the Earth, and the ſtanding ſtill of the Sun, contrary to the conſtant Phraſe of the Holy Scriptures, ſee what Force is in their excuſe, as if the Scriptures in theſe things, ſpake only according to our Common Senſes, and the Vulgar Opinion; and not the real Truth, according to Nature and Art, to inform our Judgments. I am ſure in this Point the Scripture ſpeaks a Truth, which paſſeth both the Senſe and Art of Man to apprehend. And not only in this one particular, but many other things, there are ſet down in the Creation, which are above, though not contrary to the Rules of Philoſophy: As that there ſhould be day, three days before the Sun. That the Trees and Plants ſhould ſpring and grow without the Sun, which is the natural Cauſe of their Production. That the Bird ſhould be before the Egg, which is preposterous in natural Generation: Yet theſe things we ought to believe, much more ſuch things as are more conformable to our ſenſe and common Experience.

What may be the uſe of theſe Waters, is very difficult to determine, the Scriptures herein being ſilent. Some think they were partly uſed for the drowning of the Old World: For it cannot be made good that ſo much Water could probably deſcend from the Clouds, the Waters being above the higheſt Mountain, and many of theſe Mountains being higher than the Clouds themſelves. Neither is this Phraſe uſed to ſet forth the Rain that comes from the Clouds.

Others think that thoſe Waters are placed here, for the cooling of the Firmament, which elſe would be ſet on Fire by its ſwift and continual motion; and that the daily Conſumption of theſe Waters, will be a ſecondary Cauſe of the Burning of the Heavens and the Earth at the laſt day. But theſe things are uncertain. The beſt uſe of them is ſet forth by the Prophet, Pſal. 148. 4. *Praiſe the Lord ye Heavens of Heavens, and ye Waters which are above the Heavens.*

8. *Of the firſt moving Heaven.*

Above all theſe (by the Opinion of many) there is a great vaſt Sphere incompaſſing all the reſt, which is called the *Firſt-moving Heaven*; this is ſuppoſed to move round every 24 Hours, and therewith to carry round about all the Stars and Planets every day. But as this is above our ſenſes to perceive it, ſo the Reaſons they bring to prove

it, are not of ſufficient Force. And I rather think that God hath indowed theſe Cœleſtial Bodies with an inward Power to perform their ſeveral motions. For though they are not living Creatures, as ſome have imagined; yet they may have a Propenſity to move to a certain Point, or in a Circular motion; as we ſee the Load-ſtone doth, being fitly placed.

For ſuppoſe thoſe Heavens, or Heavenly Bodies had need of any outward Cauſe of their motions: Yet how can this *Primum Mobile* yield them any help therein. For though it might move the higheſt Firmament wherein the Stars are, and which keep a Uniform motion among themſelves; yet how can it move the Planets which are farther from it, and have ſo many ſeveral motions? They anſwer to this, that the diverſity of their motion proceeds from their diſtance from this Firſt Mover, which thereby hath the leſs Force over them, to hinder them in their proper Motions from the Weſt to the Eaſt. But to this it may be replied, that if they have a Power of themſelves to perform the one motion, why can they not perform the other daily motion likewiſe by their own Power.

The Reſolution of theſe motions might much better depend upon the daily motion of the Earth upon its own Axis; and did not the Scriptures ſo oppoſe this motion, I ſhould willingly aſſent thereunto. But I ſhall find this one Argument more againſt this Opinion of the *Copernicans*. For they ſuppoſing the Earth to be a Planet, how then can they think that the Earth ſhould have any other motion than the reſt of the Planets? Now that the other Planets have no ſuch diurnal motions upon their own Axis, is apparent by the Moon; which if ſhe had any ſuch motion, the ſpots in the Moon would not appear always in the ſame place of the Moon as they do, both on the Eaſt and Weſt of the Meridian: For if the Moon were turned round, theſe ſpots would be ſometimes on the one ſide, and by and by on the other. But if you obſerve theſe ſpots, you ſhall find the Picture of the Man in the Moon, continually on the Weſtern ſide of the Moon; and though ſome variation it may have in reſpect of the vertical Point, yet in reſpect of the Pole of the *Ecliptick*, it keeps the ſame place very exactly, juſt as the Horns of the Moon do in the prime and later part of the Moon.

Thus I have aſcended to the Higheſt Parts of the Viſible Heavens, and have (I hope) in ſome Meaſure performed my Promiſe.

CHAP.

CHAP. VIII.

Containing ſome of the ſtrongeſt Arguments (by way of Objection) the Maintainers of the Ptolomean Syſtem *bring againſt the* Copernican Syſtem: *With the* Anſwers *the* Copernicans *give unto them. Extracted out of the Writings of* Pythagoras, Galileus *and other later* Aſtronomers.

OBJECTION I.

THeir Firſt Objection is againſt the Motion of the Earth: *For* (ſay they) *We ſee the Sun, Moon and Stars Riſe in the Eaſt, and make a Progreſs by the South, and ſet in the Weſt: And therefore our Viſual Senſe Demonſtrates, that they move, and not We.*

ANSWER.

In Anſwer to this, it may be urged; That the Senſe of ſeeing is deceitful, and makes that ſeem to move which ſtands till, and that ſtand ſtill which moves; as is often ſeen upon the Water. For, Rowing in a Boat, the Boat ſhall ſeem to lie ſtill in the Water, and the Banks of the ſhore ſhall ſeem to ſlide away from us: And many times the Moon and Stars ſhall ſeem to go along with us, which way ſoever we go.

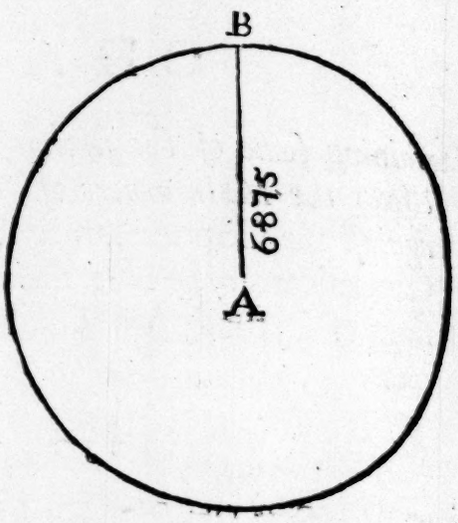
OBJECTION II.

If we ſtand upon a Hill (or on the Top of a main Maſt of a Ship at Sea) then Looking round about us, the Heavens appear on each Quarter exactly Equidiſtant from us: Therefore the Earth is in the Center of all the Heavens: For, were it in any other Place, we ſhould perceive one part of the Heavens nearer to us than another; and the Stars would ſhew Greater on that part of the Heaven, which ſhews near to us, and leſs to that part of Heaven which ſhews fartheſt from us.

ANSWER.

The Diſtance of the *Starry Heaven* from the *Sun* (which is the Center of the *Starry Heaven*) is ſo vaſt, and the diſtance of the *Earths Orb* ſo ſmall, that it ſeems in the very *Center* it ſelf, and therefore makes

makes in the *Sight* no Alteration; as by this following *Scheme* and *Example*, may be eaſily conceived: Wherein imagine this Circle, repreſenting the *Starry Heaven*, to be 20 Inches Diameter: A is the Center of the Circle, which is, in all parts, equidiſtant from the *Circumference*. This Line AB will then be 10 Inches. Now ſuppoſe the Line AB were divided into 6875 equal parts (for the *ſemidiameter* of the *Starry Heaven*, contains the *ſemidiameter* of the *Earths Orb* 6875 times) and that the *Center* were removed from its true place at A, one of thoſe equal Parts: Then may you, firſt, conſider the apparent Length of one of thoſe Parts; and ſecondly, what difference, in appearance from the *Center*, it would have: If then the whole Length A B 10 Inches, be divided into 6875 equal Parts, then ſhall every Inch be divided into 637½ Parts, which is ſo ſmall, that it exceeds not half the Breadth of a Hair of ones Head: And what difference, at the *Circumference*, half a Hair Breadth will make from the true *Center* of a *Circle* of 20 Inches Diameter, is to be conſidered. It is true, it is a difference from the true *Center*, but ſo ſmall, that no Man with the Point of a Needle can make a perceptible *Mark* of that Size. Now, though the diſtance of the *Earths Orb*, from the *Center* of the *Starry Heaven*, be ſo ſmall that it is ſcarce diſcernable, by Reaſon of the vaſt diſtance of the *Starry Heaven*, yet the diſtance of the *Earths Orb* from the *Orbs* of *Mars* and *Venus* is not ſo great, but that a ſenſible difference appears at ſeveral times in the ſight of theſe *Planets*: For *Mars* in his *Perigeum* (or when he is *neareſt* to the *Earth*) appears 60 times bigger than in his *Apogeum* (or when he is at his *greateſt* diſtance from the *Earth*) and *Venus* in her *Perigeum*, appears 40 times bigger than in her *Apogeum*: As (with good *Teleſcopes*) hath been Obſerved by *Galileus* and others ſince.



Objection III.

It is againſt Reaſon (ſay they) to think the Earth ſhould turn round; for then, we ſhould, every Converſion have our Feet turned upwards, and our Heads hanging downwards in the Air: Our Houſes would tumble over; and every looſe thing, as Stones, Animals, &c. Fall from the Earth, as having nothing to reſt their Weight upon.

Answer.

It is unreaſonable to think *Nature* ſhould create any thing to its own Deſtruction; which we muſt allow, in caſe the Place Aſſigned it by *Nature* be not quite enough to retain it; for nothing can ſubſiſt without

without a proper *Place*: But we see it customary with *Nature* to Act beyond *Vulgar Reason*: For, whither the *Earth* turn *Round*, or stand fix'd in the *Center*, it matters not in this *Argument*: For, we all allow the *Earth* to be *Round*, and few Men now are so unskilful in its *Shape* or *Figure*, as with *Lactantius*, to deny an *Antipodes* to every Place on the *Earth* and *Sea*. Which *Antipodes*, according to his *Argument*, is as much subject to these *Casualties*, as the whole *Earth* in its *Conversion*: Nay, the whole *Earth*, by admitting *Antipodes*, runs equal hazard of dropping all loose things into the *Air*, as admitting of the *Earths* Conversion does.

Objectiog IV.

If according to the Copernican *Doctrine, the Earth move round from West to East in* 24 *Hours, then (admitting the Circumference thereof to be* 21600 *Miles) it must in one Hour move* 900 *Miles, and in a Minute* 15 *Miles, and in the fifth Part of a Minute* 3 *Miles. So that if a Stone be let fall from an High Steeple, and it be the fifth part of a Minute in falling, by that time the Stone comes to the Ground, the Earth should have passed from West to East three Miles, and the Stone must (by consequence) fall three Miles to the Westward of the Bottom of the Steeple. And to this Objection may be added another*; Viz. *That* Pigeons *may by strength of their Wings fly three Miles to the Westward from their Dove-House in one quarter of an Hour: But in this time should the Dove-House, by the Earths motion be receded Eastward* 225 *Miles. which with the acquired motion of* 3 *Miles, makes* 228 *Miles, and then they should alight* 228 *Miles off from their Dove-House: Both these Arguments, and several other of the same Nature, experience contradicts; and therefore the Earth stands still.*

Answer.

By the *Earth*, is not meant only this bare *Ball* of *Earth* and *Water* whereon we dwell; but the whole *Body* as it is Cloathed with the *Elements* of *Air* and *Fire*: For these *Elements* are frequently convertible into each other: And as *Philosophers* teach us, are never so purely *one*, as not to have a mixture of *all*: Therefore, whatsoever is within this *Sphere* of *Elements*, we say is in the *Earth*, though part of these *Elements*, for their Purity, be Elevated above the Surface of that solid Body of *Earth* wherein we go: And therefore when we say the *Earth moves*, we do not mean only that part of the *Elementary Sphere* which we call *Earth*; but by the *Earth* we mean the whole *Body* of *Elements*, which is according to this *Doctrine* agitated by one single Activity; and what ever moves in any part of this *Sphere*, moves in *all*: *As for Example*, The *Stone* and the *Doves* being *Elementary Bodies*, are inclosed within the *Sphere* of the *Earths* Activity; *Viz.* Within the *Air* of the Highest, and therefore have the *Motion* impress upon them that the *Earth* hath: For which way soever the *Stone falls*, or

or is *thrown*; or which way ſoever the *Pigeon* Flies, they move along *Eaſtwards* with the *Earth*, though they fly *Weſtwards*. Thus Experience ſhews, That if a *Ship* Sails with a *ſwift* motion *Weſtwards*, it may in one *ſecond* of *Time* run about 9 Foot; and if one let fall a *Stone* from on high into the *Hold*, that *Stone* ſhall not fall 9 Foot *Eaſtwards* from the Perpendicular Point, but juſt upon the ſame Point in the Bottom of the *Hold* it would have fallen, if the *Ship* had lain ſtill, *Viz.* on the Point Perpendicular to the *Point* above it, from whence it was let fall; whereas, if the *Stone* had not received the Impreſt motion of the *Ship*, it muſt have fallen 9 Foot to the *Eaſtward* of that Point.

Objection. V.

The Earth *is a* Heavy Body, *and therefore unfit for* Motion, *eſpecially through the pure Heavens; for ſuch is the ſublimity of them, that the* Weight *of the* Earth *would preſs through them, and ſo it ſhould loſe its Place.*

Answer.

If the *Earth* be a *Heavy Body*, yet it is not unfit for *Motion*, if the moving Power be ſtrong enough to ſet it going: But *Heavy Bodies* are more unfit to lie ſtill in the ſublime Heavens, than to move in them: For thus we ſhall ſee, a *Stone* thrown into the *Air*, whilſt it hath *Motion* it will abide there, but when the *motion* dies, the *Stone* falls.

Theſe are ſuch *Objections* as are Vulgarly made againſt this *Opinion*. There are others, which by ſome, may ſeem of more weighty Concern, as being againſt *Senſe*, *Reaſon*, *Experiment* and *Demonſtration*. Now ſuch as deſire farther ſatisfaction in theſe particulars, may Read *Copernicus* himſelf, *Kepler*, *Bullialdus*, *Lanſbergius*, *Gallilæus*, *Helvetius*, &c. There are others alſo that condemn this *Hypotheſis* as *Heretical* (as formerly that of the *Antipodes* was) and diſconſentanions to ſeveral Places of *Scripture*; for they alledge. 1. That *Text* in Pſalm. 104. 5. *He ſet the Earth upon her Foundations, ſo that it ſhall never move.* And, 2. Pſal. 24. 2. *He hath founded it upon the Seas, and eſtabliſhed it upon the Floods*; and ſeveral other Texts. And for the *Motion* of the *Sun*, they alledge; 3. *Eccleaſiaſt.* 1. 5. Were it is Written, *The Sun ariſeth, and the Sun goeth down, and haſteth to his Place where he aroſe*, Alſo, 4. Pſal. 19. 5, 6. Where the *Pſalmiſt* ſpeaking of the Sun, he ſaith, *Which is as a Bridegroom coming out of his Chamber, and Rejoyceth as a ſtrong Man to Run his Race. His going forth is from the end of the Heavens, and his Circuit unto the ends of it; and there is nothing hid from the Heat of it:* But in *Anſwer* to theſe Texts, or any other quoted in the *Sacred Volume*, take this pithy Anſwer of St. *Auguſtine*, who ſaith, *If any one ſhall object the Authority of Sacred Writ, againſt clear and manifeſt Reaſon; he that doth ſo, knows not what he undertakes; for he Objects againſt the Truth, not the Senſe of the Scripture, which is beyond his Comprehenſion: Not what is in it, but what finding it in himſelf he fancied to be in it.*

Now

Now to conclude this Discourse; for those that desire to make farther Scrutiny into this *Hypothesis*, I would have them to Read (besides the above named *Authors*) Dr. *John Wilkins*, late Lord Bishop of *Chester*, his Learned *Discourse* concerning a New PLANET, tending to prove, that 'tis probable our EARTH is out of the PLANETS; Wherein he insists upon these following Particulars; *Viz.*

I. That the seeming Novelty and Singularity of this Opinion, can be no sufficient Reason to prove it Errroneous.

II. That the Places in Scripture which seem to intimate the Diurnal Motion of the Sun, or Heavens, are fairly capable of another Interpretation.

III. That the Holy Ghost in many places of Scripture, does plainly conform his Expressions to the Error of our Conceits; and does not speak of sundry things as they are in themselves, but as they appear unto us.

IV. That divers Learned Men have fallen into great Absurdities, whilst they have looked for the Grounds of Philosophy from the Words of Scripture.

V. That the Words of Scripture, in their Proper and strict Constructions, do not any where affirm the Immobility of the Earth.

VI. That there is not any Argument from the Words of Scripture, Principles of Nature, or Observations in Astronomy, which can sufficiently evidence the Earth to be in the Center of the Universe.

VII. 'Tis probable that the Sun is in the Center of the World.

VIII. That there is not any sufficient Reason to prove the Earth incapable of those Motions which *Copernicus* ascribes unto it.

IX. That it is more probable the Earth does move, than the Heavens.

X. That this Hypothesis is exactly agreeable to common Appearances.

CHAP. IX.

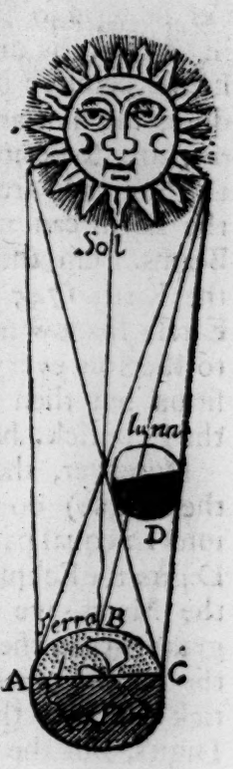
Of the Eclipſes *of the* Sun *and* Moon, *the cauſes of them, and how and when they happen.*

I. *Of the Eclipſe of the* SUN.

THE Eclipſe of the Sun is nothing elſe but the direct Interpoſition of the body of the Moon between our ſight and the Sun, for the Moon being a denſe, ſolid, and opacious Body, and not tranſparent, coming betwixt the Sun and the Earth, (which is always in a conjunction) doth thereby hide more or leſs of the Suns bright ſhining Body from our ſight; but yet the Sun is not Eclipſed in every conjunction, but when it falleth in, or near to the Head or Tail of the Dragon. And it may ſometimes ſo fall out, that the Sun may ſuffer two Total and Central Eclipſes in the ſpace of 6 months, one of which happening before the Suns Apogæon, and the other after, in which caſe there may happen no Full Moon Eclipſed that Year, according to the Rule: *Fieri poteſt ut duo interlunia nodis proxima & fere centralia unius ſemeſtris intervallo accidant; unum quidem ante ſolis Apogæum, alterum poſt illud; quo caſu intra totum annum lunarem nullum continget plenilunium Ecliptcum.* But it cannot be Univerſal (as the Moons Eclipſe is) but may appear in one Climate a great Eclipſe, in another it may be leſſer, and in other ſome no Eclipſe at all, and that at the ſelf ſame Inſtant, becauſe the Eclipſe of the Sun dependeth principally on the Parallax of the Moon, which is different in every Climate, (as hereafter, God willing, in another Work) and ſeeing that the Sun far exceedeth the Earth in bigneſs, and the Earth far exceedeth the Moon, therefore the Cone of the Earths ſhadow cannot take away, or hide the whole body of the Sun from all parts of the Earth; but one part only ſhall obſerve the ſame to be total, or of the like quantity. Hence it may appear that the Sun loſeth no Light, but only we are deprived of the ſame by the interpoſition of the Moons Terreneal Body, (as is ſaid before) and therefore Aſtronomers define, that it is *Interceptio luminis ſolaris profecta ex interpoſitu Lunæ inter ſolem atque aſpectum noſtrum.* So that *Suidas* well obſerves, That *ſolis Eclipſis ſit, quando Luna in ipſum incurrit*; and for the better underſtanding of what is ſaid, behold this Figure following.

A Typ

A Type of the Sun's *Eclipſe according to* Ptolomy.



By this Type it appears, that to him that ſtands in C, the whole body of the Sun is hidden from his ſight; to him that ſtands in B, half the Sun is only hidden; but to him that dwelleth in A, there is no part of the Sun hid from his ſight. Now there are ſome that think, that in the ſpace of 19 years all the Lunations and Eclipſes return again to their old courſe, and happen the ſame again: And of this Opinion is *Pliny*, in the 2d Book 13th Chap. of his *Natural Hiſtory*, where he ſays, *Elapſis ducentis viginti duobus menſibus Eclipſes in orbes ſuos redire.* Now ſome there be, who ſays that *Pliny*'s meaning was, That all the variety of Eclipſes are finiſhed in that ſpace of time. But this is but an Idle gloſs; for albeit in the ſame ſpace of time, the Head of the Dragon goeth once about the Zodiack, and the Eclipſes may happen thereby near the ſame time and place again, yet notwithſtanding they are not therefore the ſame, that is to ſay, of the ſame quantity and duration; for certain is the Rule, *Cæli motus inter ſe ſunt incommenſurabiles, nec unquam Phænomena eodem prorſus modo recurrere poſſunt.*

II. *Of the Eclipſe of the* MOON.

THE Moon having no Light but what ſhe receiveth of the Sun, can never be Eclipſed but at the Full, but when ſhe is Diametrically oppoſite to the Sun, and the Earth, in the midſt between them both. Hence the Philoſophers rightly defined it, that is, *Privatio Luminis in Luna orta à diametrali terræ inter Solem & Lunam oppoſitione.* And *Lucan* alſo, *Lib.* 1. *Pharſal.* ſheweth the cauſe of the Moons Eclipſe.

——cornuque coacto,
Jam Phœbe toto cum redderet orbe
Terrarum ſubita percuſſa expalluit umbra.

Therefore the Moon being full of Light, and exactly oppoſite to the Sun in the Head or Tail of the Dragon, may be totally Eclipſed, and deprived of Light by the ſhadow of the Earth; for the Earth being a thick and ſolid Body, caſteth its ſhadow to that part which is oppoſite to the Sun (whom it is that enlightneth the Moon, and all the Ætherial Region (*ſuo Lumine*) unleſs that point oppoſite to him.) When therefore the Moon at her Full, and oppoſition to the Sun, entreth into that obſcure part, on which the ſhadow of the Earth falleth, ſhe doth thereby loſe the Suns Light, and is obſcure and dark; which we call an Eclipſe. And here obſerve and be aſſured, that there is no Star whatſoever that can enter this ſhadow but the Moon alone, in regard of her propinquity to the Earth, for it is gathered

by ſundry Obſervations made by the beſt Aſtronomers, that the ſhadow of the Earth is Conical the quantity of the extent (*a centro Terræ*) being 250 Semi-diameters, and the Moon being fartheſt remote in *Apogæo*, is diſtant from the Earth but 58 Semi-diameters; from hence therefore it appears, that the Moon may enter into the ſaid ſhadow, but not any of the other Planets; and although *Venus* and *Mercury* are ſometimes within the ſaid limits, yet they are always nigh the Sun, and are never oppoſite to him; ſo that this Maſſy Globe of the Earth can never interpoſe thereby, to hide the Suns glorious Beams from them. As for *Mars*, he is (in his mean motion) from the Earth 1745 Semi-diameters; to which place the Cone of the Earths ſhadow never extendeth. And although the Moon is oppoſite to the Sun every month, ſhe is not therefore Eclipſed in every oppoſition but then only, when at the oppoſition ſhe is found in or near the Ecliptick, having then little or no Latitude.

Moreover, the Aſtronomers (the better to meaſure the quantity of the Eclipſe) do divide the Diameter as well of the Sun as of the Moon, into 12 equal parts, which they call Points or Digits, and the more Digits the Eclipſe is, the greater is the Eclipſe. Yet in the Eclipſes of the Moon, we muſt know that the Diameter of the ſhadow is far greater than the Diameter of the Moon. Hence the Moon, being in the middle of the ſaid ſhadow which anſwereth the Line of the Ecliptick exactly, the whole body of the Moon is not only obſcured 12 Digits, but the obſcuration is inlarged much by the plunging of the Moon (as it were) into the ſaid dark ſhadow, and her long continuance there; and therefore in Lunar Eclipſes, we reckon the number of Digits according to the quantity of the ſhadow of the Earth, which may ſometimes extend to almoſt 23 Digits.

A Type of the Moon's *Eclipſe according to* Ptolomy.

Wherefore we uſually reckon three ſorts of Eclipſes, the firſt partial, when her whole Body is not darkened, but ſome part thereof. The ſecond, when her whole Body is darkened exactly; and this is called a total Eclipſe, but without continuance. The third, when ſhe happens into the ſhadow of the Earth more than 12 Digits; and this is called a total Eclipſe with continuance, or ſtay in darkneſs: All the difference whereof ariſeth principally from the Latitude of the Moon, and her diſtance from the Ecliptick Line. But of theſe things (*Deo permittente*) more amply hereafter in another Work.

In this Diagram you may obſerve when the Eclipſe is total, when partial, and when there is none at all in the oppoſition.

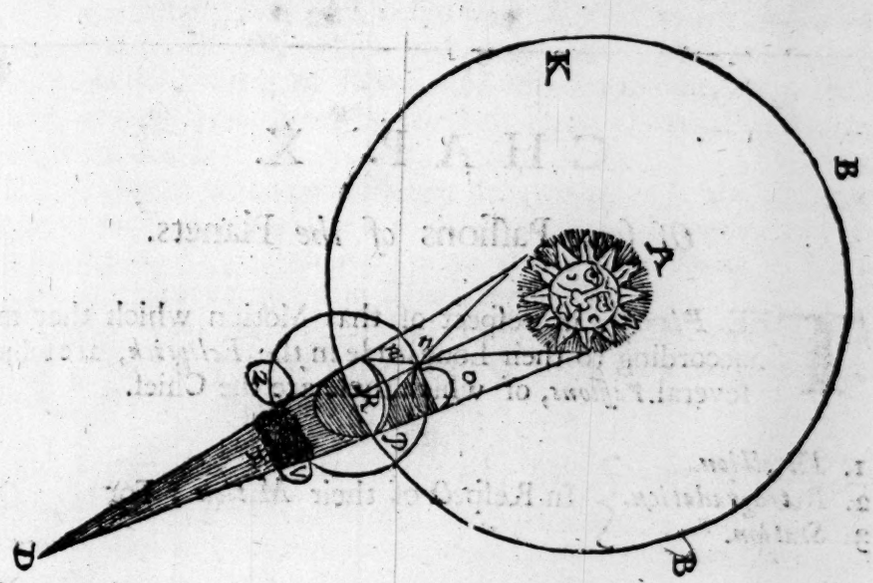
More concerning the Eclipſes.

The two foregoing Schemes declare the reaſon of the

Eclipſes

Eclipſes of the Sun and Moon according to the *Ptolomean* Syſtem, where the Earth is ſuppoſed to be the Center of the Univerſe: But this Scheme demonſtrates the ſame by the *Copernican* Syſtem, where the Sun is ſuppoſed to be the Center of the Univerſe, in which Scheme or Figure,

A Type of the Sun *and* Moon's *Eclipſe according to* Copernicus.



I. *For the* Eclipſe *of the* Sun.

A, Repreſents the *Sun*, and Center of the *World*.
B, K, R, *B*, The *Annual Orb* of the *Earth*.
R, The *Body* of the *Earth*.
S, Z, V, S, The *Orb* of the *Moon*.
O, The place of the *Moon* when ſhe is in *Conjunction* with the *Sun*.
V, X or Z, Her *Place* when ſhe is in *Oppoſition*.
D, The *Conical Point* of the *Earths Shadow*, which is *greater* or *leſſer*, according to the diſtance of the *Earth* from the *Sun*.

Now, by this *Figure* it appears, that to him that ſtands upon the *Earth* at N, the whole Body of the *Sun* is hidden from his ſight by the interpoſition of the *Moon* at O. But to him that is at R, there is but half the *Sun* obſcured. And laſtly, To him that lives at M, there is no part of the *Suns* Body hid from him; but he, at the ſame Inſtant, may behold his whole Body free from any interpoſition of the *Moon*, as is plainly repreſented in the *Figure*.

II. *For the* Eclipſe *of the* Moon.

When the *Moon* is in exact *Oppoſition* to the *Sun*, and void of *Latitude*, as ſhe is when placed at X, for then the Center of the *Earths* Shadow, whereby ſhe loſeth the whole Light of the *Sun*, by the direct interpoſition of the *Earth* between the *Sun* and *Her*; whereby *She* is totally

tally *Eclipsed* with *Contrivance*: But if she be in the Points ♈ or ♎, then her *Eclipse* is *Partile*; and only a *Part* of her *Body* is darkned; and if her *Latitude* (at the time of her *Opposition* with the *Sun*) be so great *Northward* or *Southward*, that it be out of the Verge of the Cone of the *Earths Shadow*, as at C or D. Then she is free from all obscurity thereby, and so suffers no *Eclipse* at that *Opposition*: All which Positions the *Figure* plainly Demonstrates.

CHAP. X.

Of some Passions *of the* Planets.

I. THE *Planets*, in respect of that Motion which they make, according to their Longitude in the *Ecliptick*, are subject to several *Passions*, of which these are the Chief.

1. *Direction.*
2. *Retrogradation.* } In Respect of their *Motion*; For
3. *Station.*

First, A Planet is said to be *direct* in Motion, when it moveth forward, agreeable to the Order and Succession of the *Signs*; as from *Aries* to *Taurus*, *Gemini*, &c.

Secondly, A Planet is said to be *Retrograde* in Motion, when its Motion is contrary to the Order and Succession of the Signs; as when a Planet moveth from *Cancer* to *Gemini*, and so to *Taurus*, &c.

Thirdly, A Planet is said to be *Stationary*, when in its change between *Direction* and *Retrogradation*, the change is not in a small time perceptible, and seemeth to continue in the same place for some days together; and so is said to be *Stationary*.

II. The Planets among themselves, in respect of their Longitudes in the *Ecliptick*, are subject to several *Aspects*, as principally these five.

1. *Conjunction.*
2. *Opposition.*
3. *Trine.*
4. *Quartile.*
5. *Sextile.*

First, *Conjunction* is when any two Planets are in one and the same *Sign* and *Degree* of the *Ecliptick*, as if *Saturn* be in 16 deg. of *Gemini*, and *Mars* be there also; then there is said to be a *Conjunction* of *Saturn* and *Mars*, which the Astronomers do thus Characterize ♂ ☌ ♄.

Secondly, *Opposition* is when any two Planets are in opposite *Signs* of the *Ecliptick*, or are just *six Signs*, or a half Circle, distant from each other; as if *Jupiter* be in 10 deg. of *Aries*, and *Venus* in 10 deg.

of

of *Libra*, then there is said to be an *Opposition* of *Jupiter* and *Venus*, which is thus expressed in Characters ☍ ♃ ♀.

Thirdly, *Trine*, Two Planets are said to be in *Trine*, when they are four Signs (or one third part of a Circle, which is 120 deg.) distant from each other. As if the Sun be in 25 deg. of *Taurus*, and *Mars* in 25 deg. of *Virgo*, then there is a *Trine* of the *Sun* and *Mars*, which they thus express △ ☉ ♂.

Fourthly, *Quartile*, Two Planets are said to be in *Quartile Aspect*, when they are distant from each other *three Signs* (or one *Quadrant*, or quarter of the *Ecliptick*, which is 90 deg. distant from each other,) as if the *Moon* be in 6 deg. of *Pisces*, and *Mercury* in 6 deg. of *Gemini*, then there is a *Quartile Aspect* between those two Planets, and is thus Characterized □ ☾ ☿.

Fifthly, *Sextile* is when two Planets are two *Signs* in the Ecliptick distant from each other; as if *Saturn* be in 12 deg. of *Cancer*, and *Jupiter* in 12 deg. of *Virgo*, then do those two Planets make a *Sextile Aspect*, and are thus expressed in Characters ⚹ ♄ ♃.

Venus and *Mercury* cannot make any of these Aspects with the Sun. Neither can *Venus* and *Mercury* make any of them between themselves, except the *Sextile*, which they often do.

III. The Planets Places in the *Ecliptick*, being compared with the *Suns* Place, they are either,

1. *Combust.*
2. *Oriental.*
3. *Occidental.*

First, A Planet is said to be *Combust*, when it is so near the *Sun*, that it cannot be seen, by Reason of the *Suns* Bright Rayes.

Secondly *Oriental*: A Planet is said to be *Oriental*, when it Rises in the Morning, and may be seen in the *East* before the *Sun* Rises.

Thirdly *Occidental*: A Planet is said to be *Occidental*, when it may be seen in the *West* some time after the *Sun* is set.

IV. Of the *Poetical* Rising and Setting of the *Stars* and *Planets*, and they are principally three, *Viz.*

1. *Cosmical.*
2. *Acronical.*
3. *Heliacal.*

First, A *Star* or *Planet*, is said to *Rise Cosmically*, when it *Riseth* with the *Sun*, or with that degree of the *Ecliptick* in which the *Sun* then is: And it is said to *set Cosmically* when it *setteth* in the *West* in the *Morning*, at such time as the *Sun* is Rising in the *East*.

Secondly, A *Star* or *Planet* is said to *Rise Acronically*, when it *Riseth* in the *East* Horizon, at such time as the *Sun* is going down in the *West*: And the *Acronical setting*, is when a *Star* or *Planet* goeth down, or *setteth* with the *Sun*.

Thirdly, A *Star* or *Planet* is said to *Rise Heliacally*, when having been some time *Combust* (or hidden under the *Sun* Beams) begins to appear: And *Heliacal setting* is some small time before it comes to be *Combust*.

Now

Now to know when a Star or Planet begins to become Combuſt, and when to be freed from his Combuſtment of the Suns Beams, no certain Rule can be given; for the Magnitude of the Star; the Difference of the Climate; the Cloudineſs or Serenity of the Air may much alter: But the Opinion of the Ancient Aſtronomers was; That

A Star of the	First, Second, Third, Fourth, Fifth, Sixth	Magnitude may be ſeen, when the Sun is but	12, 13, 14, 15, 16, 17 Degrees below the Horizon.

But thoſe which are only Nebulous cannot be ſeen until the Sun be 18 Degrees below the Horizon.

CHAP XI.

Containing the Rudiments of Aſtronomy, put into plain Rhyme, by Mr. John Palmer, M. A.

I. Of the Circles of the SPHERE.

SIX Great Circles *mark you ſhall*;
Which Equally *divide this Ball*:
Juſt in the midſt, between the Poles,
From East *to* Weſt *th'*Equator *Rolls*,
*Th'*Ecliptick *cuts him, and doth ſlide*
Scarce Twenty Four *Degrees aſide*:
Horizon *even with the Ground*,
From Stars *below our ſight doth bound*:
Meridian *Upright doth Riſe*,
Parting the East *and* Weſtern Skies:
Two Colures *through the* Poles *do Run*,
Quart'ring the Circle *of the* Sun,
One where the Spring *and* Fall *begin*,
Th'other where Longeſt Days *come in.*

Four Leſſer Circles (*Mark them well*)
*Are to th'*Equator *Parallel*:
Two Tropicks *Bound the* Sun's *High-way*,
Shewing the Longeſt *and* Shorteſt *Day*:
The Arctick Circle *cuts the* Bears,
*Th'*Antartick *Oppoſite Appears.*

Meridian

Meridians *half Twenty Four*,
For Hours, *and for* Degrees, *Nine Score*,
Through both Poles *of the World do pass*,
And th' Equinoctial *downright cross*;
And Nine Score Parallels *hath that Line*,
By which Stars North *and* South Decline.
*Th'*Ecliptick *hath his* Longitudes,
And Parallels *of* Latitudes
For Stars; *but in* Geography,
The Towns *beside th'* Equator *lie.*

Over our Head, *and under* Feet,
The Nine Score Azimuths *do meet*;
And here as many Parallels
Of Altitude Horison *tells.*

Longitudes *and* Meridians *all*,
And Azimuths Great Circles *call*;
But all their Parallels *in Heaven*
Being lesser, *cut the* Globe uneven.
Degrees *Three Hundred and Three Score*,
Hath every Circle; *and no more.*

II. Of the Constellations of the Fixed Stars.

THE *Army of the Starry Sky*,
Declare the Glory of God most high;
Seen and perceived of all Nations,
In Eight and Forty Constellations.

First, *Near unto the* Northern Pole,
The Dragon *and two* Bears *do Roll*,
Whose hinder Parts and Tails *contains*
The lesser and the greater Wains:
The Hare, *the* Bearward, *and the* Crown,
And then comes Hercules *kneeling down*;
And next below a place doth take,
Great Serpentarius *with his* Snake,
Under the Harp *of* Orpheus,
The Eagle *and* Antinous;
The Silver Swan *her* Wings *doth spread*
Above the Dart *and* Dolphins *Head*;
Then Pegasus *comes on amain*,
Andromeda *follows in her* Chain;
The Triangle *below her stands*,
And at her Feet is Perseus *Hands.*

The Gorgons *Head above are ſeen*
Her Parents, Cephus *with his* Queen
Caſſiopea: *Not far below*
Heniochus *his* Goat *doth ſhew*
On his Left Shoulder, *in his Hand*
He doth the Stormy Kids *Command.*

Here in the Zodiack *begins*
The Lamb, *the* Bull, *the Loving* Twins;
The Crab, *the* Lion, *and* Virgin *Tender,*
The Ballance, Scorpion *and* Bow bender;
Goat, Waterman *and* Fiſhes *twain,*
Shall bring you round to th' Rain *again.*

Fifteen Images *appear*
In the Southern Hemiſphere;
The Monſtrous Whale *before the reſt,*
Eridanus *ſcarce wets his Breaſt*;
Over the Hare, Orion *bright*
Sparkles in a Winters Night;
Then comes the Great Dog *at his Tail,*
And famous Argo *ſpreads his Sail*;
Above the Little Dog *doth flame,*
For whom the Latines *had no Name.*
Long Hydra *on her* Tail *a-low*
Carries the Pitcher *and the* Crow;
The Centaur, *holds the* Wolf *by th'* Tail;
The Altar *and* Ixions Wheel
Are never ſeen of us; *But here*
The Southern Fiſh *brings up the Rear.*

III. Of the Seven Planets.

UNder *thoſe* Fixed Stars *above*
Seven Planets *in their* Orbs *do move.*
The high'ſt is Saturn; *Thirty Year*
He ſpends in Compaſſing his Sphere.
Twelve Jupiter; *and* Mars *in twain*
Sets forward, and comes round again.
Then in one Year the Sun *diſplays*
Three hundred Sixty and Five days;
And near a Quarter, which in Four
Encompaſſing, makes one Day more.
Between the Sun *and us there fly*
Fair Venus *and ſwift* Mercury.
Theſe always near the Sun *we find,*
Not far before, nor far behind.

The

The Moon's *the lowest, who in Seven*
And Twenty Days goes round the Heaven;
And above two Days more does Run,
Before she overtakes the Sun:
So Twenty Nine Days and an half in all,
Do make a Month Synodical.
These Planets *make their Course to th'* East,
Though they be faster hurled West;
And Six Degrees *the rest may stray,*
Besides the Suns *Ecliptick way.*

SANDYS upon *Psal.* 8.

WHen I, pure Heav'n, thy Fabrick see,
The Moon *and* Stars *create by thee;*
O! what is Man, or his frail Race,
That thou should'st such a shadow grace?

CHAP. XII.

Of the Constellations *of the* Fixed Stars, *Giving an Account of their* English, Greek, Hebrew, Latine, Arabick, Caldee, Syriack, Turkish, *&c.* Names; *and of the most Eminent* Stars *in each of them: With the* Poetical Fables, *declaring how these* Asterisms *came to be placed in the* Heavens; *and also of the* Galazia.

I. Of the Zodiacal Constellations.

ARIES.

THis Sign is called by the Greeks Κριὸς *Chrysomallus, Jupiter Ammon*; by the Ægyptians or Copties *Tamesouro Ammon*, i. e. *Regnum Ammonis*.; in Hebrew *Tele*; in the Syriack *Emro*; by the Arabians *Al Hámal*; by the Persians *Bèrri*, or *Bère*; by the Turks *Kaza*: All signifying a *Lamb full grown*. In this *Constellation* are reckon'd 17 Stars, whereof 4 *informes*, i. e. Placed without the Figure. This, according to the Tradition of the Ægyptians, was made a *Constellation* in Honour of *Cham*. But as *Nigidus*, for discovering to *Bacchus*, and his thirsty Army in the Desarts of *Africa*, a Fountain of Water; Or, as *Pherecides*, for transporting of *Phryxus* and *Helle* over the Sea;

flying from their Step-Mother *Ino*. *Schillerius* will have this Conſtellation Attributed to St. *Peter*; and *Schickardus* to *Abrahams* Ram offered in the Room of *Iſaac*. This *Sign* was firſt diſcovered by *Cleoſtratus* and *Tenedian*; and it comes to the Meridian at Midnight, about the end of *October*, and beginning of *November*.

TAURUS.

THis Sign is called by the Ægyptians, *Jo*, *Iſis*, *Apis*, and *Orias* i. e. *Statio Hori*; by the Greeks and Latines, Ταῦρος and *Taurus*; in Hebrew, *Shôr*; by the Arabians, *Al Taur*; by the Syrians, *Thauro*; by the Perſians, *Ghau*; and by the Turks, *Ughuz*, *i. e.* Bos.—This *Conſtellation* conſiſts of 44 Stars, whereof 11 ſhapeleſs: Of theſe there is one of the firſt Magnitude, called by the Greeks λαμπαδίας; by *Ptolomy*, ὑπόνυξος; by the Arabs *Aldebaran*, i. e. *Stella Dominatrix*, and *Ain Al Thaur*, i. e. *Oculus Tauri*; by the Ægyptians, *Piorion*, i. e. *Statio, ſeu Dominium Hori*; in regard of the Power of the Sun in Conjunction with that Star; by the Romans *Palilicium*, becauſe heretofore it roſe at *Rome* on the Feaſt Day of *Pales*—It was Tranſlated into Heaven in Memory of the Raper of *Europe* by *Jupiter* in that ſhape; or in Honour of *Jo*, or *Iſis* Transformed by *Juno* into a *Cow*, and *Conſtellated* by *Jupiter*. Others will have it to be the Symbol of *Oſyris* or *Meſoris Mizraim*, the Son of *Cham*, who firſt taught the Ægyptians *Tillage*; or rather, of the Patriarch *Joſeph*, for his preſerving *Ægypt* in the time of Famine. *Schillerius* will have this Conſtellation Attributed to St. *Andrew*; but *Harſdoſius*, to the *Offering* of *Burnt-Sacrifice* commanded, *Levit.* 1. 3. About the end of *November*, and beginning of *December*, this *Conſtellation* may be ſeen upon the Meridian at Midnight.

GEMINI.

THoſe are called by the Greeks and Latines Δίδυμα, *Tindaridæ* and *Dioſcuri*; in the Coptick, *Cluſos*; i. e. *Cluſtrum Hori*; in the Hebrew *Termin*; in Syriack, *Tôme*; in Arabick, *Taw Amân*, i. e. *Gemelli*; they are likewiſe by the Arabs called *Giauzu*; others derive it from *Gianz*, ſignifying a *Nut*; and therefore the Turks call this Sign Kûs *Siphetlu Burgi*, i. e. *Nucem, vel Nuces referens Signum*; by the Perſians *Ghirdegân* to the ſame ſenſe. There are reckoned in this *Conſtellation* 25 Stars, whereof 7 Informes: The Star in the Head of the Weſtern *Twin* is called by the Arabs *Ras al Tawum Almukeddem*, i. e. *Caput prioris Geminorum*; the other, *Ras al Tawum Munacchet*, i. e. *Caput poſterioris Geminorum*: The two Stars oppoſite one to the other in the *Feet* of the ſaid *Gemini* are by the Arabs called *al Hen'a*; *Varro* will have them to be *Apollo* and *Hercules*; others will have them to be *Triptolemus* and *Jaſon*; ſome *Amphion* and *Zethus*: But with greateſt probability they are conceived to be *Caſtor* and *Pollux*; Δίκωρα, i. e. *Trabalia*, being no other than *two Wooden Poſts ſet parallel one to another, and joyned together at each end by two other tranſverſe Beams*. Hence, ſaith the Learned *Palmerius*; Aſtrologers make uſe of the like *Figure* or *Character*, to denote this *Twyn Sign*; which they derived from

from the *Lacedemonians*—— *Schillerius* will have this *Conſtellation* to ſignifie St. *James* the Elder; but *Schickardus*, *Jacob* and *Eſau*—It comes to the Mid-Heaven at Midnight, in the end of *December* and beginning of *January*.

CANCER.

THis Sign is in the Greek called Καρκῖνος and Ὀκτανυς, i. e. *Oſtipes*, it is likewiſe called *Nepa Aſtracus*, *Camarus*; in Arabick, *Alſertem*; in Hebrew *Sartan*; in Syriack *Sartóno*; in Perſian *Chercjengh*; by the Turks *Lenkutch* or *Lenkitch*; and *Tenkutch*, or *Tenkitch*, and *Tilenkutch*, or *Tilenkitch*; i. e. *Cancer*; in the Coptick it is called *Klaria*, i. e. *Beſtia, ſeu ſtatio Typhonis*. The whole Conſtellation is made up of 13 Stairs, of which 4 ſhapeleſs. Amongſt which the firſt Star is called in Arabick *Malaph*, i. e. *Præſepe*, or the *Mainger*; in Greek φάτνη; it is likewiſe by the Arabs called *Al Nethra*; in Chaldie *Peſebre*; and it is a *Cloudy Star*; by *Galileus* diſcovered to conſiſt of 36 ſmaller ones—— This *Crab* was made a *Conſtellation* at the Intreaty of *Juno*, being kill'd by *Hercules*, for biting him by the Foot, when he encountred *Hydra*. The *Aſinegoes* with their *Mainger*, were *Conſtellated*, becauſe of the Fight with the Gyants, *Bacchus* and *Vulcan* charged upon *Aſſes*, who with their Brayings Frighted, and ſo put to Flight their Enemies. *Schilerus* will have this *Conſtellation* aſcribed to *St. John* the Evangeliſt.—— This *Sign* is Famous according to the *Chaldaick* and *Platonick* Phyloſophy, for being ſuppoſed the *Gate* by which *Souls* deſcended into Humane Bodies. This *Conſtellation* may be ſeen about Midnight, almoſt all the whole Month of *January*.

LEO.

THis *Sign* is by the *Greeks* called λέων; in Hebrew *Ar'je*; by the Arabs *Al Aſad*; in Syriac *Ar'yo*; in the Perſian *Shir*; the Turks call it *Arſlan* or *Arſlin*, i. e. *Leo*; the Ægyptian Copties call it *Pimentekeon*, i. e. *Cubitus Nili*. The Conſtellation conſiſts of 39 Stars, whereof 8 inform. The firſt Star in this Conſtellation is by the Arabs called *Minchir al Aſad*, i. e. *Nares Leonis*; The third *Ras al Aſad, al Chemali*, i. e. *Caput Leonis Boreale*: The fourth *Ras al Aſad, al Giennubi*, i. e. *Caput Leonis Auſtralis*: The fifth, ſixth and ſeventh Stars, are called by them *Gieb'ha*, i. e. *Frons*: The eighth they call *Melichi*, to which the Latine *Regulus* Anſwers. They give it likewiſe the Name of *Kalb al Aſad*, i. e. *Cor Leonis*, being a Star of the firſt Magnitude. The ſeven and twentieth Star they call *Serpha*, i. e. *Mutatrix*, from the Change it brings of Heat from Cold; and *Danab al Aſad*, i. e. *Cauda Leonis*, and is likewiſe of the firſt Magnitude. The *Lion* was made a *Cœleſtial Sign* by *Juno* to ſpite *Hercules*, by whom he was ſlain, and is ſaid to have been bred in the *Moon*, and from thence to have fallen neat the *Nemæan* Grove in *Arcadia*, from whence called *Nemæus*. *Schillerius* will have this Conſtellation attributed to St. *Thomas*, but *Schickardus* to *the Lion of the Tribe of Juda*. This Conſtellation will be ſeen in the Meridian at Midnight in the Month of *February*.

VIRGO.

VIRGO.

THis *Conſtellation* in Greek bears the Name of παρθένος, to which the Latine *Virgo* Anſwers. In Hebrew it is called *Bethula*; in Syriac *Bthulto*; in Arabic *Adra* and *Adrenedepha*; and in the Perſian *Duſhiz á Pakiza*; all to the ſame ſenſe with the former. In the Ægyptian or Coptick it is called *Aſpholia*, i. e. *Statio Amoris*. It is likewiſe, in reſpect of the chief Star by which it is ſignified, being one of the firſt Magnitude in her Left Hand, called by the Hebrews *Shibboleth*, by the Syrians *Sh volto*; by the Arabs *Súmbela*; by the Perſians *Chûſhe*; and by the Turks *Salkim*; all ſignifying a Spike or Ear of Corn. In this Conſtellation *Ptolomy* reckons 32 Stars, of which 6 inform. The ſixth and ſeventh Stars in this Conſtellation are by the Arabs called *Min al Auwa, de latratore*; and ſo likewiſe the Tenth: The ſeventh is by them called *Zawija al Auwa*, i. e. *Angulus Latratoris*: The Thirteenth, which is alſo one of the firſt Magnitude in her Right Wing, called by *Proclus Prævindemiator*, is by the Arabs called *Mukdim al Ketaph*: The Fourteenth, which is the *Spica*, *Simak al A'zal*, i. e. *Efferens Inermem, Scil. Virginem*; to diſtinguiſh it from another Star in *Bootes* called *Simâk al Ramih*, i. e. *Efferens Haſtiferum*. The 22d, 23d, 24th and 25th Stars are called *Min al Gaph'r*, i. e. *Ex al Gaphr*, which ſignifies *Velamen, Ventrem & Tecturam, quod Stellæ ejus obtectæ ſint*. This Sign, according to the Vulgar Opinion, is taken for *Aſtræa*, or *Juſtice*; by others for *Erigone* Daughter of *Icarius*; others ſuppoſe her to be *Ceres, Quod ſpicas teneat*; others call her *Atergatis* the Goddeſs of the *Aſſyrians*; ſome will have her to be *Fortune*. *Avienus* makes her to be *Iſis*; and others again will have her to be *Concord* or *Peace*. *Schillerius* attributes her to St. *James* the Younger; but *Schickardus* will have it the Virgin *Mary*. This Conſtellation viſits the Meridian at Midnight about the end of *March*, and beginning of *April*.

LIBRA.

THis *Aſteriſm* is called by the Greeks ζυγὸς, σταθμὸς, ταλὰς ἢ Ζυγὸς; to which the Latine *Libra* Anſwers. By *Cicero* it is called *Jugum*, anſwering to the laſt of the Greek Names. In Hebrew it is called *Mozenáim*; in Syriack *Maſátho*; in Arabick *Al Mizan*; by the Perſians *Terazu*; all ſignifying *Libram, Stateram, ſeu Balancem*. The Turks uſe the Arabick Name *Mizan*, which in their Language is *Tartagick alati*, i. e. *Ponderandi Inſtrumentum*. In the Coptick it is called *Lambadia*, i. e. *Statio Propitiationis*. This Conſtellation is made up of Seventeen Stars, whereof Nine inform; among which, the firſt Star by the Greeks called Νότιος; is by the Arabs called *Zubâna Gienubi*, i. e. *Chela Auſtralis*, and *Al Kiſſa, Al Giennbija*, i. e. *Lanx Auſtralis*; The third Star called by the Greeks χηλὴ Βόρειος, is accordingly called by the Arabs *Zubâna Shemâli*, i. e. *Chilo Borealis*, and *Al Kiſſa al Shemâlija*, i. e. *Lanx Borealis*. There is no diſtinct Fa-

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ble of this Sign, it being part of *Scorpius*. Yet *Schillerus* attributes it to St. *Philip* and *Hartsdorfius*, to the *Tekel* or *Ballance* of *Belſhaſar*, Dan. 5. 27. This *Aſteriſm* mounts the Meridian at Midnight about the beginning of *May*.

SCORPIO.

THE Hebrews call this Sign *Akrab pro Akatzrab à magno oculo*: The Syrians call it *Akrevo*; The Arabs *Al Akrab*; The Perſians *Ghezdum*; The Turks *Koirughi*, *quaſi* Κατ' ἐξοχὴν, *Caudatus*, or *Uzun Koirughi*, i. e. *Longa Cauda præditus*; By *Cicero* it is called *Nepo*, which *Feſtus* ſays is an African Word; By the Greeks Σκόρπιος; and by the Ægyptians *Copties Iſias*, i. e. *Statio Iſidis*.—There are counted therein 24 Stars, whereof 3 Shapeleſs: The 6 firſt are by the Arabs called *Iclil al Gieb'ha*, i. e. *Corona Frontis*: The 6th is particularly called *Gieb'ha al Akrab Frons Scorpii*: The 8th is of them called *Kalb' al Akrab*, i. e. *Scorpii*, &c.——The *Scorpion* is Fabled to have been made a Conſtellation for having ſlain *Orion*, who boaſting he would in Hunting deſtroy all the Wild Beaſts in the Forreſt; Or, according to *Nigidius*, For that Hunting with *Diana* in the Mountain *Chillippius* in the Iſland *Chios*, he contemned and derided Her as inferior to him in Skill; Or, according to *Palæphatus* and *Nicandar* in *Theriac*, for daring to have violated her Chaſtity; for which, in Revenge, ſhe is ſaid to have ſent this *Scorpion* to Sting him to Death, as he was, for which, at her Requeſt, by *Jupiter* he was made an Aſteriſm, (*Schickardus* aſcribes it to St. *Bartholomew*) and is ſeen to crawl towards the Meridian of Midnight about the end of *May*, and beginning of *June*.

SAGITTARIUS.

THis Conſtellation is in Hebrew called *Keſheth*; In Syriack *Keſhto*; in Arabick *Alkaus*; in the Perſian Tongue *Kaman*; in the Turkiſh *Yai*; all ſignifying an *Arrow*: In Greek τόξον according to which ſignification it is likewiſe by the Arabs called *Rami*; by the Ægyptians *Pimaere*, i. e. *Statio Amœnitatis*. — It conſiſts of 31 Stars, of which the firſt is by ſome of the Arabians called *Zugo al Nuſhaba*, i. e. *Cuſpis, vel Ferramentum Spiculi*: The 8th Star in this Conſtellation is by the Arabs called *Ain al Rami*, i. e. *Oculus Sagittarii*: The 23d *Urkub al Rami*, i. e. *Suffrago*, the Hough or Poſtern: The 24th *Rukba al Rami*, i. e. *Genu*, the Knee of *Sagittarius*——*Hyginus*, from the Authority of *Soſitheus*, will have this to be *Crotus*, the Son of *Euphemis*, or *Euthemis*, the Nurſe of the Muſes, at their Inſtance, by *Jupiter*, placed in the *Zodiack*. Others will have him to be *Chiron*. This Sign, at Midnight, will be upon the Meridian about the end of *June*, and beginning of *July*.

CAPRICORN.

THE Greeks give to this Sign the Name Αιγόκερως, ἢ Αἰγοπαν; The Latines *Hircus Æquoris*: In Hebrew it is called *Gedi*; in Syriac *Gadio*; in Arabick *Al Giedi*; in the Persian *Buzeghale*; in Turkish *Oglack*; all signifying a Kid or Goat. In the Coptick or Ægyptian Tongue it is called *Hopeutus*, i. e. *Brachium Sacrificii*. It is made up (by the general assent of Astronomers) of 29 Stars, of which the first and third are by the Arabs called *Min Sad al Dabih*, i. e. *Ex fortuna Mactantis*; and simply, *Dabigh*, i. e. *Mactans*. The 23d and 24th Stars are called by them *Sad Nashira*, i. e. *Fortuna averruncantis, vel divulgantis Nuncium*; But the 24th, by a particular Name from its situation, is called *Danab al Gedi*, i. e. *Cauda Capricorni*.---This was made a Constellation in honour of *Ægipan* the Son of *Jupiter*, by the *Olenian Goat*, or rather his Foster-Brother, Son of *Æga*, the Wife of *Pan*, whence his Name; who as *Bassus*, in *Germanic*, from the Authority of *Epimenides* writes, assisted *Jupiter* in his Wars against the *Titans*, and Armed the Gods, and for that reason honoured with this Cœlestial Dignity. The reason of his being Figured half Goat half Fish, *Theon*, the Scholast of *Aratus* reports, was, That he finding on the Sea shore an empty *Murex*, or Purple Shell, is said to have wound it like a Horn, thereby striking a Panick fear into the *Titans*, and therefore they represented him with a Tail like a Sea Monster. Celebrated it is, according to the Doctrine of the *Pythagoreans* and *Platonists*, for being the Gate by which Souls ascend into Heaven, and therefore stiled *Porta Deorum*. *Schillerus* ascribes this Asterism to S. *Simon*: It ascends the Mid-heaven at Midnight about the end of *July*, and beginning of *August*.

AQUARIUS.

THis Sign is by the Greeks called ὑδροχοος, by *Appian*, *Hydriaurus*; and in the same signification by the Arabs, *Sakib al Mâ*, i. e. *Effusor Aquæ*. It is by them likewise called *Al Delu*; and in Hebrew *Deli*; in Syriac *Daulo*; in the Persian Tongue *Dul*; in the Turkish *Kugha*; all signifying an Urn, or Watering-Pot. The Ægyptians or Copties call it *Hupeutheron*, i. e. *Brachium Beneficii*. There are reckoned therein, according to *Ptolomy* and *Kepler*, 45 Stars, whereof 3 inform. Of these Stars, the 2d and 3d are called in Arabick *Sa'd al Melick*, or *Sa'd al Mulck*, the first signifying *Fortuna Regis*, the latter *Fortuna Opum & Substantiæ*. The 4th and 5th are called *Sa'd al Sund*, i. e. *Fortuna fortunarum*; under which are some other Stars of less Note, called *Al Ana*. The 6th and 7th are called *Sa'd Bula & al Bulaan*, i. e. *Fortuna Delutientis*, or *Delutientium*. The 9th, 10th, and 11th Stars are called *Sa'd al Abbija*, i. e. *Fortuna Tentoriorum*: The 14th Star in this Constellation being one of the first Magnitude, is

β in Arabick called *Diphda al Auwal*, i. e. *Rana prima*. It is likewise called *Phom al Haut al Gienubi*, i. e. *Os Piscis Australis*, and commonly, but corruptly, *Phomahant*. This Asterism is by some Fabled to be *Ganymede*, the Cup-bearer of *Jupiter*; by some *Deucaleon*: (whence by *Vomanus* this Sign is Intituled *Deucalionis Aquæ*) By others *Aristæus*. By *Schillerus* this Sign is attributed to St. *Jude*, but by *Schickardus* to *Naaman*, 1 Reg. 25. 14.——It is seen in the Meridian at Midnight, about the end of *August* and beginning of *September*.

PISCES.

THis Sign in the Greek is called Ἰχθύων ἀπειρωμὸς, and by the Jews accordingly, *Dagaim*, i. e. *Duo Pisces*; but the Arabs call it *Al Haut, & al Samaca*; the Syrians *Nano*; the Persians *Mahi*; the Turks *Balick*; which signifies a *Fish* in the singular Number; so likewise in the Coptick it is called *Pikotoron*, i. e. *Piscis Hori*. The *Northern* of these *Fishes* is in the Arabick called *Haut Alshemali*, i. e. *Piscis Borealis*, and is known by the peculiar Name of Χαλδαϊκὸς, as being represented by the Chaldeans with the Head of a *Swallow*; the reason, as *Scaliger* conceives, because when the Sun is in this Sign, the Swallow begins to appear in those Regions: The *Southern* is called *Haut al Gienubi*, i. e. *Piscis Australis*. The whole Constellation consists, according to *Ptolomy*, of 38 Stars, whereof 4 inform. The *Stream*, or *Tenuis fusio Stellarum utriusque Piscibus disposita*; *Vitruvius* calls *Mercurii Domum seu Delicias*, which *Scaliger* conceives ought to be read *Lagneam*; or as *Pliny* terms it, *Commissuram Piscium*. The Arabians call it *Cheit, vel Cheit Kottani*, i. e. *Filum Linteum*.——These are Fabled to be the *Syrian* Deities, according to *Germanicus*, *Syria duo Numina Pisces*; by which are understood *Venus* and *Cupid*, as *Hyginus* (from the Authority of *Diognetus Erythræus*) writes. For *Venus*, and her Son *Cupid*, coming to the River *Euphrates*, and frighted at the sudden appearance of the Giant *Typhon*, cast themselves into the River, and assumed the shapes of Fishes, by which means they escaped from danger: For this reason the Syrians abstain from Eating of Fish, lest they might happen to Eat up their Deities: But the Scholiast of *Germanicus* (from *Nigidius*) writes, That these were the Fishes which turn'd or roll'd up upon the Bank of *Euphrates* a great Egg, upon which a Dove sitting, Hatch'd *Venus*, the Syrian Goddess. *Schillerus* will have these attributed to St. *Matthias*; but *Schickardus* to the two Fishes in the Gospel, *John* 6. 9.——This Asterism may be seen in the Meridian at Midnight, almost the whole Months of *September* and *October*.

II. Of the Ancient Northern Constellations.

HELICE.

So Named by the *Greeks*; it is also called *Ursa Major*, &c. *Plaustrum Majus*; in Arabick *Dub Ackber*; it is likewise by the *Arabs* called *Benât al Nosh al Cubra*, i. e. *Filiæ feretri Majoris*, in regard the 4 Stars that make the *Body* of the *Bear* resemble a *Bier*, and the Three in the *Tail*, the *Virgins* or *Maids* that Attend the *Corps*: And for this Reason, the Christian *Arabs* call the four Stars *Nash Laâzar*, i. e. *Feretrum Lazari*, and the Three in the Tail, *Mary Magdalen*, *Martha* and their Maid; by the Persians it is called *Haphtûrengh Mihîn*, i. e. *Septentrio Major*; and by the Turks, *Yidigher Yilduz*, i. e. *Septenæ stellæ*, and by the Latins, *Septem Triones*. This Constellation consists of 35 Stars, whereof 8 inform. Of these Stars the 12 and 13 are called in Arabick *al Nekra al Thâlitha*, i. e. *Cotyle, Scrobs seu Cavitas ossis Tali*; the 16 *Dubr al Dub al Ackber*, i. e. *Dorsum Ursi Majoris*; the 17 *Merâk al Dub al Ackber*, i. e. *Epigastrium Ursi Majoris*; the 18 *Mag'res al Dub al Ackber*, i. e. *Uroxygium Ursi Majoris*; the 19 *Phaid al Dub al Ackber*, i. e. *Femur Ursi Majoris*; and these four last Named make up *al Na'sh al Cubra Feretrum Majus*. The 20 and 21 are called *al Phikra* or rather *al Nekra*, *al Thanija*, i. e. *Vertebra seu Cotyle secunda*; the 23 and 24 *al Nekra al Ula*, i. e. *Vertebra seu Cotyle prima*. The three Stars that make the *Tail* are called, *al Benât*, i. e. *Filiæ*; whereof the first is called *al Haun*, signifying *album Nubiculum*, the second is called *al Anâk*, or *al Anâk*, i. e. *Capella*; the third *Alkaid*, i. e. *Gubernator*. This *Constellation* was first found out by *Naupilus*, and was Anciently the *Greek* Sea-Mens Guide; as the Lesser the *Phœnicians*: The Reason is, because to the *Greeks*, who Sailed the *Mediteranean*, *Pontick* and *Euxine* Seas, this *Constellation* was still apparent: But to the *Sydonians*, *Phœnicians* and *Carthagenians*, who were more Southerly, part of the *Greater* Bear was either by the Position of the Sphere, or some other Accident, sometimes depress't, or obscured; but *Cynosura* always appeared to them: And therefore, these last chose the *Lesser*, and the *Greeks* the *Greater Bear* for their *Directress*.

Schillerus Attributes this Constellation to St. *Michael*; but *Harsdorfius* to one of *Elisa's Bears*, 2 *Reg.* 2. 24. Or the Wagon of *Jacob*, or Chariot of *Joseph*, *Gen.* 45. 27, &c. 46. 29.

CYNOSURA.

So called by the Greeks, *quasi Canis Cauda*; the Hebrews call it *Genash*, i. e. *Gallinam cum Filiis suis*; by the *Arabs* it is called *Dub Asgher*, i. e. *Ursus Minor*, and *Benat al Nash al Sughra*, i. e. *Filiæ Feretri Minoris*; by some of them it is called *Agiala*, i. e. *Plaustrum*; by the Persians it is called *Haphturengh Kihn*, i. e. *Septentrio Minor*; the *Star* in the extremity of the *Tail*, is called by the Arabs *Cancah Shemali*;

mali; i. e. *Stella Borealis*; by the Turks, *Tilduz Shemali*; and absolutely *Tilduz*, i. e. *Stella*; and by a peculiar Name in Arabick it is called *Giedi*, i. e. *Hædus*. The Italians call it *Tramontana*; and we, the *Pole*, or *North Star*. The two last and brightest in the *Feretrum*, or *Square*, are called by the Arabs *al Phercadan* or *al Pharcadein*, i.e. *Duo Vituli*. The whole *Constellation* consisting (according to *Ptolomy*) of Eight Stars, whereof one inform. Of the Fabulous *Anastrosis* of this and the former *Constellation*, *Diadorus Siculus* Reports, that these were the *Nurses* of *Jupiter*, and privately kept him from the search of *Saturn*, for which they were by him, in gratitude placed in the Heavens, and called by the Name of the two *Bears*, being Worshipped with Divine Rights, by the *Cretians* and *Sicilians*, by whom they were Styled *Dea Matres*. Others refer it to the Fable of *Calisto* and her Son *Arcas*; as *Hesiod* and *Ovid*. This *Constellation* was first discovered by *Thales* the *Milesian*, as *Hyginus* affirms, for which Reason it was called likewise *Phœnice*, from *Thales* its Inventor, being by descent a *Phœnician*, who first gave it the Name of *Arctus* or the *Bear*; but trulier so denominated from the whole Nation of the *Phœnicians*, who in their Navigations (before *Thales*) observed her as their *Directress*.

DRACO.

THis *Constellation* the Poets feign to have been the *Dragon* that kept the *Hesperides* slain by *Hercules*, and made an *Asterism* by *Juno*. Others will have the *Dragon* to be brought by the Giants, in their Flight with the Gods to oppose *Minerva*, and by her to have been strangled and thrown up to Heaven, and there fix'd as a Trophy of her Victory. *Schillerus* Attributes this *Asterism* to the *H. Innocents*; but *Schickardus* to *Draco Infernus*.—— It is called by the Greeks Δρακων ὁ Αστερισμὸς, by the Latines *Draco*; in Hebrew *Tannin*, i. e. *Draco*; by the Arabs *Tinnin* and *Tannin*, as the *Hebrew*; it is likewise by them likewise called *Taaban*, or rather *Thuban*, and in the same sense by the Persians *Asbdeha*, which is interpreted *Serpens, qui Homines ac Bestias devorat*. Some among the Arabians give it the Name of *al Haija*, which is also appropriate to the Southern *Constellation* of the same kind. It is made up of 31 Stars; of which, the first Star in the Tongue is by the Arabs called *al Rakis*, or *Arrakis*, i. e. *Saltator, seu Tripudiator*; the three next *al Awaid*, i. e. *Pulsatores Testudinis*. The 5th in the Head is called *Ras al Tinnin*, i. e. *Caput Draconis*. The 14, 15 and 16th are called *al Thâphi*, i. e. *Chytropodes*, from their Posture, representing a Skillet with Feet, Tripon or Brandiron. The 20 and 21 are called *Adphar al Dib*, i. e. *Ungula Lupi*. The 27 is called *Aldibeh*, i. e. *Victima*, as being placed before that in the Horn of *Capricorn* called *Sa'd al Dabih*, i. e. *Fortuna mactantis*. This *Constellation* is seen in the Meridian about the end of *June*.

ENGONASI.

THis *Aſterism* in Greek bears the Name of Ἐγγονασι ὁ Ὀκλαζων, i. e. *Ingeniculus*; and by ſome of the Latines *Niſus, vel Nixus, quia Laboranti ſimilis*; by the Arabs *Giathi ala Rucbeteihi*, i. e. *Incumbens Genibus.* The Number of Stars in this *Conſtellation* are by *Ptolomy* reckoned to be 29, of which the firſt is called *Ras al Giathi*, i. e. *Caput Ingeniculi*, and commonly *Aas al Aben.* That in the *Elbow* from its ſcituation, is called *Marphak*, that in his *Wriſt Mis'am*, i. e. *Carpus*, and commonly *Maaſym.*——This *Conſtellation* ſome will have to repreſent *Theſeus* or *Ixion*; others *Orpheus* or *Promotheus*; others *Thameris* or *Thamyras* a *Thraſian* Poet; who contending with the *Muſes* for Skill, and by them overcome, was puniſhed with the loſs of his Eyes; and in the Memorial of their Victory, plac'd in the Heavens in a ſupplicatory Poſture, as deprecating his Puniſhment: But *Panyaſes* will have this *Aſterism* to repreſent *Hercules.* This *Conſtellation* comes to the Meridian at Midnight in the Month of *June.*

ARCTOPHYLAX or BOOTES.

Arctophylax and *Bootes* are one and the ſame *Conſtellation*; the Firſt ſignifying *Cuſtos Urſorum*, the Latter ſo called ἀπὸ τῆς Βόος, i. e. *Bovis*, ἐλαύνειν, i. e. *pellere, quaſi Boum Agitator*; but in the Eaſtern Tongues the ſame ſeems to be deriv'd ἀπὸ τῆς Βοαῆς, i. e. *a Clamando*, whence by the Arabs called *al Auwa*, i. e. *Vociferator*, and *al Neckar*, i. e. *Foſſor ſeu Paſtinator.*——It conſiſts of 23 Stars.—— This ſome Fable to be *Lycaon*, others *Arcas*, the Son of *Caliſto* his Daughter by *Jupiter*; the Scholiaſt of *Germanicus* makes it to be the *Conſtellation* of *Icarus*; and accordingly *Propertius* ſtiles the *Septentriones*, *Icarus* his Oxen, in this Verſe,

Flectum Icarii Sidera tarda Boves.

♓

Which *Conſtellation* is ſeen upon the Meridian at Midnight about the beginning of *May.*

ARCTURUS.

SOme will have this Star ſo called, *quaſi ab* οὐρὰ Ἄρκτου i. e. *a Cauda Urſæ*; but truſier, *ad* οὐρὸς Ἄρκτου, i. e. *Cuſtos Urſæ*, in the ſame ſenſe as *Arctophylax.* This the Arabs call *al Simak al Ramih*, i. e. *Efferens Haſtiferum*; in the common Globes and Hemiſpheres falſly *Huzme.*

ARIADNES CROWN.

THis by the Greeks is called στέφανος βορεῖος, i. e. *Corona Borealis*, & *Prima*; and accordingly by the Arabs, *al Iclil Almani*, i. e. *Corona Borealis*, and ſimply *Al Iclil*, i. e. *Corona*: It is by them likewiſe called *al Phecca*, i. e. *Apertio*; in Hebrew *Kir Schetali*, i. e. *Corona Siniſtra*; and in Chaldee, *Melphelcarti*, i. e. *Sertum Pupillæ.* The Conſtellation is in Form of a Circle not compleated, and therefore by

by the Vulgar Arabs called *Káse Shekeſte*, i. e. *Scutella fracta*: The brighteſt in this Circle being of the ſecond Magnitude, is called *Lucida Corona*, and by the Arabs, *Nair Phecca*, i. e. *Lucida Phecca*. This *Crown*, conſiſting of Eight Stars, ſome Fable to have been of Gold: *Athenæus* ſays it was made of a Flower or Herb called *Theſeus*; others will have it to be of *Lawrel* or *Myrtle*; *Bayerus* deſcribes it to have been compoſed of *Elder Leaves* mixed with *Berries*; *Photius* gives this *Fable* of it: They report (ſays he) that a certain Nymph Named *Pſalacantka* in the Iſland of *Icaria* being in Love with *Baccus*, endeavoured to procure *Ariadne* to his Bed; on condition he would likewiſe be kind to her; which *Baccus* refuſing, ſhe plotted to do *Ariadne* a miſchief: This the God diſcovering, he in paſſion Transform'd her to an *Herb* bearing her Name: But afterwards, repenting the Fact, by way of Recompence and Honour, he caus'd the *Flower* to be entwin'd about *Ariadnes Crown*, which he had already fix'd in the Skie. Now moſt make only *Ariadnes Crown* to be *Conſtellated*, yet others place *Ariadne* her ſelf in Heaven, as *Scaliger* hath obſerved; and for proof whereof they add the Teſtimony of one of *Nero's*, and another of *Trajan's* Silver Coins, having on their Reverſe, the Figure of *Ariadne* carried up to Heaven in the ſame manner as is repreſented by *Propertius*. *Schillerius* will have it our Saviours Crown of Thorns, but *Harſderſius* will have it to be Queen *Heſters* Crown, *Heſt.* 2. 17.

ORPHEUS His LYRE.

THis *Conſtellation* is by *Ptolomy* called λύρα Ἀσίεσμος, and by *Aratus* λύρα καπωρεφὴς, i. e. *Lyra, deorſum pendens*; it is likewiſe by the Greeks called χέλυς, i. e. *Teſtudo*; and by the Latines *Lira* and *Fidicula*, and *Vultus cadens*; for it is repreſented in form of an *Eagle* or *Vultur*, holding a *Lyra* Inverſt. By the Arabs it is called *al Lura*, from whence comes the corrupt Name *Alahore* peculiarly Attributed to the firſt great Star in this *Aſteriſm*: It is likewiſe called *Sulaphat*, i. e. *Teſtudo*; the Perſians call it *Ciengh Rumi*, i. e. *Cythara Greca*: It conſiſts of 11 Stars, whereof the firſt is called by the Arabs *Nes'r Waki*, i. e. *Vultur Cadens*: Some Attribute this *Conſtellation* to the *Lyre* of *Apollo*, others to *Orpheus*, to whom *Mercury*, who firſt invented it, bequeathed it, and it would refuſe to ſound at the touch of the Hand of any other Artiſt. *Schillerus* will have it repreſent the Manger whereon our Saviour was laid, and *Harſdanſius*, *Davids Harp*, 1 *Sam.* 16. 23. It is ſeen in the Meridian at Midnight in the end of *July*, and partly at the beginning, partly at the End of the Year.

OPHIUCUS, Or SERPENTARIUS.

THis *Sign* by the Greeks called Ὀφιοῦχος, and by the Latines *Serpentarius*; is by the Arabs called *al Hauwa*, i.e. one that keeps and Nouriſhes *Snakes*: It is commonly called *al Hangue*; by the Jews it is called *Utzerath Hajah*, i. e. *Tenens Serpentem*: It conſiſts of 29 Stars, whereof four inform. The firſt Star is called *al Rai*, i. e. *Paſtor*, and *Ras al Hauwa*, i. e. *Caput Serpentarii*; the ſecond is called *Kelb al Rai*, i. e. *Canis Paſtoris*. In this *Conſtellation* the Ninth Star is more Eminent than the reſt, and is called

called *Unuk al Haiya*, i. e. *Collum Serpentis.*——*Æsculapius* is said to have been converted into this Sign for his rare Skill in Physick, and particularly for the Cures by him done, by help of an *Herb* shewn him by a *Serpent*. *Polizelus Rhodius* Reports, that this *Ophiucus* was *Phorbus* Prince of *Rhodes*, by *Hytilla* the Daughter of *Myrmidon*, who, when that Island was extreamly infested with *Serpents*, and especially by a great *Dragon*, which devoured many of the Inhabitants, is said to have clear'd the Island of the Venemous Beasts, and to have slain the *Dragon*, and in Memorial thereof to have been Constellated by *Apollo*. *Kepler* will have this *Constellation* to represent *Laocoon* the Trojan mentioned by *Virgil*. This *Constellation* reaches the *Meridian* at Midnight about the Beginning of *June*; the former part of the *Serpent* about the beginning of *May*; the hinder part about the end of *June*.

The *SWAN.*

THis *Sign* is called by the Greeks Κύκνος, by *Ptolomy* Ὄρνις, i. e. *Volucris*; by *Ovid Milvius*, and by others of the Latines *Gallina*; according to which last it is called by the Arabs *al Dediagie*; to which Answers the Hebrew Name *Tharnigoleth*. It is likewise by the Arabs called *al Tair*, i. e. *Volucris*, and *Katha*, which is properly an *Aquatick Fowl* resembling a *Pigeon*. The Persians call it *Isspherud*; and the Turks *Baghirtlik*; *Ptolomy* reckons in this *Constellation* 19 Stars, whereof two inform. The first of which is by the Arabs called *Minkar al Degjagic*, i. e. *Rostrum Gallinæ*; the 4th *Sadr al Degjagic*, i, e. *Pectus Gallinæ*; the 5th *Danab al Degjagic*, i. e. *Cauda Gallinæ*; and *Hiersym*, i. e. *Rosa, aut Lilium redolens*, as also, *al Rid'ph*, i. e. *quæ pone est seu sequens*; because it follows four others, whereof two in the Left *Foot*, and two others in the Left *Wing*. The *Fable* of this *Asterism* is, by *Theon*, who makes this *Swan* to be placed in the Heavens not in Memorial of *Jupiter*, but in Honour of *Apollo*, as particularly Dedicated to him, being a Musical Fowl. It is upon the *Meridian* at Midnight in the Month of *July*.

The *ARROW.*

IT is by *Ptolomy* called Οἰστὸς Ἀστερισμὸς, i. e. *Asterismus Sagittæ seu Teli*; in Arabick it is called *al Sah'm*. i. e. *Sagitta*, and in the Globes *al Hance*; in the Hebrew *Chetz*: *Kircher* says it is in Hebrew Named *Satan seu Demon*; and that the Turks call it *Orfertalem*: It consists of 5 Stars. This was the *Arrow*, with which *Hercules* slew the *Vulture* that Fed upon *Promethius* his Liver: And *Promethius* being received into favour with *Jupiter*, the *Arrow* in Memorial was made an *Asterism* in Heaven; by *Cicero* called *Musator*, by others *Temo Meridianus*: It Transperses one of the *Wings* of the *Eagle*; and passes the Meridian at Midnight about the Middle of *July*.

The *EAGLE.*

IN the Greek this *Constellation* is called Διὸς Ὄρνις, i. e. *Aquila, & Jovis Ales*; it is likewise called in Arabick *Okab*, i. e. *Aquila Nigra*; by *Gesner* called *Leporaria*, which in the Persian Tongue is called *Alak*; in Turkish, *Tausbangjil*, i. e. *Aquila Leporaria*; in Hebrew it is called *Nescher*

Neſcher, which ſignifies an *Eagle*: It conſiſts of 9 Stars, of which the third is called by the Arabs *al Nes'r al Tair*, i. e. *Vultur volens*; it is alſo called *Lucida Aquila*; the 9th Star is called *Danab Alakab*, i. e. *Cauda Aquilæ*.——This *Eagle* (according to the Poets) was Conſtellated by *Jupiter* for the Rape of *Ganymed*; or (according to *Agloaſthenes*) for that *Jupiter* going from *Naxus*, to War againſt the *Titans*, an *Eagle*, as he was Sacrificing, appeared to him in a happy Auſpice; whereupon, after his *Victory*, he took that *Fowl* into his particular Patronage. But *Hero* gives us the *Fable* quite otherwiſe; for he Writes that the *Eagle* was *Conſtellated* by *Jupiter*, for ſuckling him when an Infant in *Crete*, with *Nectar*, which ſhe firſt Drank from a Rock, and then brought it to the God. About the middle of *July*, at Midnight it may be ſeen on the Meridian.

The *DOLPHIN.*

BY the Greek it is called Δελφῖνος Ἀστερισμος, i. e. *Aſteriſmus Delphinis*; by others ἱερὸς ἰχθὺς, i. e. *Piſces Sacer*; in Hebrew *Dagaim*, i. e. *Piſces Maris*; the Arabs call it *Dulphin*; in which the moſt Eminent Star is one in the Tail, called by the Arabs *Danab al Dulphin*, i. e. *Cauda Delphinis*. This *Aſteriſm* is by *Cicero* called *Cirrus*; and by *Pliny Hermippus*, and by ſome others *Rhomboides*. This *Conſtellation* is famous for the Number of Stars in it being equal to the Number of the *Muſes*; and alſo Famous; *Quod fuit Occultis fœlix in amoribus Index*; as being Inſtrumental to *Neptune* in his Amorous purſuit of *Amphitrite*, who fled, and conceal'd her ſelf from him: Nor is this *Aſteriſm* leſs Signal for the Fabulous Tranſport of *Arion*, in *Ovid. Faſt. l.* 2. It paſſes towards the Meridian at Midnight about the end of *July*.

PEGASUS.

THis *Conſtellation* is called *Pegaſus*; by the Greeks, Ἡμιτολὸς ἵππος, It is called likewiſe *Equus Major*, *Meduſæus*, *Gorgonius*, *Bellerophontæus* and *Menalippe*; by the Arabs *al Pharas Adam*, i. e. *Equus Major*, and *Alpharas al Thani*, i. e. *Equus Secundus*; in Hebrew it is call'd *Haſus chail Kerzim*, i. e. *Equus Cornutus*: It conſiſts of 20 Stars; amongſt which the chief, by the Arabs called *Sirra al Pharas*, i. e. *Umbilicus Equi*: That in the joyning of the Wing is called *Marchab*, i. e. *Equitandi vel vehendi Locus, Sagma, Ephippium*. Some Fable this to be *Bellerophons*, others *Perſeus* his Horſe: *Callimachus* and *Catullus* call him *Unigenam Memnonis*, Brother to *Memnon* and Son of *Aurora*; The Greek Commentators make him to have been preſented to *Aurora* by *Jupiter*: But *Lycophron* deſcribes him to be the *Winged Steed* of the *Morning*, upon which ſhe is ſaid to *Ride*. *Palupharus* and *Artemidorus* make *Pegaſus* to be a *Ship* and not a *Horſe*: And according to *Voſſius*, the Name *Pegaſus* ſeems to be derived a πήγνυμι i. e. *compingo, quia Navis e multis componitur lignis*. It is ſeen in the *Meridian* at Midnight about the middle of *Auguſt*, and beginning of *September*.

ANDRO-

ANDROMEDA.

THis *Asterism* is called by the Arabs *al Mara al Mosalsala*, i. e. *Mulier Catenata*; In Hebrew *Ishà Shalahajalu Baal*, i. e. *Fœmina carens viro*. It consists of 27 Stars; among which the 12th called by the Arabs *Giemh al Mosalsala*, i. e. *Latus Catenatæ*; that in her *Girdle* is called *Izar* and *Mizar*; that in the Border of her *Vest* they call *al Deil, vel Addeil*, i. e. *Syrma seu Lacinia Vestis*; the 19th is called *Rigil al Mosalsala*, i. e. *Pes Catenatæ*. This Constellation comes to the Meridian about the middle of *October*. As to the Fable of *Andromeda*, see after in the Whale.

PERSEUS.

THis *Constellation* is by the Arabs called *Chelub* or *Chelub*, i. e. *Deceptor*; and from the Greek Name *Perseus*, *Bersbaush* and *Bersheush*; it is likewise called by them *Himel Ras al Ghul*, i. e. *Portans caput Larvæ*: It consists of 29 Stars, of which 3 inform. The first is called *Misam al Thuraiya*, i. e. *Carpus Pleiadum*, and *al Giemh Bersbaush*, i. e. *Latus Persei*; the 12th is called *Ras al Ghul*, i. e. *Caput Larvæ*; by the Jews, *Rosh ha Satan*, i. e. *Caput Diaboli*, the 24th called *Menkib al Thuraiya*, i. e. *Interscapillium Pleiadum*. This *Perseus* was the Grand Child of *Acrisius* King of the *Argives* begotten by *Jupiter* on his Daughter *Danae*, placed in Heaven by favour of *Minerva*, for having slain *Medusa* or the *Gorgon*, and freed *Andromeda* from the devouring Sea Monster. This *Asterism* is on the Meridian at Midnight in the Month of *November*.

DELTOTON.

CAlled also *Trigones* and *Delta*; by the Latines *Triangulum*, and *Nili Donum*; by the Arabs; *Mothallath*, i. e. *Triangulum*; in Hebrew, *Himmoschlush*, i. e. *Tripartitus*. It consists of four Stars, that in the top of the Triangle is called in Arabick, *Ras Almothallath*, i. e. *Caput Trianguli*. This Constellation is said to have been placed in Heaven by *Mercury*, in Memorial of the first Letter of *Jupiters* Name, Διὸς, *Bassus* in *Germanicus*, and *Hyginus* Write, that *Mercury* at the Command of *Jupiter*, placed it over the Head of *Aries*, as a Mark the better to discern that Sign of it self.

——— *Obscuro lumine labens.*

As *Cicero* in *Aratæis*. Others will have it to be the Figure of that part of *Ægypt* Constellated, which *Nilus* after that manner encompasses. This Asterism at Midnight comes to the Meridian in *October*.

CEPHEUS.

CEPHEUS.

THE true Name of this Aſteriſm by the Arabs is *Keiphus*; In Hebrew it is called *Baalath Hala*, i. e. *Domina Flammæ*; and in Arabick *Multahab*, i. e. *Inflammatus*. This Conſtellation conſiſts of 13 Stars, whereof two inform: Among which there is one in the Foot called *Al Rai*, i. e. *Paſtor*; and between his Feet another called *Al Kelb*, i. e. *Canis*; and upon his Hands certain others called *Al Agh'nâm*, i. e. *Pecudes*. The 3d, 4th and 5th, are by *Ulugh Beigh* called *Cawakih al Thirk*, i. e. *Stella Gregis*.——This *Cepheus* was Son of *Belus*, by *Anchinoe* the Daughter of *Nilus*, from whence the Perſians were heretofore called Κηφῆνες, over whom he was King, as likewiſe of *Phœnicia*, and Reigned both in *Babylon* and *Joppa*, reckoned among the Royal Fautors of Aſtronomy. It is beheld in the Meridian at Midnight about the end of *Auguſt*, and beginning of *September*.

CASSIOPEA.

IT is likewiſe called by the Greeks γυνὴ τῆ θρόνε, i. e. *Mulier ſedis, ſive Throni*; by the Arabs *Dât al Curſa*, i. e. *Inthronata*. It is alſo known by the Latin Name of *Cathedra*, *Thronus & ſedes Regia*; it conſiſts of 13 Stars according to *Ptolomy*, but *Tycho* hath obſerved therein no leſs than 45, beſides the *New Star* which appear'd in the Year 1573, and vaniſhed the Year following. It is reſembled by *Aratus* to the form of a *Laconian* or a *Carian* Key.——The firſt Star in this Conſtellation is by the Arabs called *Caph al Chadib*, i. e. *Manus tincta*. The ſecond Star is called by the Name of the whole Conſtellation, *Dât al Curſa*. The fifth is called *Rucba Dât al Curſa*, i. e. *Genu Inthronata*. The bright one in the Breaſt is called *Sadr*, i. e. *Pectus*. This *Caſſiopea* was the Wife of *Cepheus*, and Mother of *Andromeda*, who contending for Beauty with the *Nereides*, was, as a Puniſhment, and in Memorial of her Arrogance, placed in Heaven with her Heels upwards. But *Tycho* gives us a better ground of the Fable, who writes, That *Cepheus* was a great Aſtronomer, or, at leaſt, a Favourer of the Profeſſors of that Science, who in a grateful acknowledgment of his Encouragement of their Studies, gave to ſeveral Conſtellations the Name of his *Wife*, *Daughter*, and *Son-in-Law*, which he received from *Cicero*. He likewiſe reports, That in the time of *Cepheus*, thoſe Stars which make the Conſtellation of *Caſſiopea* did riſe with the firſt degree of *Aries*; and that under that Conſtellation the *Æthiopeans* did ſolemnize the Inauguration of their ſucceeding Kings, in Memorial of their firſt Mother *Caſſiopea*, whom he ſuppoſes more probably to have been called *Caſſiepea*. This Aſteriſm is diſcovered in the Meridian, partly in the end of *March* and beginning of *May*; partly at the end of *September* and beginning of *October*.

CAPUT ALGOL:
OR,
MEDUSA's HEAD.

THE Fable concerning this Conſtellation is this. *Perſeus* coming into Mans Eſtate, and being furniſhed with Sword, Hat and Wings, of his Brother *Mercury*, was ſent by his Foſter Father *Polydectes* to kill the Monſter *Meduſa*, who was to have devoured *Andromeda*, who for the Pride of her Mother *Caſſiopea*, was bound with a Chain to a Rock by the Sea ſide for that purpoſe; but *Perſeus* pitying her caſe, undertook to fight with the Monſter upon condition that *Andromeda* might be his Wife: Whereupon he ſlew the Monſter, and Delivered and Married *Andromeda*, and returned home to the Iſle of *Seriphus* with the *Gorgons* Head. For which Exploit, *Jupiter* his Father took him up into Heaven, and there placed him in the form he overcame *Meduſa*, with his Sword in one Hand, the Head of *Meduſa* in the other, and the Wings of *Mercury* at his Heels. *Schillerius* attributes it to St. *Paul*; *Shickardus* to *David* with the Head of *Goliah*.

HENIOCHUS.

THis by the Greeks is called Ἱππολαμος, Ἐλάσσιππος, Ἀρμιλλάτης, καὶ Διφρηλάτης; by the Jews *Ha Roah ſcobidò Hareſan*, i. e. *Paſtor tenens frenum*; and in the ſame ſenſe by the Arabs, *Maſik al Inan*, i. e. *Tenens Habenam*, or *Mumſik al Amna*, i. e. *Tenens Habenas*, to which the Greek Name *Heuiochus* anſwers, *i. e. Habenifer*.——It conſiſts of 14 Stars, among which the 4th is called *Menkib Dil Inan*, i. e. *Humerus Heniochi*: The 11th *Ca'b Dil Inan*, i. e. *Talus Henioce*.——This Conſtellation the Scholiaſts of *Germanicus* will have to be *Mirtillus*: The *Trezenians* are for *Hippolitus*; others for *Ericthonius*, who *Pliny* makes the firſt that join'd Four Horſes in a Chariot. *Euſebius* makes *Trochilus* the *Argive*, who was the Son of *Callithea* the Prieſteſs of *Juno*, the firſt Inventer thereof: Others will have him to be *Oenomaus*. This Sign attains the Meridian at Midnight about the middle of *December*.

HÆDI, *or the* KIDS.

THeſe are two Stars in the Left Arm of *Heniochus*, called by the Arabs *Sadateni*, i. e. *Brachium ſequentes*. They are likewiſe called *Giedyân*, and *Maazein*, i. e. *Duo Capri*. Theſe *Cleoſtratus* the *Tenedian* is ſaid firſt to have diſcovered: They are ſaid both at their

Riſing

Rising and Setting to cause Storms and Tempests, and therefore by the Poets called *Horrida & insana Sydera.*

The AMATHÆAN GOAT.

THis is a bright Star in the Shoulder of *Heniochus*, of the first Magnitude, called by the Arabs *Aiyuk*, and commonly, instead thereof, *Atud.* In Hebrew, *Aſh* or *Aiſh*; in Syriack *Jyutho*; all signifying *Capellum.* This the Poets Fable to have been the Mother of the two *Kids*, and Nurse to *Jupiter*; tho others report him to have been Suckled by a *Sow*; the *Cretans*, for that cause, honouring that Creature as Sacred. But the more General Opinion is, That he was Suckled by a Goat, and from thence he deriv'd the Title of *Ægiochus*, or the *Goat-Nurſt.* And to this effect, in some Medals of the Emperor *Valerianus*, he is represented in the Figure of a Child mounted on the back of a Goat, with this Inscription, JOVI CRESCENTI. I shall hereunto only apply an Ingenious Epigram of *Crinagoras*, in Greek, upon a Goat, whose Milk *Augustus Cæsar* used to Drink.

Thus *Englished:*

When Cæsar *did our full Bags Nectar taste,*
Whose Spring th'Exhausting Pole could never waste;
Me, that he might not want that Milkey store,
To Sea, with him, in his own Ship he bore.
Streight 'mong the Stars, shall I be made to shine,
For he I serve, than Jove's, *no less Divine.*

PLEIADES and HYADES.

SEven Stars on the back of the *Bull*, by the Latines, from the time of their Rising, called *Vergiliæ*; by the Greeks ἀπὸ τῦ πλεῖν; by the Arabs *Al Thuraiya*, i. e. *Multus seu Copiosus.* They are likewise by them called *Al Negim*, i. e. *Astrum*; by the Syrians they are called *Chimo*; by the Persians *Peru*, and *Peruin*; by the Turks *Ulger*; by the Jews *Chima*, and *Succoth Benoth.* These are said to have been the Daughters of *Atlas* and *Pleione*, whom *Mero* makes the Nurse of *Jupiter*, who fed him with *Ambrosia*; but commonly they are reputed the Nurses of *Baccus*, and for that Constellated. Their Names are *Maia*, *Sterope*, *Taygeta*, *Celeno*, *Electra*, Merope; or, according to the Scholiast of *Theocr. Coeymo*, *Glaucia*, *Protii*, *Parthenia*, *Maia*, *Stonyehia*, *Lampado.* *Michael Florentius* adds to them two other Stars, which he calls *Atlas* and *Pleione.* *Galileus* hath observed in this Constellation above 40 Stars; and *Ricciolus* no less than 40.

The seven Stars in the Head of the *Bull*, called by the Greeks *Pleiades*, the Latines call *Succulæ*, from the Greek Letter υ *Upsilon*, which they resemble, or from their Mother *Hya*, Daughter of *Oceanus*, and Wife of *Atlas*; by *Ulugh Beigh* they are called *Al Debarân*,

however that Name is peculiarly applied to the brighteſt of them, commonly called *Oculus Tauri*; in Hebrew they are called *Chima*, from the number of Stars of which they conſiſt. Theſe exceſſively lamenting the Death of their Brother *Hyas*, who in Hunting was ſlain by a Lion, were, by the commiſerating Gods, converted into Stars, whoſe Names are *Ambroſia*, *Eudora* (or *Eudoxa*) *Pheſile*, (or *Paſithea*) *Phileto*, (or *Pytho*) *Thyene*, (or *Tuke*) *Coronis* and *Prolixo* (or *Plexauris*.)

III. Of the Ancient Southern Constellations.

ORION.

THis Conſtellation was firſt by the *Bœtians* called *Candaon*, afterwards Ὠάριον, called by the Latines *Hyriades* and *Hyrides*, from his Father *Hyries*. It is by *Plautus*, *Feſtus* and *Varro*, called *Jugula*, *eo quod armatus ſit ut Gladius*; by the Jews it is called *Gibbor*, i. e. *Gigas*, and *Kelb Ha Giebbor*, i. e. *Canis fortis*, and *Bellator Fortis*; by the Arabs *Al Gianza*, and alſo *Al Giebbar*, i. e. *Gigas fortis*; in Syriack it is called *Gavoro*; in Caldie *Niphla*. It conſiſts of 38 Stars, amongſt which the firſt is called *Hecka*, i. e. *a white Circle*; by which Name the 3 Stars in the Head are denominated. The ſecond is called *Menkigh al Gianza*, i. e. *Humerus Orionis*; and *Jed al Gianza al Jumna*, i. e. *Manus dextra Orionis*. The 3d is called *Mirzam al Nagiid*, i. e. *Leo Strenuus*. The 17th and 25th are in the Arabick called *Al Tagis*, and *Al Dawáib*, the firſt ſignifying *Tiara*, the other *Antiæ ſeu Lemniſci*. The 26th, 27th and 28th, are called *Mintaka al Gianza*, and *Nitak al Gianza*, i. e. *Cingulum ſeu Baltheus Orionis*. The 29th, 30th, 31 and 32, are called *Saiph al Giebbar*, i. e. *Enſis Gigantis*. The 35th is called *Rigil al Gianza al Juſra*, i. e. *Pes Gigantis Siniſter*; and *Rai al Gianza*, i. e. *Paſtor Orionis*. The 38th is called *Rigil al Jumna*, i. e. *Pes Dexter*. — The Fable of this Conſtellation is by ſome thus related: *Orion* being a great Companion of *Diana*'s in her Hunting Diverſions, *Apollo* grew Jealous of his too much familiarity with his Siſter, and to be Revenged, ſeeing *Orion* one day ſwimming in the Sea, his Head appearing above the Water like a black Mark, he ſhew'd it to his Siſter, and told her ſhe could not hit it; whereupon, ſhe preſently drawing her Bow, let fly, and kill'd him, not knowing who he was, till the Sea had caſt him on the Shore; which perceiving, and much troubled, to make amends, ſhe plac'd him in Heaven near the *Dog* and the *Hare*, where he ſeems ſtill to Hunt. The Perſians will have this Aſteriſm to repreſent *Nimrod*. It is ſeen in the Meridian at Midnight in the Month of *December*.

SIRIUS, or the GREAT DOG.

THis is Fabled to be *Orion*'s Dog, Named *Lelaps*; others make it *Iſis* her Dog: Some again make it *Cæphalus*'s. By *Ovid* it is called *Canis Icarius* and *Erigonus*. By the Greeks Κυνος Ἀστερισμὸς, i. e. *Canis Aſteriſmus*. By the Latines it is called *Canicula*; the Ancient Ægyptians believed this Conſtellation to be the Soul of *Iſis*. *Germanicus* and *Hyginus* give it the Name of *Mæra*; the Æthiopians give it to *Nilus*, as if it were *Sydus Niloticum*, by reaſon of the great affinity between *Nilus* and that Star, for in the Dog-Days, that River hath its greateſt Inundation. By the Arabs it is called *Kelb Acbur*, i. e. *Canis Major*; by the Syrians *Kelbo Gavoro*, i. e. *Canis Gigantis*; by the Ægyptians it was called *Sothis*, in Memory of a King of that Name, Father to *Rhameſes*, who was a great Erecter of Obelisks, and Reſtorer of the Ægyptian Learning. This Conſtellation contains 29 Stars, whereof 11 inform: It is ſeen in the Meridian at Midnight about the end of *December*.

LEPUS the HARE.

THis Conſtellation is by the Greeks called λαγως, ἡ λέπορις; by the Latines *Lepus*. The Arabs call it *Arneb*; and the Jews *Arnebeth*, i. e. *Lepus*. It conſiſts of 12 Stars, whereof the 7th, 8th, 9th and 10th, are called in Arabick *Arſh al Gianza*, i. e. *Solium Orionis*. Some will have this placed in Heaven in Memorial of the Chaſe affected by *Diana* and *Orion*; Others make *Mercury* the Author of this Conſtellation, in Teſtimony of the fruitfulneſs and pregnancy of this Creature. *Hyginus* and *Baſſur* write, That Anciently in the Iſland *Hiero* there was no Hares, until one of the Iſlanders brought thither from beyond the Seas a Female big with Young; and that from thence, in a ſhort time, they increaſed, and grew ſo numerous, as wanting ſufficient Food, they deſtroy'd all the Crop in the Iſland, and brought a Famine upon the Place. In Memorial of which, this Aſteriſm was Figured in the Heavens.——To this purpoſe there is an Ingenious Epigram of *Cæſar Germanicus* in Greek.

Thus *Engliſhed*.

A Hare *by* Hounds *purſu'd, them having ſcap'd*
Met on the ſhore a Dog-fiſh, *and was ſnap'd:*
Then tries, Us Earth *and* Seas *are bent t'undo,*
Heaven's only left; yet there is a Dog *too.*

This Conſtellation is ſeen in the Meridian at Midnight in the Month of *December*.

PROCYON,

PROCYON, or the LITTLE DOG.

CAlled likewiſe *Canis Minor*; *Procynis* and *Præcanis*; but *Cicero* and *Auſonius*, *Anticanis*. *Pliny* ſays that the Romans had no Name for it, unleſs *Caniculam*. By the Arabs it is called *Kelb Aſgher*, i. e. *Canis Minor*; it is likewiſe by them called *Shirâ al Shamija*, i. e. *Syrius Shamenſis*. The Arabs make the greater and leſſer *Dogs* to be the Siſters of *Canopus*; but the Poets Fable this *Little Dog* to have been *Erigone*'s Dog, who mourned to Death for the loſs of his Miſtriſs, who Hanged her ſelf for Grief that her Father *Icorus* was ſlain by his Drunken Peaſants. This Conſtellation conſiſts only of two Stars, as *Ptolomy*, of which, that in his Shoulder is by the Arabs called *Al Mirzam*, and *Al Dira al Meſbuta*, i. e. *Brachium expanſum*; the other at the Root of the Tail they call *Al Shira*, *Al Shamiya*, i. e. *Syrius Shamenſis*, and *Al Ghomeiſa*. It is ſeen in the Meridian in the Month of *January* at Midnight.

ARGO, NAVIS.

THis Conſtellation is by *Ptolomy* called Ἀργῦς Ἀστερισμὸς, and by ſome ſimply Ναῦς, i. e. *Navis*; by the Arabs *Mercab*, i. e. *Currus ſeu vehiculum*; for ſo, by the Poets, the Ship *Argo*, which this Aſteriſm repreſents, is called *Currus volitans*. It is likewiſe in Arabick called *Al Sephina*, i. e. *Navis*. It conſiſts, according to *Ptolomy*, of 45 Stars; by *Sayerus* of 63; and by *Kepler* of 53. There are ſeveral Stars of ſecond Magnitude not far from it, which by the Arabs are called *Soheil Telkin vel Belkin*; or (as Mr. *Hide* reads it) *Belkis*, that being the Name of the Queen of *Sheba* that came to viſit *Solomon*. The Poets feign this to be the Ship wherein *Jaſon* did fetch the *Golden Fleece* from *Colchis*; ſome will have it to be the *Ark of Noah*. *Navidius* ſaith it is the Ship wherein the Apoſtles were when Chriſt appeared unto them walking on the Sea. The Poets do generally agree that this Ship was the firſt that ever Sail'd the Seas, but *Diodorus Siculus* plainly affirms the contrary, for ſpeaking of *Jaſon*, he ſays that he firſt, under the Mountain *Pelius*, built a Ship of far greater bulk than any that were then us'd, for at that time Men only Sail'd in ſmall Barks or Skiffs; ſo that this *Argo* ſeems not to be the firſt Ship, but rather the firſt of its kind. This Ship arrives to the Meridian at Midnight about the end of *January*.

DRACO.

THis Serpent is by *Ptolomy* called Ὕδρα Ἀστερισμος, i. e. *Hydri Aſteriſmus*; by the Greeks it is called Ἀσπὶς, and by the Arabs *Alſhugia*, i. e. *Serpens tenuis*. *Ricciolus* ſays it is in Arabick called *El Hawick*; and *Kepler*, *Aphaak*; in Hebrew *Hajah*, i. e. *Serpens*. In this Conſtellation there are 32 Stars, 7 inform, whereof the firſt Star is called *Minchir al Shugjâ*, and the others, from that to the ſeventh, inclusive, *Min al A'zal*, i. e. *Ex Inermi*, as if appertaining to the Sign *Virgo*. The 12th Star called by the Latines *Cor Hydræ*, is in the Perſian Tables called Ὕδρα Ἀυχὴ, and accordingly in *Ulugh Beigh*, *Unuk al Shugja*, *Collum Serpentis*; and *Pherd al Shugja*, i. e. *Solitaria Hydri*. The Head of this Conſtellation is ſeen in the Meridian at Midnight about the beginning of *February*; its Middle about Mid-*March*; and its Tail in the beginning of *April*.

CORVUS *the* CROW.

THis by the Greeks is called Κορὺξ ἢ Κόραξ, i. e. *Corvus, & Corvi Aſteriſmus*; by the Jews it is called *Oreu*, and from thence by the Arabs *Al Gorab*, *Corvus*. It is likewiſe by them called *Al Chiba*, i. e. *Tentorium*; and *Arſh al Simak*, i. e. *Solium efferentis* (*ſcil. intermem vel Verginem*) and *Agiar al Aſad*, i. e. *Clunes Leonis*; and *Al Agimal*, i. e. *Cameli*: It is ſeated upon the Tail of the Serpent, and conſiſts of 7 Stars, of which the firſt is called in Arabick *Minkar al Gorab*, i. e. *Roſtrum Corvi*; the fourth *Gienah al Goras al Siman*, i. e. *Ala dextra Corvi*. The Fable of this Conſtellation is thus: The Crow being ſent by *Apollo* to fetch Water for a Libation, ſeeing a Fig-Tree full of Figs, but not Ripe, made ſtay there until the Figs were come to maturity (which Fable ſeems to be derived, ſays *Bochart*, from *Noah*'s ſending the Crow out of the Ark) and having ſatisfy'd his longing, went to the Fountain to fetch Water, but coming thither meets with the Serpent before-mentioned, whereat affrighted, he returns back with the empty Pitcher, telling *Apollo* there was no Water in the Fountain. This untruth being diſcovered by *Apollo*, he Prohibited the Crow for ever Drinking at that time of the Year, and in Memorial of the Fact, plac'd the Crow, Snake and Pitcher, in the Heavens. The Crow is Sacred to *Apollo*, the Preſident of Divination; foraſmuch as this Fowl, by its different Notes, is ſaid to foretel Fair and Foul Weather; or for that *Apollo*, fearing the purſuit of *Typhon*, is ſaid to have aſſum'd the Figure of that Fowl; or, in alluſion to the Suns departure, cauſing darkneſs and Night of the ſame Colour with the Crow, as his return does the Day or Light, reſembling the whiteneſs of the Swan, which is likewiſe Sacred to that God. This Aſteriſm is ſeen in the Meridian at Midnight about the middle of *March*.

The

The CUP.

Manilius appropriates this Cup to *Bacchus*; *Aratus Hyginus*, and *Baſſus*, to *Apollo*, according to the foregoing Fable. It is called by *Ptolomy* Κρατηρ & Ἀστερισμος, by others, *Hydria, Calpe, Cratera, Patera, Urna & Vas*; by the Arabs, *Batyra*, i. e. *Poculum Magnum*; by ſome it is called *Alkis*, i. e. *Cyathus*, from the Hebrew *Kus* or *Kos*, ſignifying the ſame. It conſiſts according to *Kepler* of Eight Stars, which by the Arabs are called *al Ma'laph*, i. e. *Præſepe*: It is apparent in the Meridian at Midnight, about the middle of *March*.

CENTAURE.

SOme will have this to be *Minotaur*; others *Chyron*, the Son of *Saturn*, and *Phillyra* the Daughter of *Oceanus*, who taught *Æſculapius* Phyſick; *Achilles* Muſick, and *Hercules* Aſtronomy; with one of whoſe Poyſonous Arrows, caſually falling out of his Quiver, he was Wounded in the Foot; and of that Wound died, and by commiſerating *Jupiter* was made a Sign in Heaven; called by *Ptolomy* Κενταυρος Ἀστερισμος. The Arabs making uſe of the Greek Name, by whom yet, according to *Ricciolus* it is called *Albeze*, and *Aſmeat*; by the Greeks Φης; and in Barbarous Greek *Taraopoz*: It conſiſts, as *Ptolomy*, of 37 Stars; all which, together with thoſe that make up the *Fera Centauri*, are by the Arabs called promiſcuouſly *Alſhamarich*, i.e. *Spadices*, bright dappled: The 35 Star is, by *Ulugh Beigh*, called *Rigil Kentaurus*, i. e. *Pes Centauri*. This Conſtellation paſſes the Meridian at Midnight at the end of *April*.

The ALTAR.

CAlled by the Greeks Θυμιατηριον, πυρεϊμμα & Φαεος, by the Latines *Thuribulum, Conceptaculum, Bathilius, Sacrarium, Puteus, Templum, Lar, Acerra, Ara, & Altare*; by the Arabs, *Almegramet* or *al Mugramrah*. It conſiſts of 7 Stars, This was the firſt Altar (according to the Poets) that was ever Erected: It was Fram'd by the *Cyclops*, and in Memorial of the Fact Conſtellated. *Lactantius* Reports, that the firſt Altar that *Jupiter* Erected, was in Honour to *Cælus*: But the Deities to whom *Jupiter* Sacrificed upon this Expedition againſt the Giants, we find (from *Diodorus Siculus*) to have been the Sun, Heaven, and the Earth. This *Conſtellation* comes to the Meridian at Midnight about the End of *June*.

The

The *WHALE.*

THis Asterism is by the Greeks called Κητος, πηγασος & Ορφευς, by the Latines, *Cete* and *Cetus*, *Balena*, *Pistrix*, *Leo*, or *Ursus Marinus*: By the Arabs *Alketus*: *Ptolomy* reckons in it 22 Stars; and *Kepler* 25; of which, the Bright one in the Snout of the *Whale* is called *Menkar Alketus*, i. e. *Rostrum Ceti*: That in the Tail, *Danab Alketus*, i. e. *Cauda Ceti*; and both these by the Arab; are called *al Diphdaan*, i. e. *duæ Rana*: There are two also in the Hands (for this Fish is conceived to be the same with *Dagon*, or *Derceto* the *Syrian* Idol, which was represented in the Upper part after a humane shape, in the lower, after that of a Fish; and by the Jews Named *Adir Dag*, i. e. *Piscis Magnus*: They are by the Arabs called *al Naaman* or *al Naamat*, i. e. *Struthio Cameli*: The second Star in this Constellation is called *Caph al Giedma*, i. e. *Manus Truncata*: The 21 Star is called *Danab al Ketus Shemali*, i. e. *Ceti Cauda Borealis*: The 22th *Danab al Gienubi*, i. e, *Cauda Australis*; and *al Diphda al Tha'ni*, i. e. *Rana Secunda*: This Constellation is seen in the Meridian at Midnight, from the beginning of *October* to the end of *December*.

The Southern *FISH.*

THE Poets Fable this to have been the Fish which saved *Phacetis*, or *Aphacitis*, the Daughter of *Venus* fallen into the Lake *Boeth*; and for that Reason Constellated; by the Arabs called *al Haut*, *al Gienubi*, i. e. *Piscis Australis*: *Hyginus* calls it, *Piscis Solitarius*; and *Bassus*, *Piscis Magnus*: And it is said to have Spawned the other two in the Zodiack: It consists of 12 Stars, amongst which the Bright one in the Mouth is called *al Diphda al Auwal*, i. e. *Rana Prima*, and *al Dalim*, i. e. *Agger*; and *Phom al Hont*, i. e. *Os Phiscis*; and commonly (but erroneously) *Phomahant*: It is seen in the Meridian at Midnight about the middle of *August*.

ERIDANUS, or *PADUS.*

IT is a Southern Asterism, which some will have to be made a Constellation in Memory of *Phaeton*, who was Drowned therein; the *Egyptians* Challenge it for their *Nilus*. It is called by the Greeks ποταμὸς Ωρίωνος, i. e. *Fluvius Orionis*, because it springs from the Left Foot of *Orion*, and runs from thence in a fluxious Course Southward: It is likewise called *Gyhon*, and *Vardi*, i. e. *Fluvius*; by the Moors *Guad*; and by the Arabs *Nah'r*: The *Thuscans* call it *Botignon*; the *Ligurians*, *Botigum*. It consists of 34 Stars, of which that of the first Magnitude is by the Arabs called *al Dalim*, i. e. *Agger*; it is likewise called in Arabick *Acher Nah'r*. i. e. *Ultima Fluminis*; whence the common Name *Acarnar*: It is seen upon the Meridian at Midnight in *November*.

CORONA AUSTRALIS sive NOTIA.

IT is called by the Greek Poets Ἰξίονος τεῖχος, i. e. *Rota Ixionis*: The Arabs give it different Names, as *al Cubba*, i. e. *Teſtitudo vel Tabernaculum*; and *Az'ha al Naam*, i. e. *Nidus Struthionis*. This Conſtellation conſiſts of 13 Stars. It is Fabled to be made a Conſtellation by *Bacchus*, in Honour and Remembrance of his Mother *Semele*. It comes to the Meridian at Midnight (but not viſible in our Horizon) in the beginning of *July*.

IV. *Of ſuch other Conſtellations Northern and Southern; as have been Diſcovered by Modern* ASTRONOMERS.

I. Modern Northern CONSTELLATIONS.

COMA BERENICES.

IT is by *Bayerus* called πλόκαμος, i. e. *Coma, Spicarum Manipulus*: The Arabs call it *Alhaud*, i. e. *Lacus ſeu Ciſterna*: It conſiſts of 7 Stars. The Original of this Conſtellation was from *Berenice*, the Wife of *Ptolemæus Energetes*, who Vowed, if her Husband returned Victorious from his *Aſian* Expedition, ſhe would ſhave her Head and Offer her Hair to *Venus* to be hung up in her Temple; which being accordingly performed; it was, the next day after the Offering thereof, found miſſing; whereupon *Conon*, to flatter King *Ptolomy*, diſcovered to him, that the Head of Hair was Tranſlated to Heaven, and made a Celeſtial Conſtellation.

ANTINOUS and GANIMED.

THeſe are one and the ſame Conſtellation; for the Aſteriſm which by the Greeks is feigned to repreſent *Ganymed* Rap'd by the *Eagle*, and carried up to Heaven to ſerve *Jupiter* as a Cup-Bearer; the Romans in Honour to *Antinous* (the beloved Favorite of *Hadrian* the Emperour) will have to be the Repreſentation of that Beautiful *Bythinian*, who dying a Voluntary Death for the Welfare of the Emperor, was by him Honoured with Statues, Temples, Prieſts, and a place among the Celeſtial Conſtel ations; between the *Eagle* and *Sagittarius*. It conſiſts of 7 Stars; and comes to the Meridian about the middle of *July*.

EQUU-

EQUULEUS.

THis Aſteriſm, called alſo the Leſſer Horſe; is by *Ptolemy* called ἵππου μετωπιαῖος, Απεικασμος by others τμημα τοῦ ἵππου, i. e. *Sectio Equi*: In *Chryſococca's* Tables Κεφαλὴ ἵππου, i. e. *Caput Equi*; by *Ulugh Beigh*, *Kit'a al Pharas*, i. e. *Sectio, Praciſſio, vel, Segmentum Equi*; but by others of the Arabs called *al Pharas al Awwal*, i. e. *Equus Primus*; it conſiſts of four Stars in form of a Horſes Head and Neck: It comes to the Meridian at Midnight at the beginning of *Auguſt*.

¶ To theſe Northern Conſtellations ſome late Aſtronomers have added ſeveral other *Aſteriſms*, compoſed out of the Inform Stars: As of thoſe between the *Greater Bear* and the Sign *Leo*; they have formed the River *Jordan*——Of thoſe between the North-Pole, *Perſius* and *Auriga*, an Aſteriſm called *Camelo Pardalis* and *Gyraffa*.——Of the four Stars interpoſed between the Triangle and Tail of the *Ram*, another called *Veſpa*; by ſome *Apes*; i, e. the *Waſp* or the *Bees*——Of the Tract of Stars Running between the *Swan* and the *Eagle*, as far as *Serpentarius*, they have formed the River *Tigris* or *Euphrates*——And to a ſingle Star of the ſecond Magnitude, placed in the middle between *Charles his Wain* and *Coma Berenices* (from which if a Right Line be drawn through the firſt Star in the Tail of *Urſa Major*, towards the Pole, it will point directly to the *Pole Star*) they have given in Memory of the moſt Glorious Prince and Martyr *CHARLES* the I. King of *England*, the Name of *Cor Caroli*. The Primary Invention and Denomination thereof being owing to the moſt Loyal and Learned Sir *Charles Scarborough*, my moſt Honoured Friend.

II. Modern Southern CONSTELLATIONS.

MONICERUS or *UNICORN*.

IT is placed between *Orion* the *Great Dog*, and *Hydra*.

ALECTOR vel GALLUS.

OR the *Cock*, between the *Great Dog*, and the Ship *Argo*.

COLUMBA.

OR the *Dove* of *Noah*, with an *Olive Branch* in her *Beak*, not far from the *Great Dog*.

GRUS or the CRANE.

IT conſiſteth of 13 Stars, whereof Three of the Second Magnitude; one in the *Head*, one in the *Tail* or *Train*; and another in the Southern Wing. This Aſteriſm is alſo called *Phænicopterus* and *Genaros*; and it is ſeated under the Southern *Fiſh*.

PHOENIX.

ON her *Spicy Pyre*, conſiſting of Fifteen Stars; among which there is one in the *Neck* thereof of the Second Magnitude, and two *Nebulous*; It is ſeated between the Southern *Fiſh* and *Eridanus*, under *Cauda Ceti*.

TOUCAN.

OTherwiſe called the *American Gooſe*: It is likewiſe called *Pica Baſilica, ſeu Indica*, and *Rampheſtes*: It conſiſts of Eight Stars, whereof Four of the Second Magnitude. This Aſteriſm is placed in the midſt between the *Phœnix* and *Indus*.

INDUS.

OR the *Indian*, in the Figure of an *Indian*, holding in either hand a Dart; and therefore likewiſe called *Sagittifer*: It conſiſts of Twelve Stars, and is ſeated between *Toucan*, and the Conſtellation called *Pavo*.

PAVO.

PAVO.

THE *Peacock*; or, according to the Greek ταὼς; consisting of Sixteen Stars; whereof, one in the *Head* thereof, is of the Second Magnitude, and two *Nebulous*: It is placed near to *Indus*, under *Sagittarius*.

APOUS.

OR *Apis sive Avis Indica*; *Avis Paradisi*, & *Manu Codiata*; consisting of Twelve Stars: It follows after the *Peacock*, with its *Tail* toward the *Antartick* Pole.

APIS.

CAlled also *Musca* or *Muia*; and *Crabo Indicus*; consisting of Four Stars, placed under the *Centaur*.

TRIANGULUM AUSTRALE.

TRigonium *Notius, sive Deltoton*; to whom some likewise have given the *Arabick* Name of *Almutabet Algenubi*; consisting of Five Stars; in each Angle one of the Second Magnitude, and two others. It is seated *sub fera Centauri & Ara*, called by *Schillerus*, *Signum Tau*; *sive*, *Imago Crucis*; by the Spaniards, *El Cruziero*.

CAMÆLEON.

PLaced directly in Opposition to the *Lesser Bear*; and whose Form, (according to the disposition of the Stars which compose it) it represents: It consists of Ten Stars: It is placed directly under the Constellation *Musca* or the *Fly*.

PISCIS.

PISCIS VOLANS.

VOlucris, & Volatilis; calledlike wiſe *Paſſer Marinus*; and *Hirundo Marina*; in which laſt ſence it is noted by a new Greek Name *Chelidon Thalaſſia*; It conſiſts of ſeven Stars, ſeated under the Ship *Argo*, next to *Dorado*, or the *Sword-Fiſh*.

DORADO.

PIſcis *Auratus* (as the Spaniards call it) *Chryſophris*, or the *Golden Fiſh*; called likewiſe *Xiphias ſeu Gladius*, or the *Sword Fiſh*; It conſiſts of 6 Stars, with which it Deſcribes and Circumſcribes the *Pole* of the *Ecliptick*.

HYDRUS.

BY the Dutch called the *Waſſer Schlange*, conſiſting of 15 Stars; the laſt Star in the Tail whereof was in the Year 1600, diſtant 2 deg. 30 min. from the Southern Pole; but at preſent (as *Ricciolus* Notes) at a nearer diſtance.

V. Of GALAZIA.

IT is that great *White Circle* which is ſeen in the Heavens in a cold Winters Night (and at other times alſo) and by the Greeks called *Galazia*; by the Latins *Lactea Via*, or *Lacteus Circulus*; and in Engliſh the *Milk Way*.

Concerning this Circle there are ſundry Opinions both of Philoſophers and Poets, ſome of which I have here inſerted.

The Philoſopher *Democritus* affirms the cauſe of this Circle to be the exceeding great number of Stars in that part of the Heaven, whoſe Beams meeting together ſo confuſedly, and not coming diſtinctly unto the Eye, cauſes us to imagine ſuch a whiteneſs as is ſeen.

Others are of Opinion (and the more likely) That this *Milk way* is a part of the *Firmament*, neither ſo thin as the other parts thereof, nor yet ſo thick as the Stars themſelves: If it were as thin as the other parts of the Heavens beſides the Stars, then could it not retain the Light, but the Light would paſs through it, and not be ſeen. If it were as thick as the Stars, then would the Light be ſo doubled in it, that it would gliſter and ſhine as the Stars themſelves do: But being neither ſo thin as the one, nor ſo thick as the other, it becometh of that whiteneſs which we ſee it of.

The

The Poetical Fables concerning this Milky way are many.

Ovid ſaith it is the great Cauſey, and the High-way, that leadeth to the Palace of *Jupiter*.

Others ſay, *Jupiter* having begotten *Mercury* on *Maia* the Daughter of *Atlas*, brought the Child when he was Born to the Breaſt of *Juno*, ſhe lying aſleep; but *Juno* awaking, threw the Child out of her Lap, and let the Milk run out of her Breaſts in ſuch abundance, that (ſpreading it ſelf about the Heaven) it made that white Circle which we ſee.

Others ſay, that it was not *Mercury*, but *Hercules* which was laid in the Lap of *Juno*; for *Jupiter* knowing the great hatred that *Juno* bore to the *Babe*, perſwaded himſelf, that if it were poſſible to get her to Nouriſh it, ſhe could not but (as if it were by Nature) bear the *Child* good Will: Hereupon, *Hercules* was brought unto her *Breaſts* as ſhe lay aſleep; *Juno* feeling him to draw very hard, awoke ſuddenly out of her Sleep, threw the *Child* out of her Lap, and let the Milk run down the Heaven, whereby it was Stained White.

Others ſay, That *Juno* did not let her Milk run out of her Breaſts, but that *Hercules* Suck'd them ſo earneſtly, that his Mouth ran over, and ſo this Circle was made.

Others ſay, That *Saturn* being deſirous to devour his Children, his Wife *Ops* preſented him with a Stone wrap'd in a Clout, inſtead of his Child. The Stone ſtuck ſo faſt in *Saturns* Throat, as he would have ſwallowed it, that without doubt he had therewithal been Choaked, had he not been Relieved by his Wife, who by preſſing the Milk out of her Breaſt ſaved his Life: The Milk that miſſed his Mouth fell on the Heaven, and running along made this Circle.

The End of the Aſtronomical Part.

RECREATIONS Horometrical.

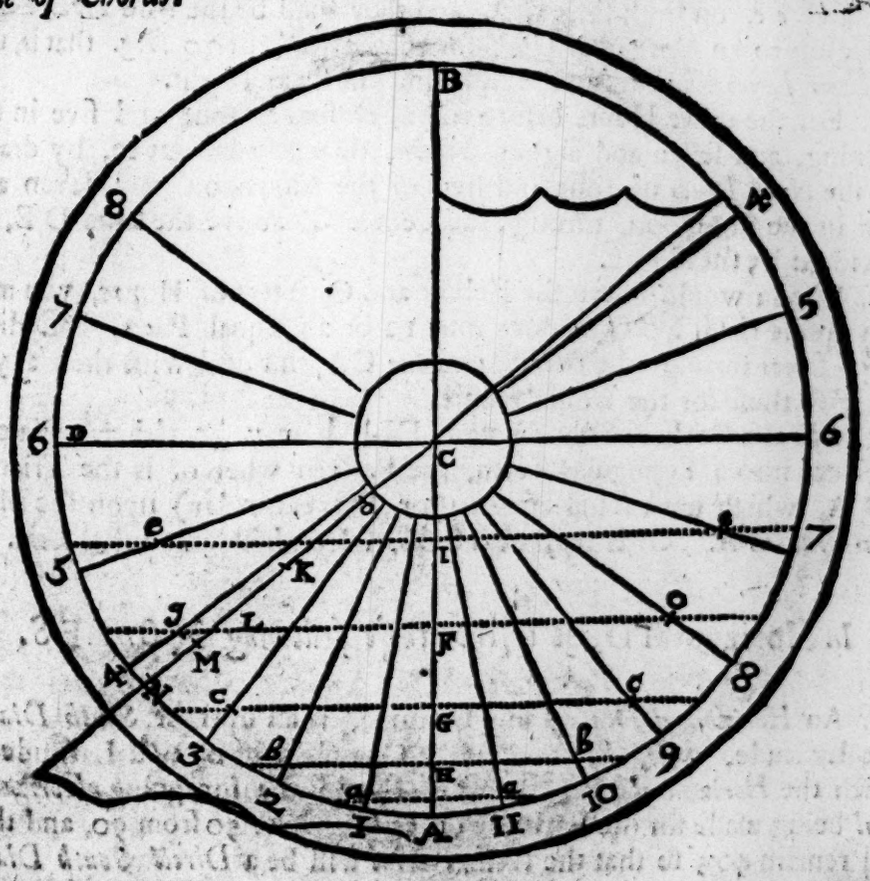
CHAP. I.

How to make an Horizontal Dial *for any* Latitude, *by help of a Line of* Chords.

HOrizontal Dials are such as are usually made on *Brass Plates*, and set upon the tops of Posts in Yards, Gardens, Windows, and other places.

A Line or Scale of *Chords* (by which this and other Dials are made) is such a Line or Scale as is described in *Geometrical Recreations*, Chap. V. Section II.

How to make an Horizontal Dial in any Latitude, *by help of this Scale of Chords.*



1. With 60 deg. of the *Chords*, upon the Center C, deſcribe a Circle, and divide it into Four equal parts, by the Lines, B C A for the *Meridian*, or Hour-Line of XII, and D E for the *Prime Vertical Circle*, or Hour Line of VI.

2. If you would make your *Dial* for the Latitude of 50 degrees; take 50 deg. out of your *Scale of Chords*, and ſet that diſtance upon the Circle from A to N, and draw the Line C N, for the *Stile* or *Cock* of your *Dial*.

3. Divide the Two Quarters of your Circle, A E and A D, each of them, into ſix equal parts, ſo ſhall you have in each Quadrant five Points, by which you may draw five *Chord-Lines*, all Parallel to D E, the Line of ſix, which will croſs the Line of Twelve C A, in the Points I, F, G, H and A.

4. Take one half of the Line A, and ſet it in the Line of the Stile C N, from C to O: From which Point O, take the neareſt Extent unto the Line C A. This Diſtance ſet from A upon the ſame *Chord-Line* A, on both ſides thereof, at *a* and *a*, through which Points *a* and *a* draw the Lines C *a* and C *a* for the *Hour-Lines* of XI and I of the Clock. In like manner,

Take half the *Chord-Line*, { H, G, F, I } and ſet it from C to { K, L, M, N } and from { K, L, M, N } take the neareſt diſtance to C A, and ſet it from { H, G, F, I } to { *b b*, *c c*, *d d*, *e e* }

Which done (which may very ſpeedily, and exactly be performed) draw Lines from the Center C through the reſpective Points *b b*, *c c*, *d d*, and *e e*, on both ſides C A, and they ſhall be the true *Hour-Lines* belonging to an *Horizontal Dial*, for the Latitude of 50 *Deg.* that is, the the *Hour-Lines* from ſix in the Morning till ſix at Night.

5. For the other Hours before and after ſix, as four and five in the Morning, and ſeven and eight at Night, they may be put in, by drawing the *Hour-Lines* of four and five in the Afternoon, and ſeven and eight in the Forenoon, through the Center C, above the Line D E, as is evident by the Figure.

6. If you would inſert the Halves and Quarters of Hours, you muſt then divide the Two Quadrants into 12 or 24 Equal Parts, and draw *Chord-Lines* through the *Meridian-Line* C A, and deal with them as you did with thoſe for the whole Hours.

7. For the Cock or Stile of your Dial, it may be either a Plate of Braſs cut into a Triangular Form, the Pattern whereof is the Triangle C S A, which muſt ſtand Square (or Perpendicular) upon the *Meridian-Line* C A. Or it may be a Rod of Iron bended to that form.

In Horizontal Dials *obſerve theſe following* NOTES.

I. An *Horizontal Dial* in any Latitude, is an upright *South Dial* in that Latitude, which is equal to the Complement of that Latitude for which the *Horizontal Dial* was made——So the foregoing *Horizontal Dial* being made for the Latitude of 50 Deg. take 50 from 90, and there will remain 40; ſo that the former *Dial* will be a *Direct South Dial* in the

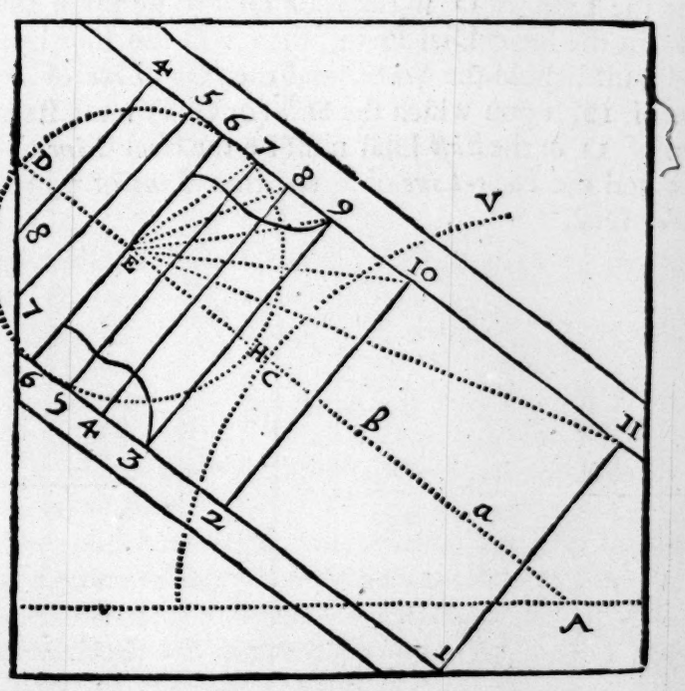
the Latitude of 40 Deg. And the Cock, or Stile, must point downwards towards the *South* Pole.

II. From an Erect Direct *South Dial*, an Erect Direct *North Dial* is deduced :——For, if you draw the *Hour-Lines* (*Stile* and all) of a *South Dial* through the Center, then will 12 at Noon on the *South* be 12 at Midnight in the *North*; and the *Cock*, or *Stile* of the *North Dial* must point upwards towards the *North Pole*; And the *Hour-Lines* about 12 at Night (*viz.* 9,10,11,12,13, 1,2,3.) must be omitted; As in *Fig.* I. all Three Dials are distinguished. So that a *North Dial* is no other than a *South Dial* inverted.

CHAP. II.

To make an Erect Direct East *or* West Dial.

THese Dials are the same in all Latitudes, and may be made *Geometrically*, after this manner.



Towards the bottom of your Plain, draw an Horizontal Line B A; At the *North end* thereof (as at A,) with 60 deg. of your *Scale of Chords*, describe an Arch of a Circle, as S C V. Then out of your *Scale of Chords* take the Complement of the *Latitude* (in this Example 40 deg.) and set them from S to C, and draw the Line A C D, quite through the Plain; which Line D A divide into five equal parts in the Points a, b, H, E; and with that distance of the Compasses, upon the

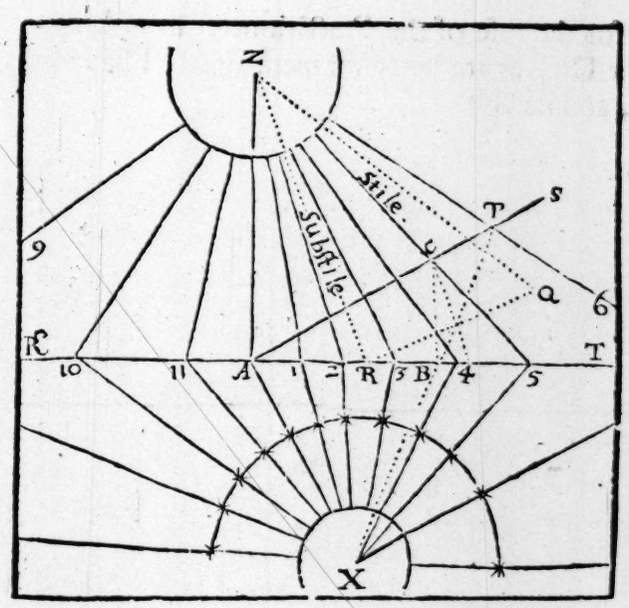
point E, describe the Circle D 6 H 6, and through the Center E, draw the Line 6 E 6, at right Angles to the Line A D, and that shall be the Hour Line of six; And by the Points 6 and 6 draw the Lines 6 11 and 6 1, both parallel to the *Equinoctial Line* A D-----Then divide one Quarter of the Circle into six equal parts, and laying a Ruler upon the Center E, and each of those parts, it will cut the uppermost parallel Line in the points 6, 7, 8, 9, 10 and 11. From which points, if you draw Lines parallel to the Line 6 E 6, they shall be the Hour Lines of an *Erect Direct* East *Plain*. For the Hour-lines before 6 in the Morning and after 6 in the Evening, they are the same with those before 6 in the Morning, and before 6 in the Evening: For the distance betwixt 6 and 5, is the same as is between 6 and 7, and the distance between 6 and 4 the same as between 6 and 8---- For *the Stile of these Dials*, it may be a Plate of Brass, so broad as is the distance between the Hour-lines of 6 and 9. and must be placed perpendicularly upon the Hour-line of six. This Plain hath Two Faces, one open to *the East* (as this in the Figure) the other to the *West*; and one being made the other is made also: For, if you prick (or draw) the Lines of the *East Dial* through the Paper, those Lines on the back side shall be a *West Dial*, only the *East Dial* Lines are those of the *Morning* from 4 till 11, and those of the *West Dial* are those of the *Afternoon* from 1 to 8 at Night.

From an East *or* West Dial, *a* Polar Dial *may be deduced.*

Suppose the Line A D in the *East* Dial to lie upon the *Horizontal Line* A B, in the same Dial Plain, then will the *East* Dial be a *Polar* Dial, and must behold the *South*, and the *Hour-Line* of 6 must be the *Hour Line* of 12, upon which the *Stile* (or *Cock*) must stand. And the *Hour-Line* of 11 in the *East* Dial must be the *Hour-Line* of 5 at Night in the *Polar*; and the *Hour-Line* of 1, the *Hour-Line* of 5 in the Morning, in the *Polar* Dial.

CHAP. III.

By help of an Horizontal Dial, *in any Latitude, to make a* Declining Dial *in that Latitude.*



SUppoſe an upright Plain in the *Latitude* of 51 deg. 32 min. to decline from the *South Weſtward.* 30 deg.

1. Draw a right line K T, repreſenting the *Baſe or horizontal Line* of your *declining Plain.*

2. In this line, aſſume any point (as A.) and from it draw another line A S, making an Angle S A T equal to 30 deg. the Plain's declination, (towards K, if the Plain decline *Eaſtward*) or towards T, if it decline *Weſt*, as here it doth.

3. Upon this point A, erect a Perpendicular to K T. for the meridian line of the *declining Plain*; ----And from the ſame point A, another perpendicular to the *Line* A S, ~~for the Meridian (or 12 a Clock line)~~ of the *horizontal Dial*, and in this line aſſume any point at pleaſure (as X) for the Center of the *horizontal Dial.*

4. Upon X deſcribe the *Horizontal* Dial K T D E, for the Latitude of 51 *deg.* 32 *min.* and draw the *Stile* thereof X B, continuing it, till it cut the Line A S in P, and all the *Hour-Lines* till they cut the Line K T, in the Points 9, 10, 11, and 1, 2, 3, 4, 5.

5. Make A Z equal to A P; ſo ſhall Z be the Center of the *Declining Dial.*

6. Upon the Point B, (where the Line X P cutteth the Line K T) erect a Perpendicular, cutting the Line A S in C.

7. Make

7. Make A R equal to B C, and draw the Line Z R for the *Substilar* Line of the *Declining Dial.*

8. From R erect a Perpendicular to Z R, making R Q equal to A B, and draw the Line R Q for *the Stile* of the Declining Dial.

9. From the Center of the Declining Plain Z, draw right Lines from the Center T to the ſeveral Points 9, 10, 11. and 1, 2, 3, 4, 5, *&c.* (where the Hour-Lines of the *Horizontal* Dial did cut the Line K T;) thoſe Lines ſhall be the true *Hour-Lines* belonging to *the Declining Plain*, and *the Dial* is finiſhed, which you may bring into what form you pleaſe, Round, Square, *&c.*

And here, for the eaſe of the Practitioner, in making of *Horizontal*, and ſuch other Dials as are hereafter mentioned, I have inſerted this following Table, and its Uſe.

A Table

A Table shewing the Degrees of each Hour-Line from 12, upon all *Horizontal* Dials, from 30 to 60 deg. of Latitude, and for all Direct *North* or *South* Reclining or Inclining Dials, where the Heighth of the Stile of such Dials is between 30 and 60 deg. of Altitude.

Hours	XI I		X II		IX III		VIII IV		VII V	
Distances from 12	deg.	m.	deg.	m.	deg.	m.	deg.	m.	deg.	m.
30	7	37	16	7	26	34	40	54	61	49
31	7	51	16	34	27	15	41	44	62	30
32	8	05	17	1	27	55	42	32	63	11
33	8	19	17	27	28	37	43	20	63	49
34	8	31	17	54	29	13	44	5	64	24
35	8	44	18	20	29	50	44	49	64	58
36	8	57	18	45	30	27	45	31	65	30
37	9	10	19	9	31	2	46	12	66	10
38	9	22	19	34	31	37	46	50	66	29
39	9	24	19	58	32	11	47	28	66	56
40	9	45	20	21	32	44	48	7	67	21
41	9	57	20	44	33	16	48	29	67	47
42	10	10	21	7	33	46	49	12	68	11
43	10	22	21	29	34	18	49	44	68	33
44	10	32	21	51	34	47	50	10	68	54
45	10	43	22	12	35	17	50	46	69	15
46	10	54	22	33	35	44	51	15	69	35
47	11	5	22	53	36	11	51	42	69	53
48	11	17	23	13	36	37	52	9	70	11
49	11	25	23	33	37	3	52	35	70	28
50	11	35	23	52	37	28	53	0	70	43
51	11	45	24	9	37	52	53	21	70	59
52	11	55	24	27	38	15	53	46	71	13
53	12	5	24	43	38	37	54	8	71	28
54	12	13	25	2	38	58	54	29	71	41
55	12	22	25	10	39	19	54	49	71	54
56	12	32	25	34	39	40	55	9	72	5
57	12	40	25	50	39	59	55	28	72	17
58	12	48	26	5	40	18	55	45	72	28
59	12	56	26	20	40	36	56	3	72	38
60	13	4	26	34	40	54	56	19	72	48

Degrees of Latitude, or of the Stiles (or Cocks) Heighth.

The Use of this Table.

Suppose you were to make an *Horizontal* Dial for the Latitude of 35 Degrees, and a Direct *South* Dial for the Latitude of 55 deg. Or (which is all one) for any Direct *South* or *North* Dial, *Reclining* where the heighth of the Stile is 35 deg.

First

First draw a downright Line for the *Hour-Line* of 12, and upon any part thereof (towards the upper end) assume a Point for the *Center* of your Dial; and upon that Point with your Compasses opened to 60 deg. of your *Scale of Chords*, describe a *Semicircle*; Then take 90 deg. of your Scale of Chords, and set that distance from the *Hour-Line* of 12, upon the *Semicircle* both ways; and through those Two Points draw another Right Line, which will pass also through the Center, and be the *Hour-Line* of 6 in the *Morning* and 6 in the *Evening*.

Then go to this Table, and look in the first Column thereof for the Latitude 35 deg. and in the Line against 35, towards the Right Hand, you shall find,

deg.	min.		Hours	
8	44	to stand under the hours of	XI	I
18	20		X	II
29	50		IX	III
44	49		VIII	IV
64	58		VII	V

Which Degrees and Minutes taken out of your *Scale* of *Chords*, and set upon the *Semicircle* on both sides of the Line of 12, Lines drawn from the Center, through those Points, shall be the true *Hour-Lines* proper for an *Horizontal* Dial for the Latitude of 35 deg. or of a *South* or *North* Dial in the Latitude of 55 deg. and of any Direct *North* or *South* Reclining Dial, where the *Stile*'s Heighth is 35 deg.

CHAP. IV.

How, Mechanically, *by help of an* Horizontal Dial *for any Latitude, to describe the* Meridian, Substile, Stile, *and the other* Hour-Lines *upon any other Plain in that* Latitude, Regular *or* Irregular.

1. UNder the Plain upon which you intend to make your Dial, let there be erected a Scaffold of one or more Boards broad (according to the bigness of the New Dial you intend to make,) which Scaffold must be exactly Level, or parallel to the *Horizon*.

2. Being provided of an *Horizontal Dial* with Hours, Halves and Quarters (the larger the better) set it upon the Scaffold, the Center towards the *South*, and the Stile pointing up towards the *North* Pole; then

3. Having attained the true *Hour of the Day*, either by some true *Sun-Dial*, a *Solar Observation* by *Quadrant*, *Ring*, or other *Instrument*, or by a good *Minute-Watch* well rectified;

4. Bring

4. Bring *the Horizontal Dial* upon the Scaffold to the True Hour and Minute of the Day, at such convenient distance from the *Plain* as your Judgment will direct you; and there with small Tacks, or Cement, fix your Dial upon the Scaffold, so that it move not, there being a small string fixed in the Center of the Dial. Then,

5. Extend the Thread, which is in the Center of the Dial, along, justly, to touch the Top of the Stile of the Dial, till it doth touch the New Dial Plain, and note that Point; for that shall be the *Center* of the New Dial: The Thread it self is *the Stile* or *Axis*, and the Line under it (which may be found by help of a *Carpenter's* Square applied to the Plain and the String) is the *Substilar Line* upon the Plain, by help of which the Stile may be made and fastned.

6. To draw the Hour Lines: Extend the Thread fixed in the Center of the *Horizontal Dial*, over every Hour (Half and Quarter, if you will) till the Thread do touch the *Plain* upon the *Horizontal Line* thereof, (which will always be in the same Level with the upper part of the Scaffold,) and where they intersect this Line (or the Plain,) make Marks, noting them with the same Figures as the Thread passed over in the *Horizontal* Dial: So Lines drawn from the Center of the New Dial, through these Points, shall be the true Hour Lines of the New Dial.

But some Hour-Lines may run beyond the Limits of the Plain, or the passage of the Thread may be obstructed by some Objects, or Irregularities in the way, that it cannot come to touch the Plain; Then to help these, or the like Impediments,

7. Upon the *Horizontal* Scaffold, draw as large a *Square*, or *Parallelogram* as you can (or you may extend the Hour-Lines of the *Horizontal Dial* upon the Scaffold it self) and transfer (by help of the Center-Thread) all the hours of the *Horizontal* Dial into the sides of the *Square* or *Parallelogram*: And then,

8. Bring a Thread from the Center of the New Dial, and rest it upon the Hour-Point marked in the side of the *Square* or *Parallelogram*: Then the Thread in the Center of the *Horizontal* Dial carried along, so as only to touch the other Thread from the Center, will describe the Hour-Line desired, whether upon an *Even* or *Uneven Plain*, from the Center of the New Dial.

Thus far for Dials *whose Centers will fall within their Plains; but for such as decline much from the* South *towards the* East *or* West, *in such the Thread which passeth from the Center of the* Horizontal Dial, *by the side of the* Stile *thereof, the Thread will not meet with the Plain at all, or (at least) not in a great distance, so that no Center can be found upon the Plain it self. In such Cases you must,*

9. Set up a *Board* or other *Object*, over, or on that side of the *Plain* towards which the *Axis* tendeth, to receive the Center; and then, fixing a Thread there, by that, and the other Thread you may describe all the Hour-Lines, as was before shewed in *Irregular Plains*: And the Thread from the Center will be the true *Axis* or *Gnomon* of the *Dial*, and must be fixed to the Wall by two Stays.

CHAP. V.

Of Projective Dials.

BY this Artifice *Hour-Lines* may be Projected upon all kinds of *Superficies*, without any regard had to their ſtanding; either in reſpect of *Declination*, *Reclination*, or *both*; Or whether the *Plains* be *Flat* or *Curved*; *One* or *more*: For if a *Point* be aſſigned, wherein the *Hour-Lines* and *Axis* ſhall concur, *Hour-Lines* may be Projected upon ſuch *Plains*, and an *Axis* ſet up after the uſual manner, by the directions following.

Firſt, To the *Point* aſſigned (upon any ſide of it) by help of a *Semicircle*, or other *Level*, ſtretch out an Horizontal Thread, ſerving for the Horizontal Line, which *Line* (or *Thread*) need not be one ſingle Line, but may be returned by Two, Three, or more Angles, provided that all the parts of it do lie in the ſame Superficies, and parallel to the Horizon.

Secondly, With a Perpendicular Thread held up, project the Sun into the aſſigned Point, and into the Horizontal Thread alſo, and there ſtick in a Pin, or make a Mark upon the Horizontal Line, through which the Shadow cutteth; and at the ſame Inſtant alſo take the Sun's *Altitude*.

Thirdly, By the *Altitude* taken, find out the Sun's *Azimuth* for that time. This *Azimuth*, whatever it be, is repreſented by the *Mark* before made in the Horizontal Line or Thread.

Fourthly, Apply a Paiſtboard to the aſſigned Point, holding it flat, that it may lie in the ſame Plain with the Horizontal Thread; and upon this Paiſtboard protract your *Azimuth*, by a Thread extended from the Point aſſigned for the *Center*, to the *Mark* made upon the Horizontal Thread. This done,

Fifthly, By help of that *Azimuth* upon the *Paiſtboard*, protract the *Meridian Line*, obſerving the true *Coaſt*; and to the *Meridian* thus found, deſcribe an Horizontal Dial for the place.

Sixthly, Apply the *Paiſtboard* to its place again, all things ſtanding right as before, project all the Hour-Lines into the Horizontal Thread from the *Paiſtboard*, and make Marks upon the Horizontal Line from the Points of each ſeveral Hour.

Seventhly, Project the *Meridian Point* by a Perpendicular Thread upon ſome Object, into that place whereabouts you imagine the *Axis* of the World would paſs, whether it be above or below, from the Point aſſigned for the *Center*.

Eighthly, By help of a *Semicircle* (or a *Quadrant* applied to a String) elevated, or depreſſed (as occaſion offers) from the *Point* aſſigned for the *Center*, according to your *Latitude*, project the *Pole of the World*.

Ninthly, Extend a *Thread* from the Point aſſigned for the *Center* to the *Pole of the World*, which *Thread* will repreſent the *Axis*.

Tenthly, By the Points upon *the* Horizontal *Thread*, and *the Axis*, (either by your *Eye*, laying *the Axis* to the Hour-Points, or laying the Hour-Points to *the Axis*,) you may project all the Hours ---- Or without *the Axis* you may content your ſelf with the Pole-Point projected into *the Meridian*: For, if from the Point aſſigned for *the Center*, or meeting of the Hours and *Axis*, you extend a *Thread* to each Hour-Point in the Horizontal Line, and do repoſe, (by your *Eye*) the ſame *Thread* upon the Pole Point, then ſhall the Shadow of *the Thread* give you that Hour Line; and ſo you may do for all the reſt.

Eleventhly, Your *Thread* or *Axis*, lying in its true ſituation, you may eaſily fit an *Axis* to the ſame poſture. If your Dial be deſcribed upon a Plain *Superficies*, you may then (by one ſide of a *Square*, applied to *the Thread* or *Axis*, and the other ſide lying upon the Plain,) find out the *Subſtile*. ---- But if the *Dial* be deſcribed upon a *Curved Superficies*, or upon more than *One*, then muſt you be content to ſet up your *Axis* by the direction of *the Thread* only.

Twelfthly, The Point aſſigned for *the Center* being a Point of *the Axis*, is, as it were, *the Apex* of *the Gnomon*, unto which all *the Work is projected*. But if it be required to ſet up an *Axis* to ſuch a *Superficies* upon which the Hours and *Axis* will not meet at any tolerable diſtance; in ſuch a caſe ſet up any point (of *Wyre*, or ſuch like) of ſuch diſtance from the *Superficies*, as that the Hours and *Axis* may be diſtinct, and through that *point* let it be reqnired to make *the Axis* paſs. You have no more to do, but only to project to this *point*, as before, by letting the *ſhadow* of a perpendicular Thread paſs through that *point*, and noting the ſame upon the Horizontal Thread; aud counting that end of the *Wyre* as your *Center*, proceed as before; for the *Thread* that lies to project the Hours is a *Pattern* for *the Axis*.

Now this is a *General* way to project Hour-lines upon *one*, or *many Superficies*, be they *plain* or *curved*, *one* or *more*, and that without any laborious Inquiſition of any of their ſituations, in reſpect of *Declination*, *Reclination*, or *both*.

CHAP. VI.

Of Reflected Dials.

IF a piece of *Looking-Glaſs* (of about one third part of an Inch diamiter) be Horizontally fixed upon *the Soyle* or *Tranſome* of any *Window*, unto which *the Sun* hath free acceſs; you may draw *Hour-lines* upon any Plain, whether *Horizontal*, *Reclining*, or *Inclining*, be they *one* or *more*; by which the true Hour of the day may be diſcovered by *the ſpot of Light* which the Sun ſhall reflect from *the Glaſs* upon any of *the Plains:* And that by theſe Precepts following.

Firſt, The *Glaſs* being plac'd truly Horizontal, obſerve *the ſpot of Light* that the Sun caſts, and make a Mark at it.

Secondly, And at the ſame time take the Sun's *Altitude*, and find its *Azimuth*.

Thirdly, Extend an *horizontal Thread* in the ſame *Level* with the *Glaſs*, but within the Room.

Fourthly. Project *the Azimuth* into *the horizontal Thread*, by holding up a perpendicular Thread in ſuch a place, that, though it hang at liberty, you may at once diſcern both *the Mark of the ſpot of Light*, and the *Glaſs* likewiſe; and then obſerve where *the perpendicular Thread* ſeems to cut *the horizontal Thread*; and at that apparent interſection, make a *mark* upon *the horizontal Thread* for *the Azimuth*.

Fifthly. Apply a *Paiſtboard* to *the Glaſs*, ſo that it may be ſtayed upon ſome *Reſt*, that after it is taken away, it may be reſtored to its place again, without any alteration. Let it be alſo placed *horizontal*, ſo that it may have full relation to the *horizontal Thread*.

Sixthly, At the *Glaſſes Center* make a Point for *the Center* upon the *Paiſtboard*, and extend a *Thread* from the *Center* upon the *Paiſtboard*, till it touch the *Mark* made for the *Azimuth* upon *the horizontal Thread*, and draw upon *the Paiſtboard* that Line which the extended *Thread* figures out thereupon: Afterwards, unto the ſame *Azimuth* upon the *Paiſtboard*, draw a *Meridian Line*, and to it an *horizontal Dial* for the *place*; and applying the Paiſtboard again to its former ſituation, project *the hours* thereon unto *the horizontal Thread*, and there make *Marks*, or tie *Knots*.

Seventhly, Then project *the Meridian* (by a *perpendicular Thread*, covering, in appearance, both *the Mark* at 12, and *the Glaſs*) unto the contrary *Coaſt*, to that wherein the Pole is elevated above the Horizon; that is to ſay, In our *Northern* Climates you muſt project *the Meridian Southward* from *the Glaſs*, becauſe the *North Pole* is elevated: And in this *Meridian* elevate your *Semicircle* or *Quadrant* from *the Glaſs Southwards*, till the *Plummet* fall upon the *Latitude*; ſo ſhall it point out, upon ſome Object ſet to receive it, the *North pole Reflected*.----Or elſe, if this be not convenient, (becauſe in Windows which look towards the *South*, the *North Pole* will be without the Room, and ſo the *Axis* above the *Glaſs* extended towards that *Pole*, will be without alſo,) you may

may in such Cases, find out *the opposite Pole* to it; that is to say, that Pole which the former *Reflected Axis*, being extended through the *Glass*, and below it, would sign out; and that may be effected in this manner.

Eighthly, Project the *Meridian-line* towards the Pole that is elevated; that is with us, towards the *North* Pole; and then (because the *North* Pole is elevated by *Reflection*, towards the *South*, so, by the same Reason, the *South* Pole must be depressed towards the *North*) by your *Semicircle* or *Quadrant*, and String directed even with *the Center of the Glass*, express or project your Latitude downwards (but towards the *North*,) so shall the *Semicircle*, or *Thread*, point out the *Reflected South* Pole in the *Meridian*. Now, whether you will, or can (most conveniently) use the *Reflected North* Pole above the *Glass*, or the *Reflected South* Pole below it, you are to take your choice; for both the one and the other of them do represent the *Reflected Axis of the World*.

Ninthly, By this *Reflected Axis*, and the *Hour-points* signed out upon the *horizontal Thread*, you may easily project the *Reflected Hours*, upon any kind of *Superficies*, *one* or *more*, whatever they be that stand in the way.

CHAP. VII.

How (Mechanically) *by help of a* Trigon, *to inscribe the* Equinoctial, Tropicks, *and the other Signs and* Parallels *of* Declination, *upon all sorts of* Sun-Dials: *As also the* Azimuths, *or Points of the Mariners Compass; the* Almicanters, *or Circles of the* Sun's Altitude, *&c.*

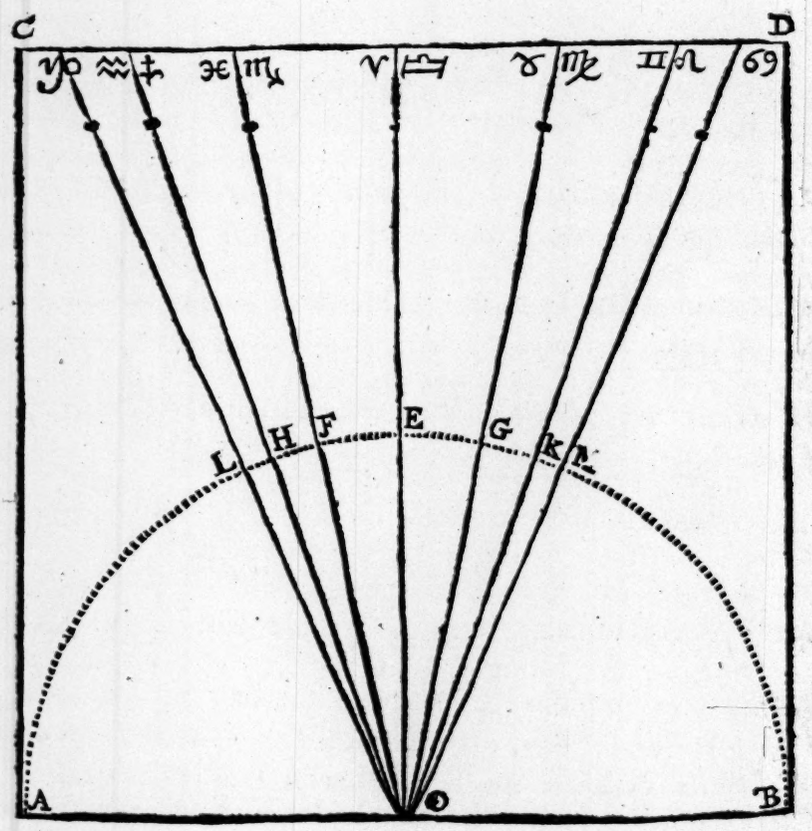
I. *How to make the* Trigon.

THE *Trigon* may be made upon a piece of thin Board, or Sheet of Paistboard, as is the Figure A B C D; Through the middle whereof draw the Right Line ⊙ E ♎, representing the *Equinoctial Circle* in the Heavens; then upon ⊙, as a *Center*, with 60 deg. of your *Scale of Chords*, describe the Semicircle A E B; Then out of your *Chord* take 23 deg. 30 min. the quantity of the Sun's greatest *Declination*, and set them from E, on both sides of the *Equinoctial* to L and M, and draw the Two lines ⊙ M ♋, and ⊙ L ♑, for the Two *Tropicks*; Then for the other intermediate Signs, set off 11 deg. 30 min. from E, both ways, to F and G, aud draw the lines ⊙ F ♏, and ⊙ G ♉, for the lines of the Sun's Entrance into the Sign ♏ and ♐, and into ♉ and ♒---- Also, take 20 deg. 12 min. from the Scale of *Chords*, and set them from E,

E, on both ſides, to H and K, and draw the lines ⊙ H ♒, and ⊙ K ♑ for the lines of the Sun's Entrance into the Signs ♒ and ♓, and into ♊ and ♌---Then towards the upper ends of the ſeveral lines, ⊙ L, ⊙ H, ⊙ F, ⊙ E, ⊙ G, ⊙ K, ⊙ M, make ſmall Holes, through which to put a ſmall String: And thus is your *Trigon* finiſhed, and fitted for the inſerting of the Parallels of the Twelve Signs into all ſorts of Sun-Dials, either *Direct* or *Reclining*, *One* or *more*, *Contiguous* or *Separate*:-----But if you would put in other *Parallels* of *Declination*, as ſuch as when the Days are juſt 8, 9, 10, 11, 12, 13, 14, 15 or 16 Hours long, Then you muſt inſert into your *Trigon* ſuch degrees of *Declination* as the Sun hath when the Day is ſo many Hours long, as you would deſcribe upon your Dial: And ſo in the Latitude of *London*,

					deg.	min.	
When the day is either	8	or	16	Hours long, the Sun hath	21	40	Degrees of Declination.
	9		15		16	55	
	10		14		11	37	
	11		13		5	55	

And ſuch degrees of *Declination* you muſt put into the *Trigon*, between the *Equinoctial* and the *Tropicks*.



II. *How*

II. *How to insert the* Equinoctial, *the* Tropicks, *and other Intermediate* Parallels *of* Declination *into all sorts of* Dials.

YOU are to Note, that the *Parallels* of the *Signs*, the *Diurnal Arches*, the *Azimuths*, the *Circles* of *Altitude*, and all other *Circles* relating to the Course of the *Sun*, when they are described upon any *Sun-Dial*, are not shadowed out by the whole *Stile* or *Axis* of the *Dial*, as the *Hours* are, but by some One Point in the same *Axis*; as by a *Knob*, *Button* or *Notch*, filed in the *Stile* of the *Dial*; Or by a *hole* in a *Glass Window* for *projected Dials*; or by a piece of *Looking-Glass* for *Reflected Dials*; in all which Cases, the *Center* ⊙, in the *Trigon*, is to be applied (in all Cases,) so that the line A B thereof, must lie upon the *Stile* of the Dial (if the Dial have a *Stile*) or Parallel to the *Axis* of the World, if it be an *hole* in a *Window*, or piece of *Looking-Glass*: And now, the *Trigon* being thus disposed, so that it may be turned about the *Axis*, the *Center* ⊙ always fixed in the same point.

Now suppose you would insert the *Equinoctial* into any Dial, (for one *Rule* serves for all *Plains*:) *First*, put a long small String through the Hole which is under ♈ ♎, tying a Knot on the back side, that it slip not through the Hole in the Board; Then apply the *Center* ⊙ to the *Point* (or *Button*) in the Stile, and the Side A B to the Stile it self. Now if you extend the Thread over the Line ♈ ♎ E ⊖, till it touch the *Dial-Plain*, that point of touching shall be one point through which the *Equinoctial* is to be drawn upon the Plain——Then turn the Line A B about upon the Stile, leaning either towards the Right or Left Hand, as occasion offers it self, extend the Thread till it touch the Dial-Plain in some other point, and that other point shall be another point through which the *Equinoctial* is to be drawn upon the *Dial*. And if your *Dial* be all but one plain *Superficies*, Two points will be sufficient to draw the *Equinoctial* by, it being a *Great Circle* of the *Sphere*, and so a *Right Line* upon all *plain Superficies*; But if *the Dial* consist of more than one *Plain*, then must you (in the same manner as before) find two points at least upon each *Superficies*; which you may easily and speedily do, by turning the line A B of the *Trigon* about (or parallel to) the *Axis*, and extending the Thread over the line ♈ ♎ E ⊙, till it touch the Plain.

In like manner, if you would insert the *Tropick* of *Capricorn* into your *Dial*, you must put the String or Thread through the Hole under ♑, and then apply the Point ⊙ to the Point in the *Axis*, and the Side A B to the Stile, extend the Thread over the line ♑ L ⊙, till it touch the Plain, and that point of Touch shall be one Point through which *the Tropick* of *Capricorn* is to be drawn----Again, Move *the Trigon* (in this position) towards the Right or Left Hand (as occasion requires,) extend the Thread over the same Line ♑ L ⊙, till it touch the Plain, and that shall be another point through which that *Tropick* must be drawn; And in this manner may you find as many points upon the Plain as you please, and the more the better (for these *Parallels* will not be streight lines, as *the Equinoctial line* was, but *Conick Sections*, or *Curved lines*) through which points a line being traced, with an even Hand, shall be *the Tropick* of *Capricorn* upon your *Dial-Plain* or *Plains*.

And

And in this manner may the *Tropick* of *Cancer*, and all *the parallels* of the other Signs be drawn, if first you put the String through the respective Holes, and apply the *Trigon* to the *Axis*, and extend the Thread till it touch the Plain, as before, you may find as many points as you please, through which to draw your *parallel*. And let this suffice for the Inscription of *the parallels* of the Signs of *the Zodiack*------And if you would insert the *parallels* for the length of the Day, they are to be done in the same manner, if instead of *the Declination* for the Signs, you put into your *Trigon* the *Declinations* answering to the *length of the days* you intend to insert into your Plain, as is before directed.

III. *To insert the* Parallels *of the* Sun's Altitude *into all* Projected or Reflected Dials.

THis may be performed by a *Quadrant* drawn upon a large piece of Board, or Sheet of Paistboard, and divided into 90 Deg. after the usual manner. For if you apply a Board, or other matter to *the Glass* (if a *Reflected Dial*; or to *the Hole*, if it be a *projected Dial*) so that it may lie exactly level, or parallel to *the Horizon*; Which Board thus fixed, apply the *Center* of *the Quadrant* to *the Center* of *the Glass* or *Hole*, setting one side of *the Quadrant perpendicular*, the other *parallel* to *the Horizon*, and a small *String* or *Thread* in *the Center*: Then consider what *Circle of Altitude* you would describe upon your *Dial-plain* or *plains*, (suppose 30 degrees:) The *Quadrant* standing in the fore-said posture, upon any part of *the Horizontal Board*, extend the *Thread* by *the Superficies* of *the Quadrant* over 30 deg. thereof, till it touch *the Wall* or *Ceiling of the Room*, and where it touches any Object that is in the way, that point of touch shall be one point through which *the parallel* of 30 degrees of *Altitude* shall pass.----Again, Move *the Quadrant* (*the Center* being still kept in *the Center of the Glass* or *Hole*, but *the Horizontal side* moved to some other part of the fixed Board) extend *the Thread* again over 30 deg. of *the Quadrant* till it touch *the Wall* or *Ceiling of the Room*, and that shall be another point through which the *parallel* of 30 deg. of *Altitude* shall pass. And in this manner may you find as many points as you please, by which to describe that *parallel*----- If both *the Glass* and *Ceiling* be both *Horizontal*, then these *parallels of Altitude* will be perfect *Circles*, whose *Center* will be in *the Vertical point* directly over *the Glass*, and in the same Plain with *the Ceiling*. But if the Plains be *Upright*, or *Reclining*, then these *parallels* will be *Curved Lines*, as *the parallels* of *the Signs*, and must be drawn through the respective points found with an even Hand----And as this *parallel* of 30 deg. of *Altitude* was drawn, so may any other, as of 10, 20, 40, 50, &c. be, by extending *the String* in *the Center* over those degrees in *the Quadrant* till they meet with the Plain.

IV. How

IV. *How the* Azimuths, *or* Vertical Circles *may be described upon all sorts of Plains.*

FOR the performance of this Work, you must first find a *Vertical point*, above, tending to *the Zenith* of *the Glass* (if it be for a *Reflected Dial*; Or below, towards *the Nadir* of *the hole*, if it be a *Projected Dial.*) This Point thus found (which may be done by a Plumb Line, or perpendicular Thread from *the Ceiling* to *the Glass*, or from the *Hole* to the *Floor*) may be called *the Reflected Vertical point*, in which a *Thread* or *String* is to be fixed: And by a point found in *the Reflected Axis* of *the Horizon* the *Azimuths* may be drawn; as *the Hours* were by a point found in *the Reflected Axis* of *the Equinoctial*: In manner following:

Upon a Sheet of Paistboard, or like Material, let there be described such *Azimuth Circles* as you intend to describe upon your Plain; whether they be *the Degrees* of *the Horizon*, or *the points of the Mariners Compass*: for both are put on by the same Method, only vary in their Denomination. One being numbered by 10, 20, 30, *&c.* the other by *South, S by W, S S W, &c.*

These *Azimuth Lines* being drawn upon the Paistboard, place the *Center* of them upon *the Center* of *the Glass*, and *the South Azimuth* thereof upon *the Meridian* of the place, so that the Paistboard may be there fixed in the very Plain of *the Horizon*, that is, perfectly *horizontal* or *level.*

Then take *the Thread* which is fixed in *the Glass*, and draw it over any *Azimuth* marked on the Paistboard, till it meet with some Object in the Room, as a *Post*, *Wall*, or the like, and there fix it.

Then take *the Thread* whose end is fixed in *the Reflected Vertical point* over *the Glass*, and lead it along by *the horizontal Thread*, till it meet with *the Wall* or *Ceiling* of *the Room*; and where it toucheth, that shall be one point through which that *Azimuth* must be drawn. In like manner, move the same *Thread* higher or lower at pleasure, by the side of *the horizontal Thread*, till it be opposed by some *Wall* or *Ceiling*, and there make another Mark for a second point, through which that *Azimuth* must be drawn: And now have you Two Points, the which (if they fall both upon one and the same *Superficies*, *Wall* or *Ceiling*,) are sufficient; for *the Azimuths* being *Great Circles* of *the Sphere*, will be streight Lines; and so may you find Two Points upon each *Superficies*, if there be more than One: Or, by the first Two Points, if a *Thread* be so situated, that it may interpose between *the Eye*, and the said *Two Points*, you may make as many Points as you please upon all *the Superficies*, be they never so many: And in this manner may what *Azimuths* you please be inscribed upon any Plain or Plains.

Note, *In all the* Reflected Dials *mentioned in this* Chapter, *although the* Plains *themselves may be many, and those* Regular *or* Irregular, Contiguous *or* Separate; *yet (in all Cases) the* Glass *must lie level or parallel to the* Horizon; *which is a thing very difficult to effect by any* Instrument, *the* Glass *being so small*; *Wherefore One* Mechanical *way may be this, which I think is inferior to none:*

In the Hole made in *the Tranſome* of *the Window*, in which *the Glaſs* is to be fixed, pour ſome Drops of Fair Water, or other Liquor, till the Hole be full; then, the Sun ſhining, will caſt *the Reflex* thereof upon *the Ceiling*, which point of *Reflected Light* mark upon *the Ceiling*, and incontinently, with a Spunge, dry up the Water which is in the Hole: Aud having *Cement* (or the like) to faſten *the Glaſs* with, put it in the Hole, and *the Glaſs* upon it, and while *the Cement* is yet warm and pliable, move *the Glaſs* upon it till it caſt its ſhadow upon that point of *the Ceiling* to which *the Water* was *Reflected*; and by this means ſhall your *Glaſs* be truly placed.

CHAP. VIII.

Of Dials *which ſhall give the Hour of the Day by the* Sun, *and by the* Stars *in the Night-Seaſon: How ſuch* Dials *may be made, and how to make Uſe of them.*

THere are ſeveral ways by which any *Stars* Horary diſtance from the Meridian (call'd the *Stars* Hour) may be obtained: As,

I. *By any* Quadrant, *or other Inſtrumental* Dial, *which giveth the Hour of the Day by the* Sun.

I Will Illuſtrate this in the Uſe of Mr. *Gunter's Quadrant*, it being an Inſtrument more frequently known than any other of that kind: For,

If you obſerve the ſame Rules in finding the *Stars* Hour, as is directed for finding of the Hour of the Day by the *Sun*; that is, by ſetting the Bead to *the Stars Declination*, inſtead of *the Sun's Declination*, and then obſerve *the Stars Altitude*, as if it were *the Sun's Altitude*, the Bead ſhall then ſhew among the Hour-Lines the *Stars* Hour, or *the Stars Horary diſtance* from *the Meridian*.

But here you are to note, *That this way of finding the* Stars *Hours is peculiar to ſuch* Stars *only as are between the* Tropicks. *Wherefore, another more general way may be this:*

II. By

II. *By a* Sun-Dial *made under the* Soyl, *and on the* Jaums *of a* Jetty *Window, on the inside of a Room.*

And such a *Dial* may be made by the Rules before given in *Chap.* V. hereof; Or thus:

HAving made a small round hole in any Quarry of Glass in the Window, and darkned the other part of the same Quarry round about the hole, you must upon the Window-Board draw a Meridian Line, which Line must pass directly under the hole before made, and must be transferred to the Ceiling of the same Room, by the help of Perpendicular Threads.

Then from the hole in the Window, to the Meridian Line on the Ceiling, extend a String, till it make an Angle equal to the *Latitude* of the place you make the *Dial* in, and where the String (with this condition) so resteth fix the end of the String in that point of the Ceiling, letting the other part of the String hang at liberty.

This done, by help of an *Horizontal Dyal*, whose Center (for the present) must be placed in the hole in the Window; the Lines of which *Dial* must also be extended by a Thread fixed in the Center thereof, by which Line extended over each Hour, and the String before fixed in the Ceiling, the Hour Lines may be transferred and marked upon, or under the Window Board; and also upon the *Jaums* and *Cheek Posts* of the said Window, and there numbred by Letters or Figures.

Now such a *Dial* being made, I shall shew,

How to find the Hour by the Sun *in the day time, and any* Stars *Hour (or* Horary *distance from the Meridian) in the* Night *Season.*

1. *By the* Sun.

THE *Sun* shining through the hole before made in the Window, move the String, whose end is fixed in Ceiling, along the Hour Points which are marked about the Window, until such time that the Spot of Light that cometh through the hole shineth upon the String, and then see upon what Hour, or part of an Hour the string resteth, for that is the true time of the Day.

2. *By the* Stars.

THis differeth little from the former; for when through the Window you see a *Star* you know, and would know his Hour, move the String along the Hour Points as before, till such time as you bring your Eye, the String, the Hole, and the *Star* all in one and the same Plain or right Line; for then will the String rest upon that *Stars* Hour, or his *Horary* distance from the Meridian.

In the Hole made in *the Tranſome* of *the Window*, in which *the Glaſs* is to be fixed, pour ſome Drops of Fair Water, or other Liquor, till the Hole be full; then, the Sun ſhining, will caſt *the Reflex* thereof upon *the Ceiling*, which point of *Reflected Light* mark upon *the Ceiling*, and incontinently, with a Spunge, dry up the Water which is in the Hole: Aud having *Cement* (or the like) to faſten *the Glaſs* with, put it in the Hole, and *the Glaſs* upon it, and while *the Cement* is yet warm and pliable, move *the Glaſs* upon it till it caſt its ſhadow upon that point of *the Ceiling* to which *the Water* was *Reflected*; and by this means ſhall your *Glaſs* be truly placed.

CHAP. VIII.

Of Dials *which ſhall give the Hour of the Day by the* Sun, *and by the* Stars *in the Night-Seaſon: How ſuch* Dials *may be made, and how to make Uſe of them.*

THere are ſeveral ways by which any *Stars* Horary diſtance from the Meridian (call'd the *Stars* Hour) may be obtained: As,

I. *By any* Quadrant, *or other Inſtrumental* Dial, *which giveth the Hour of the Day by the* Sun.

I Will Illuſtrate this in the Uſe of Mr. *Gunter's Quadrant*, it being an Inſtrument more frequently known than any other of that kind: For, If you obſerve the ſame Rules in finding the *Stars* Hour, as is directed for finding of the Hour of the Day by the *Sun*; that is, by ſetting the Bead to *the Stars Declination*, inſtead of *the Sun's Declination*, and then obſerve *the Stars Altitude*, as if it were *the Sun's Altitude*, the Bead ſhall then ſhew among the Hour-Lines the *Stars* Hour, or *the Stars Horary diſtance* from *the Meridian*.

But here you are to note, *That this way of finding the* Stars *Hours is peculiar to ſuch* Stars *only as are between the* Tropicks. *Wherefore, another more general way may be this:*

II. By

II. *By a* Sun-Dial *made under the* Soyl, *and on the* Jaums *of a* Jetty *Window, on the inside of a Room.*

And such a *Dial* may be made by the Rules before given in *Chap.* V. hereof; Or thus:

HAving made a small round hole in any Quarry of Glass in the Window, and darkned the other part of the same Quarry round about the hole, you must upon the Window-Board draw a Meridian Line, which Line must pass directly under the hole before made, and must be transferred to the Ceiling of the same Room, by the help of Perpendicular Threads.

Then from the hole in the Window, to the Meridian Line on the Ceiling, extend a String, till it make an Angle equal to the *Latitude* of the place you make the *Dial* in, and where the String (with this condition) so resteth fix the end of the String in that point of the Ceiling, letting the other part of the String hang at liberty.

This done, by help of an *Horizontal Dyal*, whose Center (for the present) must be placed in the hole in the Window; the Lines of which *Dial* must also be extended by a Thread fixed in the Center thereof, by which Line extended over each Hour, and the String before fixed in the Ceiling, the Hour Lines may be transferred and marked upon, or under the Window Board; and also upon the *Jaums* and *Cheek Posts* of the said Window, and there numbred by Letters or Figures.

Now such a *Dial* being made, I shall shew,

How to find the Hour by the Sun *in the day time, and any* Stars *Hour (or* Horary *distance from the Meridian) in the* Night *Season.*

1. *By the* Sun.

THE *Sun* shining through the hole before made in the Window, move the String, whose end is fixed in Ceiling, along the Hour Points which are marked about the Window, until such time that the Spot of Light that cometh through the hole shineth upon the String, and then see upon what Hour, or part of an Hour the string resteth, for that is the true time of the Day.

2. *By the* Stars.

THis differeth little from the former; for when through the Window you see a *Star* you know, and would know his Hour, move the String along the Hour Points as before, till such time as you bring your Eye, the String, the Hole, and the *Star* all in one and the same Plain or right Line; for then will the String rest upon that *Stars* Hour, or his *Horary* distance from the Meridian.

III. *By a* Dial *made in a Yard or Garden.*

IN ſome convenient open place erect a Pole perpendicular to the *Horizon*, about 10 or 12 foot high; then provide a Frame of Wood in form of a *Parallelogram*, of what bigneſs you pleaſe, (but the ſides being 2 foot broad, and 3 foot long, is a competent bigneſs;) within the *Area* of this Frame make the true Hour-Lines of an *Eaſt* and *Weſt* Dial; which Hour-Lines may be of reaſonable big Wyre; and upon the Edges of the Frame, ſet the Numbers of the Hours, the Forenoon Hours on the *Eaſt* ſide, and the Afternoon Hours on the *Weſt* ſide, and over the Hour-Line of Six erect an *Axis* (of a competent Length) as if it were a *Sun-Dial:* Which Dial being thus prepared, if you ſet it upon the former erected Pole, ſo that the Two Ends of the Frame may ſtand due *North* and *South*, and the Stile thereof Parallel to *the Axis* of the World, then is it fit for uſe, either to find the Hour of the Day by the *Sun*, or *the Stars* Hour in the Night.

1. *By the* Sun.

THis is all one as if it were a *Dial* made againſt a Wall; for the ſhadow of *the Axis* upon the Frame will ſhew the Hour of the Day.

2. *By the* Stars.

WHen you ſee a *Star* you know, and would find that *Stars* Hour; move your ſelf about the Dial-Poſt (coming near to it, or going farther from it) as occaſion offereth, till you bring your Eye, the *Axis*, and the *Star* in the ſame Plain or Right Line, and then mind what Hour-Line (or between what Hour-Lines) is intercepted by that view, for that is that *Stars* Hour; and by this *Dial* you may at any time know what *Stars* are upon *the Meridian.*

IV. *To know what* Stars *are upon the Meridian.*

IF you go behind *the North* End of the Frame, and look by the ſide of the Frame, you ſhall ſee what *Stars* are then upon *the South* part of *the Meridian.*

And if you go behind *the South* End of the Frame, and look by the Side of the Frame, you ſhall there ſee what Stars are upon *the North* part of *the Meridian.*

And thus the Stars Hour *may be found by any of the forementioned ways: which obtained, the true* Hour of the Night *may be found: But firſt you muſt know the* Right Aſcenſion *of the* Sun, *and alſo of the* Star, *whoſe* Hour *you found by your* Dial. *And to that end I have inſerted the Two following Tables.*

A Table of the Complement of the Sun's Right Aſcenſion at Midnight, every Night in the Year.

	Jan.		Feb.		March		April		May		June	
Days	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.
1	4	25	2	18	0	32	10	39	8	46	6	41
2	4	21	2	14	0	28	10	35	8	42	6	37
3	4	17	2	10	0	24	10	31	8	38	6	33
4	4	13	2	6	0	21	10	27	8	34	6	29
5	4	9	2	2	0	17	10	24	8	30	6	24
6	4	4	1	58	0	14	10	20	8	26	6	20
7	4	0	1	54	0	10	10	16	8	22	6	16
8	3	56	1	50	0	7	10	13	8	18	6	12
9	3	51	1	46	0	3	10	9	8	14	6	8
10	3	47	1	43	11	59	10	6	8	10	6	4
11	3	43	1	39	11	55	10	2	8	6	6	0
12	3	38	1	35	11	52	9	58	8	2	5	56
13	3	34	1	31	11	48	9	54	7	58	5	52
14	3	30	1	27	11	45	9	50	7	54	5	48
15	3	26	1	24	11	41	9	47	7	50	5	43
16	3	22	1	20	11	37	9	43	7	46	5	39
17	3	18	1	16	11	34	9	39	7	42	5	35
18	3	14	1	12	11	39	9	35	7	38	5	31
19	3	10	1	8	11	27	9	31	7	34	5	27
20	3	6	1	5	11	23	9	28	7	30	5	22
21	3	2	1	1	11	19	9	24	7	26	5	18
22	2	57	0	57	11	16	9	20	7	22	5	14
23	2	53	0	54	11	12	9	16	7	18	5	10
24	2	49	0	50	11	8	9	12	7	14	5	6
25	2	45	0	47	11	5	9	9	7	10	5	0
26	2	41	0	43	11	1	9	5	7	6	4	58
27	2	37	0	39	10	57	9	1	7	2	4	54
28	2	33	0	35	10	54	8	57	6	58	4	50
29	2	29			10	50	8	53	6	54	4	46
30	2	25			10	46	8	50	6	49	4	41
31	2	22			10	43			6	45		

Days	July h.	m.	August h.	m.	Septemb. h.	m.	October h.	m.	Nov. h.	m.	Decem h.	m.
1	4	37	2	35	0	41	10	52	8	53	6	45
2	4	33	2	31	0	37	10	48	8	49	6	40
3	4	29	2	27	0	34	10	45	8	45	6	35
4	4	25	2	23	0	30	10	41	8	41	6	31
5	4	21	2	20	0	27	10	38	8	38	6	26
6	4	17	2	16	0	23	10	34	8	33	6	22
7	4	13	2	12	0	19	10	30	8	29	6	18
8	4	9	2	9	0	16	10	26	8	24	6	13
9	4	5	2	5	0	12	10	22	8	20	6	9
10	4	1	2	2	0	9	10	19	8	15	6	4
11	3	57	1	59	0	5	10	15	8	11	6	0
12	3	53	1	54	0	1	10	11	8	7	5	55
13	3	49	1	50	11	58	10	7	8	2	5	51
14	3	45	1	46	11	54	10	3	7	58	5	46
15	3	41	1	43	11	51	10	0	7	53	5	41
16	3	37	1	39	11	47	9	56	7	49	5	36
17	3	33	1	35	11	43	9	52	7	45	5	32
18	3	29	1	32	11	40	9	48	7	41	5	27
19	3	25	1	28	11	36	9	44	7	37	5	23
20	3	21	1	25	11	33	9	40	7	32	5	19
21	3	17	1	21	11	29	9	36	7	28	5	15
22	3	13	1	17	11	25	9	32	7	24	5	11
23	3	9	1	14	11	22	9	28	7	20	5	6
24	3	5	1	10	11	18	9	24	7	15	5	2
25	3	2	1	7	11	15	9	21	7	11	4	57
26	2	53	1	3	11	11	9	17	7	7	4	53
27	2	54	0	58	11	7	9	13	7	3	4	49
28	2	50	0	56	11	3	9	9	6	58	4	44
29	2	46	0	52	10	59	9	5	6	54	4	40
30	2	42	0	49	10	56	9	1	6	49	4	35
31	2	39	0	45			8	57			4	30

A Table of the Right Aſcenſion of 46 Eminent Fixed Stars throughout the *Zodiack*.

The Names of the STARS.	R. Aſcen. H.	M.
South Star in the Whale's Tail	0	27
The Girdle of *Andromeda*	0	51
The formoſt Horn of the Ram	1	36
Whales Belly	1	46
South Foot of *Andromeda*	1	43
The Whales Jaw	2	45
The Brighteſt of the Seven Stars	3	28
Aldebaron, or the Bull's Eye	4	17
Capella, or the Goat	4	52
Formoſt Shoulder of *Orion*	5	8
Orion's Head	5	17
Middlemoſt in *Orion's* Belt	5	20
Cyrius, or the Great Dog	6	30
Procion, or the Leſſer Dog	7	22
The Lowermoſt Head of the Twins	7	26
North Aſcellus	8	23
South Aſcellus	8	25
Cor Leonis or the Lyon's Heart	9	50
Cauda Leonis, or the Lyon's Tail	11	32
Vindemiatrix	0	46
Spica Virginis	1	8
Arcturus	2	1
Left Shoulder of *Bootes*	2	19
South Ballance	2	23
North Ballance	3	3
Bright Star of the Crown	3	25
Antares, or the *Scorpion's* Heart	4	9
Hercules Right Shoulder	4	11
Hercules Head	5	0
Head of *Ophyncus*	5	20
Lyra, or the Harp	6	22
Vultures Tail	6	51
The Swan's Bill	7	18
The Vulture	7	35
Lowermoſt Horn of the Goat	8	30
Swan's Breaſt	8	11
Swan's Tail	8	31
Lowermoſt Wing of the Swan	8	33
Girdle of *Cepheus*	9	25
Pegaſus Mouth	9	28
Right Shoulder of the Water-bearer	9	47
Fomahaunt	10	39
Scheat	10	48
Mercha	10	49
Head of *Andromeda*	11	32
Caſſiopea's Chair	11	53

V. *To find the* Hour *of the* Night *by the* Stars.

Example.

OBserving by ſome of the ways before preſcribed, that upon the laſt Day of *December*, I found that the *Great Dog*'s Horary diſtance from *the Meridian* was 9 Hours, 22 Minutes; which ſet down as followeth:——Then (by the Firſt *Table*) I find the Complement of *the Sun*'s *R. Aſcenſion* on the 31*ſt* of *December*, to be 4 Hours 30 Minutes; which ſet down under the former------And (by the Second *Table*) I find *the R. Aſcenſion* of *the Great Dog* to be 6 Hours, 30 Minutes; which ſet down under the Two former: All which being added together, do make 20 Hours, 22 Minutes. Which (becauſe it is above 12 Hours) ſubſtract 12 Hours from the Sum, the Remainder will be 8 Hours, 22 Minutes for the true Hour of the Night.

	H.	M.
The *Great Dog's* Hour	9	22
The Comp. of the *Sun's* R. Aſcenſion, *December* 31	4	30
The *Great Dog*'s R. Aſcenſion	6	30
The Sum	20	22
Subſtract	12	00
There Remains	8	22

For the true Hour of the Night.

VI. *Another way to find the* Hour *of the* Night, *by ſeeing any* Star *upon the* Meridian.

I have already intimated, how you may know by *the Dial* upon a Poſt before mentioned, what *Stars* are upon *the Meridian* at any time of *the Night*; It reſteth now to ſhew how (from thence) to find *the Hour* of *the Night*.

Example.

Suppoſe, therefore, that upon the 20*th* Day of *January*, I find *Aldebaron*, or *the Bull's Eye*, to be upon *the South* part of *the Meridian*, then (by the Firſt *Table*) I find the Complement of the *Sun's R. Aſcenſion* on the 20*th* of *January*, to be 3 hours, 6 minutes-----And (by the Second *Table*) *the R. Aſcenſion* of *the Bull's Eye* (or *Aldebaron*) to be 4 hours, 17 minutes. Theſe Two added together make 7 hours, 23 minutes, for the true Hour of the Night.

	h.	m.
The Comp. of *the Sun's R. Aſcen.* Jan. 2	3	6
The R. *Aſcenſion* of *the Bull's Eye*	4	17
Their Sum	7	23

Which is the true *Hour* of the Night, *Jan.* 20.

CONCLUSION.

THere are ſeveral other ways to find the Hour of the Day or Night, by the *Sun*, *Moon*, or *Stars*; as by *Quadrants*, *Rings*, *Cylinders*, *Walking Staves*, *Nocturnals* of ſeveral Kinds: By the Shadow of the Moon upon a Sun-Dial, and many other ways. But foraſmuch as theſe Inſtruments are to be had at any Shop where *Mathematical Inſtruments* are Made, or Sold, I ſhall forbear to ſay any thing of them in this place: Only I ſhall exhibit to the Practitioners View, Two Tables, by help of which the Uſual Furniture may be put into all ſorts of Sun-Dials, *Plain*, *Concave*, *Convex*, *Regular* or *Irregular*. And alſo all the foremention'd *Quadrants*, *Rings*, and other *Inſtruments*, which ſhew *the Hour* and *Azimuth* of the Sun by *the Altitude* thereof, are made.

A Table shewing the *Altitude* of the Sun for every Hour and Quarter, at his Entrance into every of the XII Signs of the *Zodiack*, in the Latitude of *London*, 51 deg. 32 min.

Hours	♋ Altit.		♊ ♌ Altit.		♉ ♍ Altit.		♈ ♎ Altit.		♓ ♏ Altit.		♒ ♐ Altit.		♑ Altit.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
XII	61	59	58	41	49	59	38	28	26	57	18	15	14	57
1	61	49	58	34	49	52	38	22	26	52	18	10	14	52
2	61	23	58	8	49	32	38	4	26	37	17	56	14	39
3	60	40	57	27	48	57	37	36	26	12	17	34	14	18
I XI	59	42	56	33	48	10	36	50	25	37	17	3	13	48
1	58	29	55	25	47	11	36	5	24	53	16	24	13	9
2	57	4	54	6	46	1	35	4	24	1	15	36	12	23
3	55	29	52	34	44	40	33	55	22	59	14	19	11	29
II X	53	45	50	55	43	11	32	36	21	49	13	36	10	28
1	51	53	49	7	41	52	31	8	20	31	12	24	9	19
2	49	54	47	12	39	47	29	34	19	6	11	6	8	3
3	47	51	45	13	37	57	27	53	17	36	9	42	6	41
III VIII	45	42	43	7	35	59	26	5	15	57	8	11	5	13
1	43	31	40	58	33	57	24	12	14	13	6	34	3	39
2	41	15	38	45	31	49	2	25	12	23	4	51	1	59
3	38	59	36	30	29	40	20	13	10	30	3	5	0	15
IV VII	36	42	34	14	27	28	18	7	8	32	1	13		
1	34	23	31	56	25	13	15	58	6	31				
2	32	4	29	37	22	56	13	46	4	25				
3	29	43	27	16	20	37	11	32	2	16				
V VII	27	23	24	56	18	18	9	17	0	5				
1	25	4	22	37	15	59	6	58						
2	22	43	20	17	13	39	4	39						
3	20	28	17	57	11	19	2	20						
VI VI	18	12	15	42	9	0								
1	15	58	14	0	6	44								
2	13	46	11	13	4	22								
3	11	37	9	1	2	8								
VII V	9	30	6	52										
1	7	25	4	46										
2	5	24	2	42										
3	3	27	0	43										
VIII IV	1	34												

A Table of the Sun's *Azimuth* from the *South* for every Hour and Quarter, at his Entrance into every of the XII Signs of the *Zodiack*, in the Latitude of *London*, 51 deg. 32 min.

Hours	♋ Azim.		♊ ♌ Azim.		♉ ♍ Azim.		♈ ♎ Azim.		♓ ♏ Azim.		♒ ♐ Azim.		♑ Azim.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
XII	00	0	00	0	00	0	00	0	00	0	00	0	00	0
1	7	10	5	53	5	37	4	55	3	54	3	53	3	52
2	14	22	13	24	11	10	9	40	7	57	7	40	7	10
3	21	27	19	53	16	44	14	13	12	11	11	10	10	37
I XI	27	56	26	0	22	13	18	52	16	22	14	45	14	13
1	34	14	32	1	27	32	23	22	20	15	18	17	17	38
2	40	12	37	40	32	41	27	41	24	6	21	53	20	59
3	45	39	43	4	37	32	32	8	29	44	26	30	24	25
II X	50	51	47	57	42	5	36	24	31	50	28	52	27	49
1	55	31	52	41	47	4	40	30	35	35	32	18	31	0
2	51	43	57	11	50	51	44	26	39	10	35	39	34	17
3	64	11	61	18	54	33	48	13	42	36	48	50	37	34
III IX	68	10	65	16	58	47	51	56	46	2	42	4	40	38
1	71	52	69	56	62	33	55	33	49	25	45	16	43	40
2	75	26	72	37	66	10	58	59	52	43	48	21	46	42
3	78	48	76	5	69	28	62	22	55	59	51	22	49	40
IV VIII	82	0	79	20	72	57	65	40	59	0	54	22		
1	85	8	82	42	76	12	68	53	62	8				
2	88	10	85	38	79	23	72	6	65	12				
3	91	9	87	50	82	28	75	7	68	12				
V VII	94	15	91	34	85	30	78	6	71	10				
1	96	51	94	25	88	27	81	9						
2	99	38	97	16	91	23	84	6						
3	102	25	100	7	94	22	87	4						
VI VI	105	8	102	54	97	12								
1	107	22	104	58	100	2								
2	110	36	108	26	103	0								
3	113	88	111	14	105	54								
VII V	116	3	114	3										
1	118	50	116	54										
2	121	41	119	47										
3	124	29	122	37										
VIII IV	127	24												

An Eaſie Way to make a Spot-Dial *within a Room, without any Calculation, or Uſe of Threads, &c.*

THis may be done Two Ways, either 1. By the direct Rays of the Sun thorough a little Hole in the Glaſs: Or, 2. By the Reflected Rays from a ſmall ſhow of a *Looking Glaſs*, ſet in a convenient place in the Window. The firſt way I account not ſo convenient, becauſe the Hour-Lines will many of them fall on the Floor, Tables, *&c.*

The other way is this, 1. Take a little piece of *Looking-Glaſs* (common Glaſs may ſerve, but not ſo well) about the breadth of a ſixpence, faſten this ſo in the bottom of the Window (or ſides if you ſee good) that the Rays of the *Sun* may not be hindred from coming upon it (or as little as may be).

2. It is neceſſary that you have with you, upon this occaſion, a rightly ſet Sun Dial Quadrant, *&c.* The like to find the ſeveral hours of the Day by being ſo provided, upon a clear Sun-ſhine Day, watch the Sun-Dial till the ſhadow is juſt upon ſome Hour-Line, then immediately obſerve where your ſpot of light falls on the Wall or Ceiling; and there with black Lead, or the like, make a mark, and alſo ſet a Figure by it of what hour it is. Do the ſame by the next hour, and ſo on as long as the *Sun* ſhines, ſo as to give a ſpot in the room.

3. About a Month or more after, obſerve in like manner by your *Sun Dial* where the ſpot falls on the Wall or Ceiling at the ſeveral hours, and make marks and figures as you did before, and you will find the hour ſpots now to fall far diſtant from where they were a Month ago.

4. This done with a long Ruler and black Lead, Ink, or the like, draw a long line joyning each pair of marks that have the ſame figure annexed to them, and each ſuch line produced at both ends ſhall be the hour line for that hour; to which put, where you ſee fit, its proper figures, and in like manner do by the reſt.

Note, 1. The beſt time of the year for the Operation is the *Spring*, or *Autumn*; other times of the year may ſerve, ſo as the times of obſervations be either *both before*, or *both after the ſolſtizes*, *June* 11, *December* 11.

Note 2. When an hour-line drawn upon the Ceiling comes to the Wall, there will be then needful a third obſervation where the ſpot falls on the Wall at that Hour; at which having made a mark, draw a ſtraight line from the end of the line on the Ceiling, down to your mark, for that hour on the Wall; ſo will you have that hour-line compleat. In like manner you may do by the reſt if there be need.

Note, 3. Your Glaſs, and what it ſtands on muſt be very firm; for if it be moved ever ſo little, 'twill not agree with your hour-lines.

They who pleaſe and have a convenient Room and Window, may by the very ſame method draw hour-lines to a ſpot of light coming throw a little hole in a Quarry of Glaſs, but then it will be convenient that the reſt of the Quarry be darkned by paſting a piece of black paper, or the like, on the inner ſide, leaving the hole in the middle.

FINIS.

RECREATIONS
Cryptographical.

THIS *Cryptographical Art* is of great uſe in many reſpects, but principally in *War*. And if we conſider to what neceſſities great Princes and Potentates have been put unto to communicate their Minds or Intentions to their Correſpondents before the improvement of this Art, it would make a man but of a reaſonable underſtanding admire: Wherefore I ſhall here give you a very brief account of ſome of them, and afterwards ſhew ſome other more Artificial and abſolute ways whereby Communication may be had both ſecretly and ſwiftly.

I. *Of ſome Antient ways.*

1. When *Hiſtæus* the *Mileſian* was kept Priſoner by *Darius*, and deſpairing of his return home, unleſs he could find out ſome way to ſend to *Ariſtagoras* (who was his Subſtitute at *Miletum*) to perſwade his Revolt from *Darius*, but knowing that all Paſſages were ſtopt, and all Meſſengers ſtrictly examined and ſearched; he at length found out this (both tedious and weak) courſe. He got a truſty Servant of his, the Hair of whoſe Head he cauſed to be ſhaved off, and then upon his bald Pate he writes his mind to *Ariſtagoras*, keeping his Servant privately about him till his Hair was ſomewhat grown, and then bad him haſte to *Ariſtagoras*, and bad him cauſe him to be ſhaven again, and then upon his Head he ſhould find what his Lord had writ unto him.

2. When *Harpagus* had a mind that *Cyrus* ſhould haſten his Invaſion on *Media* (*Cyrus* being then in *Perſia*,) his Letter he ſent to him he incloſed in the belly of a Hare.

3. The *Lacedemonians* uſed this way to communicate their Letters to their Generals abroad, which in caſe of Interception, the Contents ſhould not be diſcovered. They had made two round Sticks or Cylinders of the ſame length and thickneſs, one of which was delivered to the General when he ſet forth, and the other was kept at home by the chief Magiſtrate. When occaſion was, they wound about the Stick a long narrow ſcrowl of Paper or Parchment, in ſuch manner, like the threads of a Screw, ſo that the edges of the Paper or Parchment ſhould always lie cloſe together; the Paper thus fitted to the Stick (or Cylinder) they wrote their Letters upon the tranſverſe junctures of the Paper, ſo that one

one part of each Word or Letter was in one place, and the other in an other: When they had finiſhed their Writing, they took off the ſcroll and ſent it to the General, who knew how to apply it; and if it had been intercepted, the Enemy could make nothing of it; and this manner of Writing they called *Scytale*.

4. The *Mileſians* uſed to Write their Letters, and incloſe them in a Cake, or Loaf of Bread.

5. The *Romans* into the hilt or ſcabord of a Sword.

6. The *Perſians* uſed to fix them to Arrows, and ſhoot them to places appointed near hand.

7. *Toroſthanes* by a Pigeon, ſtained with Purple, gave notice of his Victory at the Olympick Games, to his Father in *Ægina*, the ſame day that he obtained it. And this way of conveyance by Pigeons at great diſtances is of wonderful Celerity: For in *Aleppo* they have Pigeons which will fly (with Letters fixed to their Legs, or about their Necks) from *Babylon* to *Aleppo* in 48 hours, which is accounted 30 days journey. The manner of tutering their Pigeons is this: They take them when they ſit on their Neſts, tranſporting them in open Cages, and return them with Letters; who will never give reſt to their Wings until they come to their Young. But let this ſuffice for ſome of the ways uſed by the Antients, I will proceed now to more Artificial ways of effecting the ſame.

I. *By the 24 Common Letters of the Alphabet.*

THE *Letters* of the *Alphabet*, by which you would repreſent your Mind to your *Correſpondent*, may be ſo diſpoſed in the form of a Right Angled *Parallelogram* (with any Number of *Cyphers*, *Intervals*, or (rather) *ſuperfluous Letters* between every *ſignificant Letter*;) ſo that *the Writing* may be read by your Correſpondent any of theſe ways, which of them ſhall he agreed upon between you; As,

1	In upright Columns	Deſcending with 4 intervals.	I
2		Aſcending with 3 intervals.	II.
3		Deſcending and Aſcending alternately 2 inter.	III
4		Aſcending and Deſcending alternately 5 inter.	IV
	Or,		
5	Horizontally in Lines	Direct with 7 intervals.	V
6		Retrograde 5 intervals.	VI
7		Direct and Retrograde alternately 2 Inter.	VII
8		Retrograde and direct alternately 3 inter.	VIII

(As in Example.)

Example I.

Suppoſe that I would write to my Correſpondent theſe Words,

Your Uncle is now Dead.

Wherefore I write it in *Upright Columns* deſcending, with 4 *Intervals:* (Which my Correſpondent is ſuppoſed to know:) Wherefore I firſt make 4 Pricks, one under another, and under them I make the firſt Letter of my intended Writing [*y*], under that four Pricks more, and under them the next Letter [*o*], then two Pricks under, and (becauſe your Column will hold no more Pricks below) two above in the next Column, and uuder them [*u*]; *&c.* As in the *Parallelogram* following.

Points

.	.	u	.	.	.	.	a
.		.	.	i	.	.	.
.	u	.	.	.	.	d	.
.	.	.	l	.	.	.	.
y	.	.	.	.	o	.	.
.	.	n	.	.	.	.	d
.	.	.	.	s	.	.	&c
.	r	.	.	.	.	e	.
.	.	.	e	.	.	.	.
o	.	.	.	.	w	.	.
.	.	c	.	.	.	.	.
.	.	.	.	n	.	.	.

Points made for the Intervals.

s	r	u	s	a	k	t	a
g	g	b	i	i	r	r	s
b	u	r	d	b	l	d	t
c	l	y	l	d	m	m	a
y	o	x	o	c	o	s	l
l	m	n	q	f	q	g	d
f	y	t	s	s	r	y	&c
t	r	p	b	e	g	e	m
f	g	o	e	g	b	p	o
o	q	g	m	i	w	l	n
m	o	c	y	h	l	g	t
b	l	m	t	n	s	q	v

Superfluous Letters to ſupply the Intervals.

I have been ſomewhat large in this, but it is becauſe I will give *Examples* in all the reſt, but not in a double Nature, as I have done here, with Points only to ſupply the ſuperfluous Letters.

Example II.

Haſte from London, for you are betrayed.

Upright Columns Aſcending, with 2 Intervals.

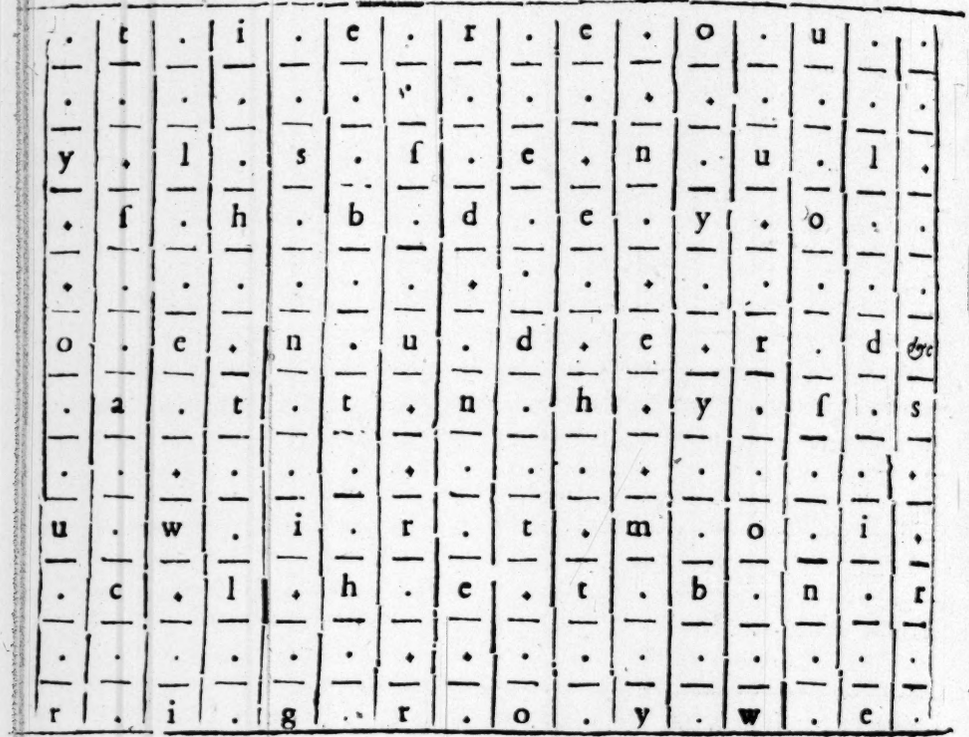
.	.	.	o	.	.	.	a	.	.	.	.
.	.	o	.	.	.	y	.	.	.	y	.
.	e	.	.	.	f	.	.	.	t	.	.
a	.	.	.	d	.	.	.	e	.	.	&c
.	.	.	L	.	.	.	u	.	.	.	d
.	.	r	.	.	.	r	.	.	.	a	.
.	t	.	.	.	n	.	.	.	e	.	.
h	.	.	.	n	.	.	.	r	.	.	.
.	.	.	m	.	.	.	o	.	.	.	e
.	.	f	.	.	.	o	.	.	.	r	.
.	s	.	.	.	o	.	.	.	b	.	.

Ex-

Example III.

Your Caſtle will this Night be Surrendred to the Enemy by your own Souldiers.

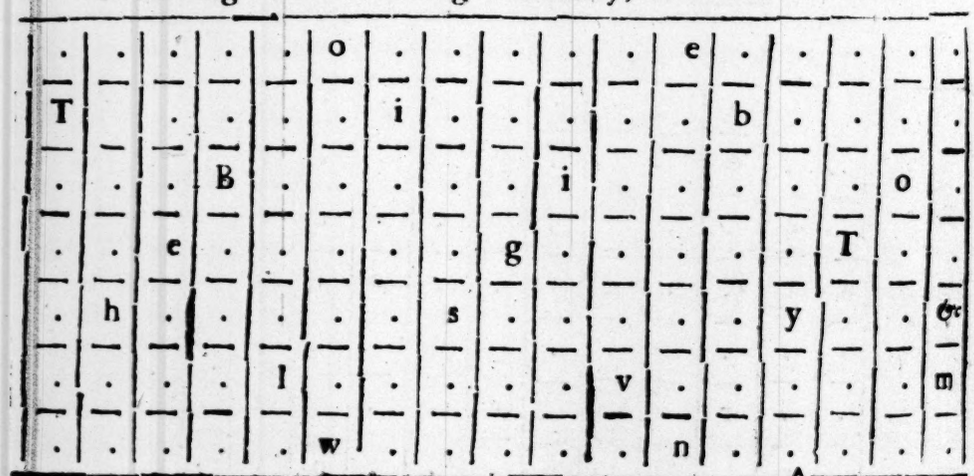
Deſcending and Aſcending Alternately, with 2 Intervals.



Example IV.

The Blow is given by Tom.

Aſcending and Deſcending alternately, with five Intervals.

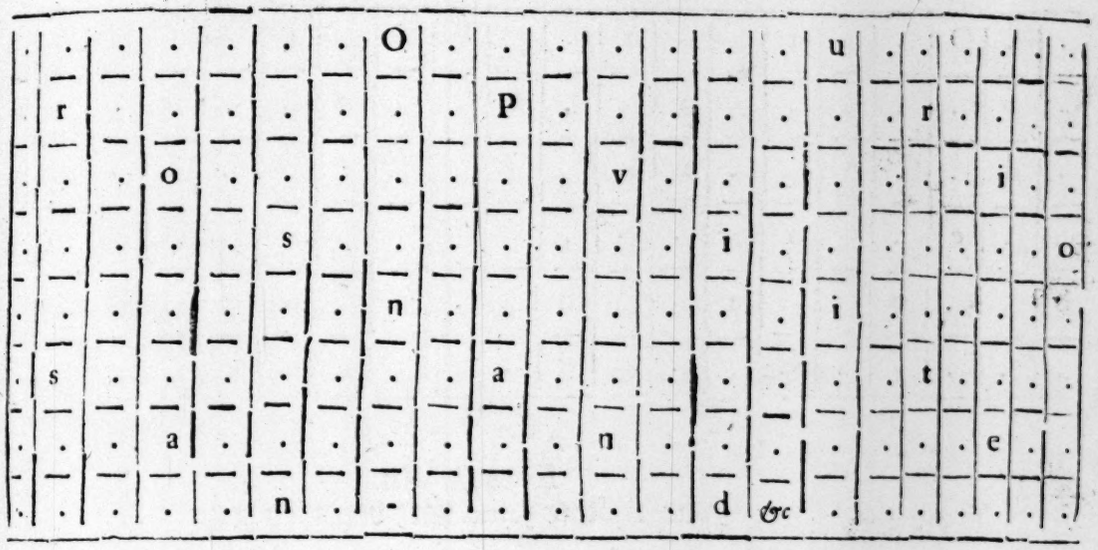


Example

Example V.

Our Provision is at an end.

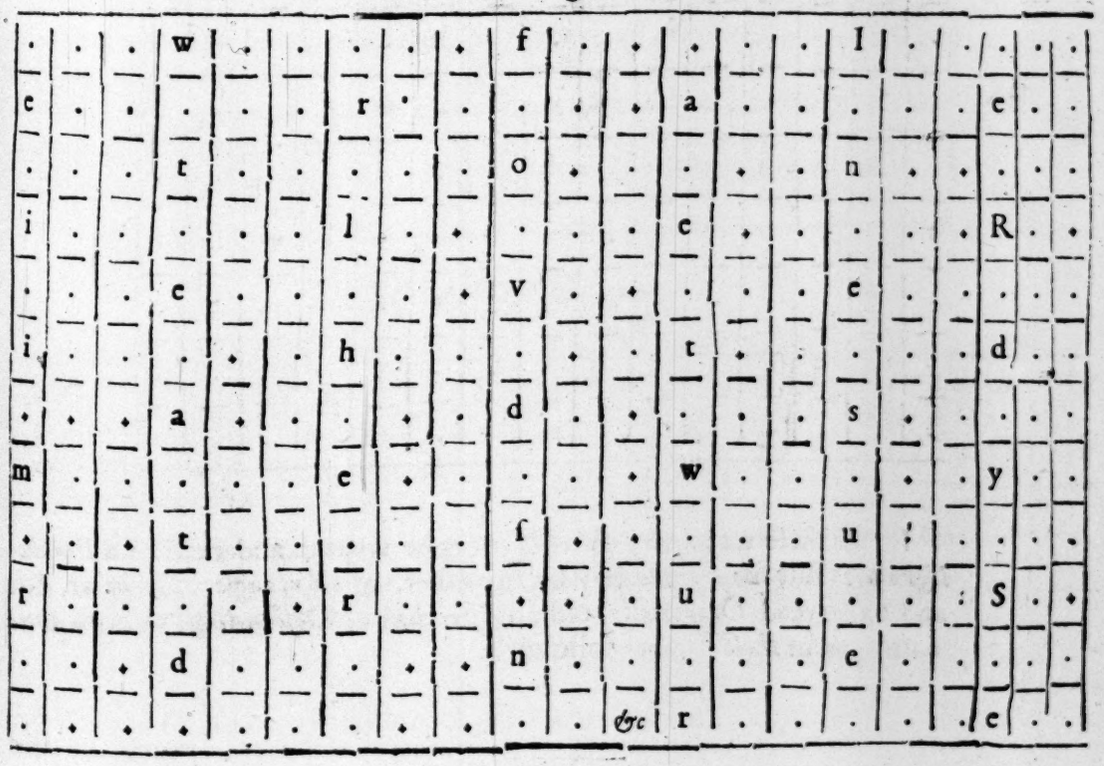
Horizontal direct, with seven Intervals.



Example VI.

If we are not Relieved this Day, we must Surrender.

Horizontal, but Retrograde, with 5 Intervals.



Example VII.

Our Victuals and Amunition are both at an end.

Direct and Retrograde alternately, 2 Intervals.

.	.	O	.	.	u	.	.	r	.	.	V	.	.	i	.	.	c	.	.
.	n	.	.	a	.	.	s	.	.	l	.	.	a	.	.	u	.	.	t
.	d	.	.	A	.	.	m	.	.	u	.	.	n	.	.	i	.	.	t
.	.	e	.	.	r	.	.	a	.	.	n	.	.	o	.	.	i	.	.
b	.	.	o	.	.	t	.	.	h	.	.	a	.	.	t	.	.	a	.
.	.	.	.	.	.	.	.	&c	d	.	.	n	.	.	e	.	.	n	.

Example VIII.

Your Brother James hath betrayed you.

Horizontal, Retrograde and Direct, alternately, with 3 Intervals.

.	.	.	u	.	.	.	o	.	.	.	Y	.	.	.
r	.	.	.	B	.	.	.	r	.	.	.	o	.	.
.	r	.	.	.	e	.	.	.	h	.	.	.	t	.
.	.	J	.	.	.	a	.	.	.	m	.	.	.	e
.	.	.	a	.	.	.	h	.	.	.	s	.	.	.
t	.	.	.	h	.	.	.	b	.	.	.	e	.	.
.	y	.	.	.	a	.	.	.	r	.	.	.	t	.
.	.	e	.	.	.	d	.	.	.	y	.	.	.	o
.	.	.	.	.	.	.	.	.	.	&c	u	.	.	.

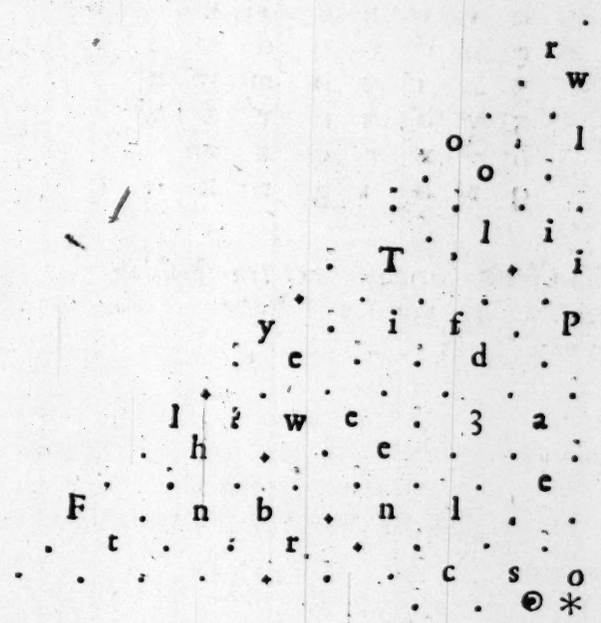
In all these ways may *secret Writing* be written and read in a *Parallelogram*: But *the Figure* may be any other, as a *Triangle*, *Rhombus*, &c. and to be read *Diagonally*, either *Ascending* or *Descending*, or both *alternately*; as in *these Figures* following.

Example

Example IX.

Fly For the Town will be fired in three places.

In a Triangle *Diagonally* Aſcending, with 2 Intervals.



II. *By* Tranſpoſing *of the* Letters *of the Alphabet.*

Julius Cæſar, when he communicated his mind to his *Correſpondent* (to whom he had before declared his *manner of Writing*,) he did it by this way of *Tranſpoſition*, putting *D*, the *fourth Letter*, for *A*, the *firſt Letter of the Alphabet*; and ſo throughout *the whole Alphabet*, as in this *Scheme*, which will ſerve for a *Clavis* to unlock *ſecret Matter* written according to *this Tranſpoſition*, which may be varied Millions of ways.

The Scheme, or *Clavis.*

a	b	c	d	e	f	g	h	i	k	l	m	n	o	p	q	r	ſ	t	v	w	x	y	z
-	-	-	-	-	-	-	-	--	-	-	--	-	-	-	-	-	--	-	-	--	-	-	-
d	e	f	g	h	i	k	l	m	n	o	p	q	r	s	t	u	w	x	y	z	a	b	c

The *Scheme* thus prepared, and *the Correſpondent* knowing *the Order of Tranſpoſition*, Suppoſe that I would write theſe words; To be read in Upright Columbs Deſcending:

As you tender your own safety get out of Italy, and repair to Paris with what speed you can.

D	g	z	k	M	v	S	l	h
w	h	q	h	x	h	d	z	g
b	v	w	x	d	s	v	l	b
r	b	d	r	o	d	m	d	r
y	r	i	y	b	m	w	x	y
x	y	h	x	d	v	z	w	f
h	v	x	r	q	x	m	s	d
q	r	b	i	g	r	x	h	q

Now by this *Clavis* (or any other *Transposition* (or varying) of the *Letters of the Alphabet*) you may frame your *Writing* to be read any of the former ways, as *Downwards*, *Upwards*, *Direct*, *Retrograde*, *Diagonal*, &c.

But this way (in my judgment) for Secrecy, is more liable to be discovered (if intercepted) than the former ways, by supplying Intervals by (any number of) superfluous Letters; for that in this way there are no more Letters used than will be required to make out the Words *intended; which makes it very liable to discovery: Wherefore I shall in the next place shew a way whereby to use other* Characters *instead of the* Letters *of the* Alphabet; *with Intervals of a different Character interposed.*

III. By Other Characters, *which shall Represent the* 24 Letters *of the Alphabet.*

NOT to invent *new*, *strange*, and *insignificant Characters*, such as your *Steniographers*, or *Short Hand Writers* use; I shall here present you with an *Alphabet* of known *Characters*; *viz.* with those of *the Astronomical Signs*, *Planets* and *Aspects*; Whereof *the Twelve Signs*, *the Seven Planets*, and *the Five Aspects* together makes Twenty Four, equal to the Number of *the Letters of the Alphabet*. And of these let *the five Aspects* represent *the five Vowels*, {a e i o u} {☌ ☍ □ △ ✶} And let *the Characters* of *the Seven Planets* represent *the Letters* towards the beginning of the *Alphabet*: And the Characters of the 12 *Signs* the *Letters* towards the end of *the Alphabet*; as in the following *Scheme*, or *Astronomical Clavis*.

The Astronomical Clavis.

a	b	c	d	e	f	g	h	i	k	l	m	n	o	p	q	r	s	t	u	w	x	y	z
☌	♄	☿	♂	☍	☉	♀	☿	□	☾	♈	♉	♊	△	♋	♌	♍	♎	♏	✶	♐	♑	♒	♓

The *Key* being thus agreed upon between you and your Correspondent, it will be very easie to communicate your mind to him in any of the forementioned Methods, *viz.* by *Parallelogram*, *Triangle*, &c.

And

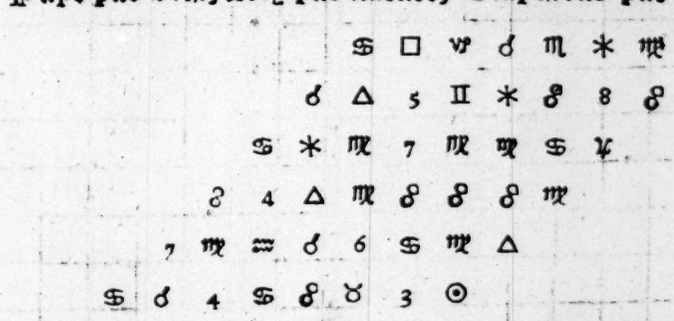
And so supposing you would write to your *Correspondent*, to be read in a *Parallelogram Descending and Ascending alternately*; and at the end of every Word (for distinction) to interpose some one of the Nine *Digits*, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, or 9, and at the close of all this *Character* ⊕.

Suppose you would write these Words, to be read in Columbs Descending and Ascending alternately, with a Digit Figure between each Word, and at the End ⊕.

I have escaped out of the Castle in Disguise, and do lodge at the Ball in the High Street, by the Name of Mary Grice.

This way of Writing is far more difficult to be discovered than the former by Letters only, though as much, or more to be suspected when intercepted. Let your Writing be in any Language, it may be by these Characters expressed, and in any form: As suppose I would write this *French* Proverb, in a *Rhomboyades* form, to be read in a *Diagonal* Descending and Ascending alternately:

Pape par voir, Roy par nature, Empereur par force.



IV. *By Knots tyed (or other Marks made) upon a String.*

FOR the effecting of this, the *Two Correspondents* must each of them have a fine thin Board, or piece of Brass, or Copper, made in the form of a *Parallelogram*, or *Lang Square*; at one end whereof let be written or engraven the Twenty Four Letters of the Alphabet, either in order, or transposed, which shall be agreed upon; and between each Letter, let there be a faint or occult Line drawn or gauged quite through the Board, or Plate, from end to end, representing Twenty Four Columbs: Also, let the Two longer sides of the Board, or Plate, have Notches cut in it, at about one quarter of an Inch distance from each other; as in the Figure is expressed.

The

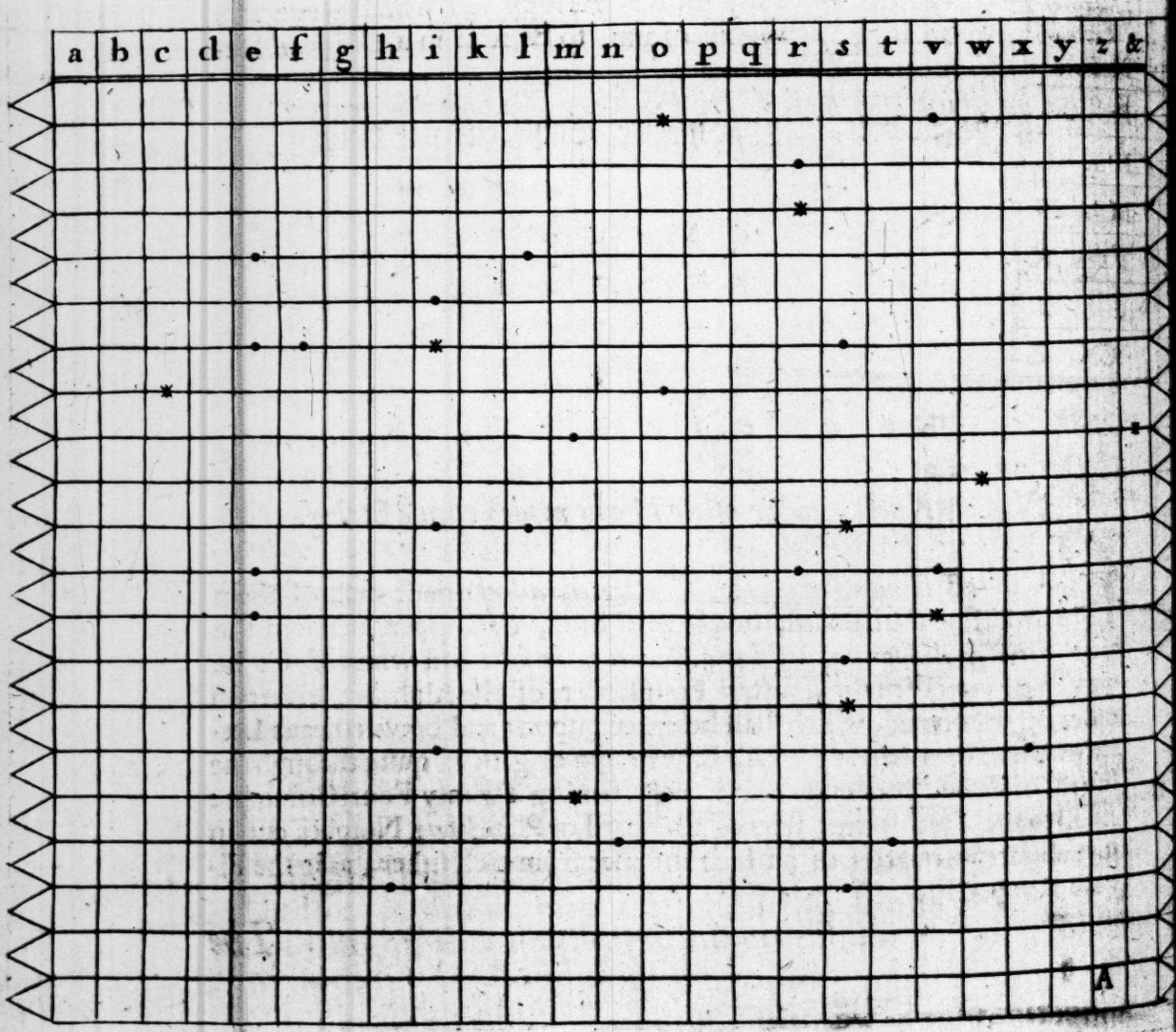
The Figure of the Board *or* Plate, *with the Alphabet.*

a, b, c, d, e, &c.

When you would communicate your mind to your Correſpondent, take a Piece of even and well-twiſted Packthread, at one end whereof tie a Nooſe to be put over an Hook towards the top of the Board on the back ſide thereof, made for that purpoſe. Then bring the *String* over at *a*, and bring it to the Notch *b*, and from *b*, over at *c*, and guide it to *d*, and ſo continue till you have as many Lines of String as will contain your intended Writing. The *Board*, and *String* being thus prepared, ſuppoſe you would expreſs theſe Words,

Our Relief is come, and will ſerve us Six Months.

Upon *the String*, under the Letter *o*, tie a Knot, or (rather) with a Needle, and ſome light coloured Thread, or Silk, ſtich through that part of *the String* which lies under *o*, do the like under *u*, and in the next line under *r*, ſo have you your firſt word *Our* expreſſed upon the String. And doing ſo with all the reſt, you will find the *Knots* or *Marks* upon *the String* when it is upon *the Board* or *Plate*, as in *the Figure*: Which being taken off, and conveyed to your *Correſpondent* (who knows well enough how to apply it to his *Board*) it will declare your Writing very plainly, and void of all ſuſpicion, if intercepted.



A NEW CHARACTER *Easie to Learn and Write, difficult for a Stranger to decipher, Legible by the Motion of the Fingers, wherein each Line of Writing shall contain Two Lines of Sense.*

An Alphabet of the First Forme. } *The Ground Line.*

a	b	c	d	e	f	g	h	i	k	l	m	n	o	p	q	r	s	t	v	w	x	y	z

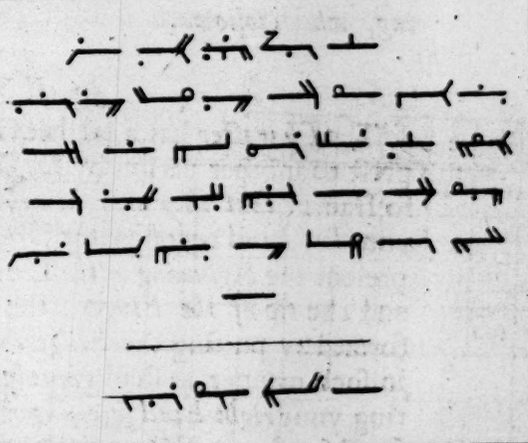
Or thus

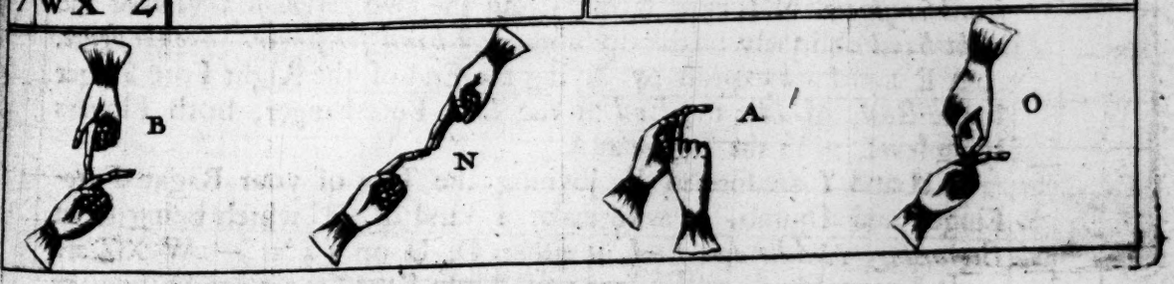
An Alphabet of the Second Forme.

·	A	E	I
·	O	V	Y
/	B	C	D
//	F		G
/	H	K	L
/·	M		N
\	P	Qu	R
\\	S		T
7	W	X	Z

An EXAMPLE

A New Character
Easy to learn and write
difficult for a Stranger
to decipher legible
by the Fingers.





EVery Letter of this Alphabet is built upon a short line, thus (—), which, for distinction sake, may be called *the Ground-Line.* The several letters are formed by the addition of a few small strokes, or points, which represent divers letters, according as they stand at the *Beginning, Middle,* or *End of the Ground-Line.*

I have given the Alphabet in Two Forms; The first as plainer to be understood, the second Form as easier to be remembred. At the Top of

of the ſecond Alphabet you have a *Point*, or *Speck*, and the letters A E I, which denotes, that a *Point* at *the Beginning* of *the Ground line* (either above it, or under it) ſtands for A, the firſt Letter of the Three. If the *Point* be over (or under) *the Middle* of *the Ground Line*, it is E, the middle Letter; and if over (or under, *the End*, it is I, the third letter at the Right Hand. So in the middle Rank is (') H K L, which ſhews that this oblique ſtroke (') ſet at *the Beginning* of *the Ground Line*, is H, at *the Middle* K, at *the End* L; and ſo of the reſt.

That your Writing may look more gracefully, let all your *Ground-Lines* be at equal diſtances from each other; and when you end a word leave one line blank, as you will ſee in the Example annext.

It will oft fall out, that Two, and ſometimes Three Letters may be expreſt ſucceſſively upon one ſide of *the Ground-Line*, *viz.* when after a Letter, whoſe place is at *the Beginning*, another follows immediately, whoſe place is in *the Middle*, or *End* of *the Ground-Line*, as you will find in the Syllable *be*, *ben*, *ſu* *ſun*, *wet*, &c. which Syllables rightly written, will ſeem but one Letter.

When you have written a Line on *the upper ſide* of your *Ground-Line*, begin again on *the under ſide*, and go on with your matter; ſo will every Line be *double written*; which will take up leſs room than other Writing, and render it very difficult to be deciphered.

Inſtead of more Directions there is an Example in the Scheme foregoing, which follows.

An EXAMPLE.

The *Character* has a farther Convenience, that it may be read or expreſt to another perſon by *the Motion of the Fingers*, the *Letters* being ſo framed *that they may be eaſily ſo expreſt*, thus. Let the *fore-finger of* your *left hand* repreſent *the Ground-line*. Let the *root* of the *Finger* repreſent the *beginning of the Line*, the *middle joynt*, the *middle of the Line*, and the *tip of the Finger*, the *end of the Line*; The ſeveral *letters* are formed by putting the *Fingers* of the *right hand* upon this *ground finger* in ſuch manner as ſhall repreſent the *Letter*. So B is expreſt by putting your *right-hand fore-finger* perpendicularly on the *root* of the *left-hand foreſinger*. N. is expreſt by putting the two *foremoſt fingers* of the *right-hand* obliquely on the tip of the *left-hand foreſinger*. *See the figure.*

A E and I are expreſt by laying the End of the Right Fore-Finger In the *Root*, *Middle* and *End* of the Left Fore-Finger, both Fingers lying level, as in the *Figure* at A.

O U and Y are formed by joyning the Tips of your Right Fore-Finger and Thumb, ſo as to make a kind of O, whtch being ſet at *Beginning*, *Middle* and *End*, is either O, U or Y; as — W X Z are made by crooking, or bending your Right Fore-Finger, to imitate the crooked ſtroke for thoſe Letters.

When you come to the End of a word, you may give a Fillip with your Finger, to intimate as much. By this means Two Perſons may talk to each other without being heard or underſtood by By-ſtanders.

Note, *If you would have this way of Writing peculiar to your ſelf, ſo as that others, who have ſeen this Alphabet, may not read it, you may if you pleaſe, alter the Alphabet, by making the additional ſtrokes, &c. ſtand for other letters than here they are put for.* Some

Some other ways of Secret Writing.

I. IF you take ſome *Salalmonack*, beaten to Powder, and ſteeped in *Vinegar*, and write what you pleaſe therewith upon White Paper, letting it be thorough dry, and nothing will be diſcerned; but hold the Paper a little while againſt the Fire, and you ſhall ſee all that was written, as black as if it had been written with Ink.----If you write with the Juice of *Limon* it will do the ſame.

II. Take good *Roach-Allumb*, and boyl (or diſſolve) it in fair Water till it be very ſtrong, then write therewith upon *Venice*, or (thin Paper) what you pleaſe, ſo that when it is thorough dry, nothing will be ſeen: But take the written Paper, and draw it through a Baſon of fair Water, till it be thoroughly wet, and then what you writ will appear as if it were written with White upon the wet Paper.

III. If you write with *Urine*, *Milk*, or ſuch other Glutinous Moiſture, and when it is dry, throw fine Aſhes or Duſt thereupon, rubbing ſoftly upon the Writing, the Words or Letters will appear----Alſo if you write upon the Back of your Hand (or any part of your Body) with *Urine*, take ſome Paper and burn it, and with the Aſhes of the Paper rub the Writing over, and it will be very legible upon your Hand or Body.

RECREATIONS Magnetical.

CHAP. I.

Of the Magnet *or* Load-ſtone; *And of Experiments performed thereby.*

I. *Of the Name.*

IT is called *Load-ſtone*, or a Leading Stone, *eſt enim Lapis cujus ductu nautæ curſum inſtituunt*: In the *Germain* Tongue it is called *Seil-Steen*: In *Low-Dutch*, *Segel-Stein*: *i. e. Lapis Navigationes. Magnet-Stein*: In French *Aimánt:* In Spaniſh *Imán*: In Italian, *Magneté*, *Magneſia*: In Latin *Magnes*, *Magneſius*: In Greek μαγνὴς μαγνῆτις, *à Magnete* Inventore. But *Lucretius* writeth that its name is Derived from the Countrey *Magneſia*:

> The *Greeks* do call it *Magnes* from the *Place*;
> For that the *Magnets* Land it did Embrace.

Plato ſaith, ſome call it *Lapis Heraclius*, from the Name of *Heraclea*, a City of *Lydia*, where it was firſt diſcovered; and upon the ſame account *the Touch-ſtone* is called *Lapis Lydiæ*. *Theophraſtes* calleth it *Herculeum* for the ſame Reaſon; and others take it from *Hercules*: *Nicander* and *Pliny*, think it ſo called from one *Magnes*, a *Shepherd*; for it is reported that he found it by chance, his Hob-nail'd Shooes and Crook, ſticking faſt to it, as he was feeding his Flock in *Indea*. Others call it *Siderites*, which in *Greek* ſignifies *Iron*. By us it is called *Load-ſtone*, alluding to *the Two Stars* in *the Tail of the Celeſtial Bear*, called *Helice* and *Cynoſma*, which were ancientiy called *the Load-ſtars*, or *Leading-Stars*; and this *Stone* performing the Office of thoſe *Stars*, taketh its Name of *Lead*, or *Load-ſtone*.

II. *Where*

II. *Where these* Stones *are found, and of their* Colour, Weight *and* Force.

THey are found in divers parts of the World, and most commonly in *Iron-Mynes*: Of them there are divers sorts, differing one from another in *Colour*, *Weight*, and *Force* in attraction; but all of them agree in *Property*.

1. The best of these *Stones* come out of *East India*, from the Coasts of *China* and *Belgana*. They are of an *Iron*, or *Sanguine Colour*, *Massy* and *Weighty*. Some of them will attract, or lift up not much above their own weight, but others (well *Capt* and ordered) will take up five, ten, and some twenty times their own weight; and these (being but rarely found) are sold for their weight in Silver in *India*, where they grow.

2. There are some found in *Arabia*, and *the Red Sea*, which grow broad and flat, like a Paving-Tile. They are of a *Reddish Coloar*, and are not so weighty as those which grow in *China*, but their Vertue near as good, and will continue long upon a Needle touched on them.

3. Some of these *Stones* are found in the *Levant*, in the Island called *Elba*, near *Parta Ferraro*, and are there called *Calamita Preta*, or *Black Magnets*; These have no great Force, neither doth the Vertue they infuse last long.

4. There are others of a *White Colour*, and spungy, like an *Honey-Comb*, found in *High Albany*; they are lighter, but their Vertue is stronger than *the Black ones* before spoken of.

5. Of these *Stones* some are found at *Long-sound*, in *the Iron-Mines* in *Norway*, their Colour *Black*, mixed with *Grey*, but of an indifferent Force.

6. Some there are found in the *Mines* of *Carraca*, and *Cantabria*, in *Spain*; Others in *Bohemia*; and some in the *West* of *England*.

CHAP. II.

That the whole Globe *of the* Earth *hath true* Magnetical *Vertue*; *Proved by Experiment.*

1. TAke any Piece of *solid Earth*, that hath some toughness to hold together, and will abide the Fire (as *Brick*, or *Tobaccopipe-Clay*) fashion it in such sort that it be uniformly extended towards both ends (the *Oval* Figure will be very fit for this purpose) put it into a Charcole Fire, increasing the Heat by little and little, and with often blowing make it thoroughly red hot; Let it remain so for half an hour or more, that all superfluous moisture, and adverse qualities may thereby be consumed and separated from it: Then take it forth, and let it cool of it self, being laid upon a *Meridian-line* first drawn upon a Free-Stone elevated according

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to the Latitude of the place: And then it is certain, that this piece of *Earth* thus *ordered*, will ſenſibly ſhew you that it hath *true Magnetical Vertue*. For one end of a *Magnetical Needle* will covet towards one end of this prepared Maſs of Earth, and fly from the other: And that End which cooled toward *the South*, will draw the *true North End* of the *Needle*; and that end which cooled towards *the North*, will draw the *true South end* of *the Needle*. And for a farther infallible argument, do thus: Mark (with a piece of Chalk, or the like) what end of *the Maſs* drew *the North* end of *the Needle*, and put it into the Fire again; and when it hath been glowing hot half a quarter of an hour, take it out and cool it, being placed with that marked end towards the *North*; and that end now will draw *the South* end of *the Needle*, and *the North* end of *the Needle* will ſhun it, which before approached unto it.

CHAP. III.

That Magnetical Vertue may be infuſed into a Needle, *without the help of a* Load-Stone.

IRon being a Mineral of *the Earth*, and having a Sympathetical Quality with *the Loadſtone*, acquiring this Verticity from *the Magnetiſm of the Earth*; as in Bars of Windows which have been of long continuance: Take ſuch a Bar, and file both the ends thereof very ſmooth; then take a ſmall *Needle*, and touch it thereupon, the *South* end of *the Needle* upon *the North* (or upper) end of the Bar, and *the North* end of the *Needle* upon the *South* (or lower) end of the Bar: Such a *Needle* you will find to have reſpect, and conform it ſelf to *the magnetical Meridian* of the *Earth*.

CHAP. IV.

Of the Attractive *Virtue of the* Loadſtone.

1. IF you apply a piece of *Iron* to either of the *Poles* of a *Load-ſtone*, it will here hold it, and at a diſtance will alſo draw, or attract a ſmall piece of *Iron*, according to the Vigor or Indebility of the Stone.

2. Whatſoever ſtrength a *Stone* hath, it may be artificially improved to be greater, by applying two ſmooth and bright pieces of *Iron* to either *Pole* of the *Stone*, unto which *Irons*, the *Stone* will immediately impart its *Vigour*; And the *Stone* by this means will become far more vigorous than it was before theſe *Irons* were applied to its *Poles*, and bound thereto by *Braſs Hoops* or *Bands*, which is called *Capping of a Load-ſtone*.

3. A Piece of *Steel* having received ſtrength from the *Stone*, that will alſo attract another piece of *Steel* to it, in proportion as the Virtue the firſt piece hath received from the *Stone*. For if you touch a *Knife* (being clean) upon the *Stone*, that *Knife* ſhall take up a *Key*, or other bright piece of *Iron* of two *Ounces* weight: And that within the *Sphere* of the *Stones* activity, ſhall deliver the virtue into another piece of *Iron*, cauſing the two pieces of *Iron* to hang one to another.

CHAP. V.

Of the Sympathetical *and* Antipathetical Property *of the* Load-Stone.

1. WHen a *Needle* is touched upon a *Load-Stone*, the *North* and *South* ends of this *Needle* will apply themſelves to the *Poles* from whence they received their *Magnetical Life*, *viz.* the *North* end of the *Needle* to the *North* end of the *Stone*, and the *South* end of the *Needle* to the *South* end of the *Stone*; which denotes their mutual *Sympathy*.

2. But putting the *North* end of the *Stone*, to the *South* end of the *Needle*, when it hangs upon a *Pin*, the *South* end of the *Needle* will immediately fly away; and if you put the *South* end of the *Stone*, to the *North* end of the *Needle*, it will there diſcover the *Antipathetical* Nature; for it will fly away from it.

3. But a contrary *Operation* there is yet in the two *Needles* to that of the *Stone*: For if one of the *Needles* being hung upon a *Pin*, if you apply the *North* end of the other *Needle*, to the *North* end of that upon the *Pin*, it ſhall immediately fly away, which denotes a contrary *operation* in the *Needle*, to that of the *Stone*; and the *South* end of the one, will come to the *North* end of the other.

4. The ſame Property of *Sympathetical Coition*, and *Antipathetical Repulſion*, may be alſo diſcovered by two *Load-ſtones*, floating in two little Boats in a Baſon of Water, the two *Poles* of either *Stone* being diſpoſed parallel to the Horizon: And if you put the two *South-Poles* together, they ſhall avoid the contact of one another by a natural *Antipathy*; But if the *North-Pole* of one, be direct to the *South-Pole* of the other, they will immediately manifeſt their natural *Sympathy* one to another, and will cleave together by a ſtrong attraction.

5. This is alſo evident between the great *Magnet* the *Earth*, and a *Load-ſtone*; For if you put a *Load-ſtone* in a *ſtring*, and hang it up in the *Air*; or to float in the *Water* in a *Wooden-Diſh*, and putting the *North* end of the *Small-Magnet*, towards the *North* of the *Great-Magnet* the *Earth*, it ſhall immediately change its Poſition, and turn its *North Pole* towards the *South Pole* of the *Great Magnet*.

CHAP. VI.

Several Conſequencies *which follow the various* Cutting, *or* Dividing *of a* Load-Stone.

1. IF a *Load-ſtone* be caſually broken into ſeveral *pieces*, every *piece* ſhall be an entire *Load ſtone*; having both its *Poles* diſtinctly in it ſelf, with all the other properties which the other *Load ſtone* had before it was broken.

2. But if a *Load-ſtone* be divided in the middle, between the two *Poles*, that is in the *Equinoctial*, then is it abſolutely two entire *Load-ſtones*; but thoſe parts which were the *Equinoctial* before, are now become two *Poles*; and the two *Poles* that were the two *Poles* before, continue to be two *Poles* ſtill.

3. But if a *Load-ſtone* be cut through a *Meridian*, and conſequently through both the *Poles* thereof; ſo that one *Axis* is now converted into two, and each of them remove into each *Stone* one: And ſo it is become two abſolute *Load-ſtones*, and the *Axis* of either of them will retire into the gravity of each part: And if you joyn the two pieces together again, the two *Axes* will again become one.

4. But if you cut off a piece of a *Stone* at the very *Pole*, in a parallel Section, the Virtue of that *piece* cut off will immediately retire from it into the main *Stone*, and will have little or no virtue in it; but applying the ſmall piece cut off, to the ſame place again, the *Stone* will forthwith impart the ſame virtue as was before into this piece ſo cut off, ſo long as it doth abide in that place; but when it is removed, it doth again loſe its virtue.

5. Likewiſe, If you apply a *Weak Stone* to the *Poles* of a *Strong one*; the *Strong Stone* will impart of his *Virtue* to the *Weak Stone*, making it to be as ſtrong as it ſelf, ſo long as it is his Neighbour; but when this weak *Magnet* deſerts this Neighbourly Propinquity, the ſtrong *Magnet* will draw its Virtue to it ſelf again.

CHAP. VII.

How to find the Poles *of a* Loadſtone.

1. TAke a thin piece of *Steel*, about an Inch in length, and a quarter of an Inch broad: This piece of *Steel* being bent Circular, and laid upon the Stone, will immediately lye parallel to the *Axis* of the *Stone*; and direct which way the *Poles* do lye. Which being thus far diſcovered, you may find them exactly thus. Take a piece of a common ſewing *Needle*, which being laid upon the *Stone* near either of the *Poles*,

Poles, it will elevate one end thereof, then move it farther and farther, till it do erect it ſelf *perpendicular*, and that very Point will be the *Pole* of the *Stone*.——And now to know which *Pole* it is that you have found, apply a *Needle* that hath been touched, and if the *Stone* draw the *North* end of the *Needle* to it, then is that the *North Pole* of the *Stone*; but if the *South* end be drawn, the contrary.——Otherwiſe, you may find the *Pole* of a *Stone* in this manner; Take a common *ſewing Needle*, and thred it, then hang it over the *Stone* where you imagine the *Pole* to be, and keep the Point a little ſhort from the *Stone*, and the *Needle* will point *directly* to the *Pole* of the *Stone*.

2. If you take a ſmall ſewing *Needle*, and touch the two ends thereof upon the two *Poles* of the *Load-ſtone*; then having a *Glaſs of Water*, lay the *Needle* gently upon the *ſuperficies* of the *Water*, and it will there *ſwim*. Then take a *Knife* that hath been touched with a *Load-ſtone*, and move it too and fro upon the edge of the *Glaſs*, and the *Needle* will follow it up and down, and will play upon the ſuperficies of the *Water*, as a *Fiſh*; then take away the *Knife*, and the *Needle* will reſt upon the *Water*, and in a ſituation *North* and *South*.

3. If you take two *ſewing Needles* touched, and put the Point of one *Northerly*, and the Point of the other *Southerly*, and put them into two ſmall pieces of *Cork*, and put them thus into a *Baſon* of *Water*, one on one ſide, and the other on the other ſide of the *Baſon*, you ſhall ſee them as it were quickened with a vital ſpirit, even ſo to move one towards another, at the firſt ſoftly, but when they draw nearer, they will ruſh together with great violence; the Point of the one ſtriking exactly againſt the Point of the other.

4. If you take a good large *Braſs* or *Pewter Plate*, and upon it place ſome *filings* of *Steel* or *Iron*, and ſome ſmall *bits* of *ſewing Needles*, and diſperſe them about the *Plate* in ſeveral places; Then take a *Load-ſtone*, and put it under the *Plate*, with one of the *Poles* upwards, and put it ſometimes to one *heap*, and ſometimes to *another*, ſo ſhall you find them *heaps* to interfere, and claſh one againſt another, being very pleaſant to behold.

CHAP. VIII.

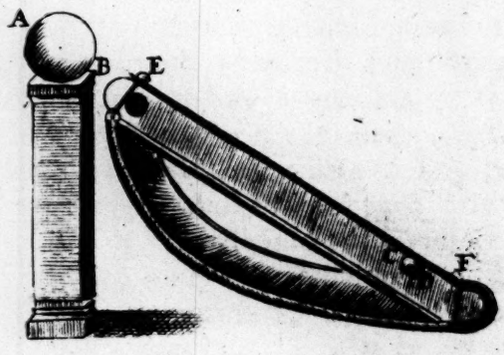
Of ſeveral Attempts that have been made to contrive a Perpetual Motion *by* Magnetical Virtues.

IT was the Opinion of *Peter Peregrinus*, That a *Magnetical Globe*, or *Terella*, being rightly placed upon its *Poles*, would of its ſelf have a conſtant *Rotation*, like the *Diurnal motion* of the *Earth*: But this his Opinion is generally exploded, by common experience.

Athanaſius Kirkerus thinks it poſſible to contrive ſeveral pieces of *Steel* and a *Load-ſtone*, that by their continual *Attraction* and *Expulſion* of one another, they may cauſe a perpetual *Revolution* of a *Wheel*; and of this Opinion were *Peter Peregrinus* above mentioned, *Cardan* and others: But Dr. *Gilbert* concludes it to be a vain and groundleſs fancy.

Of

Of theſe kind of Inventions, this which follows is the moſt likely, wherein a *Load-ſtone* is ſo diſpoſed, that it ſhall draw unto it, upon a *Reclining* Plain, a *Bullet* of *Steel*; which *Steel*, as it aſcends near to the *Load-ſtone*, may be contrived to fall down into a hole in the Plain, and ſo to return unto the place from whence at firſt it began to move; And being there, the *Load-ſtone* will again *Attract* it upwards, till coming to this *Hole*, it will fall down again; and ſo the *Motion* ſhall be *perpetual*, as may be conceived by this Figure:



Wherein ſuppoſe the *Load-ſtone* to be repreſented at *A B*, which though it hath not ſtrength enough to *attract* the *Bullet C* directly from the Ground, yet may do it by help of the Plain *E F*. Now, when the *Bullet* is come to the top of this Plain, its own *Gravity* (which is ſuppoſed to exceed the ſtrength of the *Load-ſtone*) will make it fall into the *hole* at *E*; and the force it receives in this fall, will carry it with ſuch a violence unto the other end of this Arch, till it will open the Paſſage there made for it, and by its return will again ſhut it; ſo that the *Bullet* (as at the firſt) is in the ſame place whence it was firſt *Attracted*, and conſequently muſt move *perpetually*.

But this Invention, tho' it have ſuch ſtrong probability in it, yet there are ſome particulars which may render it inſufficient.

CHAP. IX.

Of Magnetical Inclination, *or of the* Inclinatory (*or* Dipping) Needle: *And thereby to find the* Latitude *at* Sea *or* Land, *without help of the* Sun *or* Stars.

OF the Mariners *Plain Compaſs* I ſhall ſay nothing, it being ſo well known to all Perſons: But of this *Inclinatory* (or *Dipping*) *Needle* much hath not been ſaid; But it was the uſeful Invention of Dr. *Barlow*, Phyſician in Ordinary to Q. *Elizabeth*: Alſo written upon by Dr. *Gilbert*, Dr. *Ridley*, and Mr. *Edward Wright*, who hath a *Geometrical Demonſtration* of the Natural Reaſon thereof; and laſtly by Mr. *Henry Bond*, who hath applied it to the finding of the *Longitude* as well as the *Latitude*.

This Inſtrument is now contrived ſo, as to hang in a broad *Braſs Ring*, within two *Glaſſes*, to keep the *Needle* from the *Air*: In the *Braſs Ring* are two *Quadrants* divided into 90 *Deg.* and the *Needle* hangs upon an *Axis*, playing upon two *Pevets*: And the Inſtrument hangs in a Box ſo ordered, that the Ship moving up and down, the Inſtrument hangs Perpendicular in reſpect of the *Ring*, and horizontal in reſpect of the *Box* (or *Compaſs*).

The *Needle* in this *Braſs Ring* between the two *Glaſſes* (before it is touched with the *Load ſtone*) is made to hang upon its *Axis Horizontally*; but when it is touched it hath an *Inclination*, reſpecting a certain *point* of *Elevation* or *Depreſſion*, according to the *Latitude* wherein it is at any time uſed: And the *Depreſſions* are ſuch, in every degree of *Latitude*, as in his *Table* are exhibited.

A Table of Magnetical Inclination.

Latitude.	Magnetical Inclination.		Latitude	Mag. &c.		Latitude	Mag. &c.		Latitude	Mag. &c.		Latitude	Mag. &c.		Latitude	Mag. &c.	
D	D	M	D	D	M	D	D	M	D	D	M	D	D	M	D	D	M
1	2	11	16	30	45	31	52	27	46	68	24	61	79	29	76	86	28
2	4	20	17	32	24	32	53	41	47	69	17	62	80	4	77	86	48
3	6	27	18	34	0	33	54	53	48	70	9	63	80	38	78	87	8
4	8	31	19	35	36	34	56	4	49	70	59	64	81	11	79	87	26
5	10	34	20	37	9	35	57	13	50	71	48	65	81	43	80	87	44
6	12	34	21	38	41	36	58	21	51	72	36	66	82	13	81	88	1
7	14	32	22	40	11	37	59	28	52	73	23	67	82	43	82	88	17
8	16	28	23	41	39	38	60	33	53	74	8	68	83	12	83	88	33
9	18	22	24	43	6	39	61	37	54	74	52	69	83	40	84	88	47
10	20	14	25	44	30	40	62	39	55	75	35	70	84	7	85	89	1
11	22	4	26	45	54	41	63	40	56	76	17	71	84	32	86	89	14
12	24	52	27	47	15	42	64	39	57	76	57	72	84	57	87	89	27
13	25	38	28	48	36	43	65	38	58	77	37	73	85	21	88	89	39
14	27	21	29	59	54	44	66	15	59	78	15	74	85	44	89	89	50
15	29	4	30	51	11	45	67	30	60	78	52	75	86	7	90	90	00

The Use of the Table.

Suppose you were at Sea in an unknown Latitude, and would find it by this Instrument and Table.——Your Instrument being fitted, and the Fly in the Box, and the *Brass Ring* to hang directly *North* and *South*; Then look how much you find the *North* end of the *Needle* in the *Ring* to be depressed (which suppose to be 63 *deg.* 40 *m.*) find these *degrees* and *minutes* in the *Table*, under the Title of [*Magnetical Inclination*] and against it you shall find 41 *deg.* which tells that you are in 41 *deg.* of *North Latitude*, because the *North* end of the *Needle* is depressed: For if the *South* end had been so much depressed, you had then been in 41 *deg.* of *South Latitude.*

RECREATIONS
Chymical.

CHAP. I.

Of Artificial Repreſentations.

THE Grounds and Principals of *Chymical Philoſophy* go thus: That *Salt*, *Sulphur* and *Mercury* are the Principles into which all things do reſolve; and that the Radical and Original Moiſture whereby the firſt principle of *Salt* conſiſteth, cannot be conſumed by *Calcination*; but the forcible Tinctures and Impreſſions of things, as *Colour*, *Taſte*, *Smell*; nay, and *the very Forms themſelves*, are inviſibly kept in ſtore in this firm and vital Principle. To make this good by Experiment, They take a *Roſe*, *Julyflower*, or any kind of Plant whatſoever; they take this Principle in the Spring-time, in its fulleſt and moſt congruous conſiſtence; they beat the whole Plant in a Mortar, Roots, Stalks, Leaves and all, till it be reduced to a confuſed Maſs. Then after *Maceration*, *Fermentation*, *Separation*, and other workings of Art, there is extracted a kind of Aſhes, or Salt including theſe Forms and Tinctures under their Powers and *Chaos*. Theſe Aſhes are put up into Glaſſes, written upon with the ſeveral Names of the Herbs or Plants; and ſealed *Hermetically*; that is, the Mouths of the Glaſſes heated in the Fire, and then the Neck of it wrung about cloſe; this they call the Seal of *Hermes* their Maſter. When you would ſee any of theſe Vegitables again, they apply a Candle, or ſoft Fire to the Glaſs, and you ſhall preſently perceive the Herbs or Plants, by little and little to riſe up again out of their Salt or Aſhes, in their ſeveral proper forms, ſpringing up as at firſt, they did in the Field or Garden (but in a ſhorter time.) But remove the Glaſs from the Fire, and they immediately return to their own Chaos again.

Now although this went for a great Secret in the time of *Quercetan*, yet *Gaffarell* ſaith, that now 'tis no ſuch rare matter; for Monſieur *de Claves*, a moſt excellent Chymiſt of theſe days (and others) uſes to make ſhew of them at any time.

CHAP. II.

To preserve Fire as long as you will, in imitation of the inextinguishable Fire of Vestales.

HAving Extracted the Burning Spirit of the *Salt* of *Jupiter*, by the degrees of Fire, according to the Rules of *Chymestry*: The Fire being kindled of it self, break the *Limbeck*, and the Irons which are form'd at the bottom will flame, and appear as burning Coles, as soon as they feel the Air; the which Fire, if you promptly incloſe in a Vial of Glaſs, and cloſe it up with *Hermes* Wax, to prevent Air getting in; then will it keep Fire for a thouſand years; yea, if it be kept under Water, or in a Cave, Vault or Cell: But if you open it and let in the Air, the Fire quickly extinguiſhes.

CHAP. III.

To make the Philoſophers Tree.

TAke two Ounces of *Aquafortis*, and diſſolve it in half an Ounce of fine *Silver* refined in a *Cappel*: Then take one Ounce of *Aquafortis*, and two Drachmes of *Quick Silver*, which put in it, and mix theſe two diſſolved things together; then caſt it into a Viol of half a Pound of Spring Water; which muſt be well ſtopped: And then may you every day ſee it grow both in the *Tree* and in the *Branch*.

CHAP. IV.

Of the Re-animation of Simples, when (by reaſon of the great diſtance of Places) the Plants cannot be tranſported.

TAke the *Simple*, *Root*, *Stalk* and *Branches*; Burn all and take the *Aſhes* of it, which let be *Calcinated* two hours between two *Crenſets*, well *Luted*, and extract the *Salt*; that is, to put Water into it in moving of it, then let it ſettle; and do it ſo two or three times: Afterwards *Evaporate* it, that is, let the Water be Boiled in ſome Veſſel, until it be all conſumed, and then there will remain a *Salt* at the bottom; which

which afterwards Sow in good Ground well prepared, and you ſhall have the Plant grow.

CHAP. V.

Of the making of the Epitomy (or repreſentation) of the Great World.

DRaw *Salt Niter* out of *Salt Earth*, digged on the Shores (if you can) where are Minerals of Gold and Silver; Mix this *Niter* (being firſt well cleanſed) with *Jupiter*, and Calcine them *Hermetically*; then put it into a *Limbeck*, whoſe Receiver let be of Glaſs, and both well Luted together; but at the bottom of the Receiver you muſt put ſome Leaves of Gold; Then put Fire under the Limbeck, until Vapours ariſe, which will cleave unto the Gold; Augment your Fire till there aſſend no more: Then take away your Receiver, and cloſe it Hermetically; and make a Lamp Fire under it, and in a ſhort time you ſhall ſee preſented to you, almoſt all things which Nature affords; *viz.* Trees, Flowers, Fruits, Fountains, the Sun, Moon, Stars, *&c.*

To make a Perpetual Motion.

Paraſelſus and his Followers have bragged, that by their *Separations* and *Extractions* they can make a *Little World*, which ſhall have the ſame perpetual *Motions* with this *Microcoſme*, with the repreſentation of all *Meteors*, *Thunder*, *Snow*, *Rain*, the *Courſes* of the *Sea* in its *Ebbs* and *Flows*, and the like; and one of them I find in the 118 *Prop.* of the *Etteneary* of *Mathematical Recreations*; which is this, Mix Five Ounces of ☿ with an equal weight of ♄, grind them together with Ten Ounces of *Sublimate*; diſſolve them in a Celler upon ſome *Marble* for the ſpace of four days, till they become like *Oil-Olive*; Diſtil this with Fire of Chaff, or Driving Fire, and it will ſublime into a dry ſubſtance; and ſo by repeating of theſe *Diſſolvings* and *Diſtillings*, there will be at length produced divers ſmall *Atoms*, which being put into a Glaſs well Luted, and kept dry, will have a *Perpetual Motion.*

CHAP.

CHAP. VI.

Of a famous Perpetual Motion invented by Cornelius Dreble, *and made for K.* James I. *wherein was repreſented the conſtant* Revolutions *of the* Sun *and* Moon, *and that without the help either of* Spring *or* Weight.

MArcellus Vranckhein ſpeaking of the means whereby it was performed, he calls it *Scintillula animæ Magneticæ Mundi, ſeu Aſtralis & inſenſibilis ſpirittis:* Being that grand ſecret which thoſe Dictators of Philoſophy, *Democritus*, *Pythagoras* and *Plato* did Travel to the Gymnoſophiſts and *Indian* Prieſts. The Author himſelf does not reveal the way how it was performed: But one *Thomas Tymme*, (one that did often pry into his works) affirms it to be done thus; By extracting a Fiery Spirit out of the Myneral Matter, joyning the ſame with his proper Air, which included in the Axle-Tree (of the firſt moving Wheel) being hollow, carrieth the other Wheels, making a continual Rotation, except iſſue or vent be given to this hollow Axle-Tree, whereby the impriſoned Spirit may get forth.

But theſe, and ſuch like miraculous promiſes, would require as great a Faith to believe them, as a Power to perform them.

CHAP. VII.

Of the making of GOLD.

COncerning this Matter, I ſhall only ſet down (in his own Words) the *Opinion* of the Right Honourable *Francis* Lord *Verulam*, Viſcount St. *Alban*, in his *Natural Hiſtory*, Century IV. and *Experiments Solitary*, touching the *Making of* GOLD.

The World (ſaith he there) hath been much abuſed by the Opinion of *Making of Gold:* The *Work* it ſelf I Judge to be poſſible; But the *Means* (hitherto propounded) to effect it, are, in the *Practice*, full of *Error* and *Impoſture:* And in the *Theory*, full *of unſound Imaginations.* For to ſay that *Nature* hath an Intention to make all Metals *Gold:* And that, if ſhe were delivered from Impediments, ſhe would perform her own Work: And that if the Crudities, Impurities and Leproſities of *Metals* were cured, they would become Gold: And that a little *Quantity* of the *Medicine*, in the Work of Projection, will turn a Sea of the *Baſer Metal* into *Gold*, by Multiplying: All theſe are but Dreams;

Dreams; and ſo are many other Grounds of *Alchymy*. And to help the Matter, the *Alchymiſts* call in likewiſe many Vanities, out of *Aſtrology*, *Natural Magick*, Superſtitious Interpretations of *Scripture*; Auricular *Traditions*; Feigned Teſtimonies of *Ancient Authors*; and the like. It is true, on the other ſide, They have brought to light not a few profitable *Experiments*, and thereby made the World ſome amends. But we, when we ſhall come to handle the *Verſion*, and *Tranſmutation* of *Bodies*: And the *Experiments* concerning *Metals*, and *Minerals*; will lay open the true Ways and Paſſages of *Nature*, which may lead to this *Great Effect*. And we commend the Wit of the *Chineſes*, who deſpair of making *Gold*, but are mad upon making of *Silver*: For certain it is, that it is more difficult to make *Gold* (which is the moſt Ponderous and Materiate amongſt *Metals*) of other *Metals*, leſs Ponderous, and leſs Materiate: then (*viâ verſâ*) to make *Silver* of *Lead*, or *Quick-ſilver*; both which are more ponderous than *Silver*: So that they need rather a further degree of *Fixation*, then any *Condenſation*. In the mean time, by occaſion of handling the *Axioms* touching *Maturation*, we will direct a *Trial* touching the *Maturing* of *Metals*; and thereby turning ſome of them into *Gold*: For we conceive, indeed, that a perfect good *Concoction*, or *Digeſtion* of *Maturation* of ſome Metals, will produce *Gold*. And here we call to mind, that we knew a *Dutch-man*, that had wrought himſelf into the belief of a Great Perſon, by undertaking that he could make *Gold*: Whoſe Diſcourſe was, That *Gold* might be made, But that the *Alchymiſts* Over-fired the Work: For (he ſaid) the making of *Gold* did require a very temperate *Heat*, as being in *Nature* a Subterrany work, where little *Heat* cometh; but yet more to the making of *Gold*, then of any other Metal; And therefore, that he would do it with a Great *Lamp*, that ſhould carry an Equal and Temperate Heat: And that it was the work of many Months. The device of the Lamp was Folly; but the Over-firing now uſed, and the equal Heat to be required, and the making it a work of ſome good Time, are no ill Diſcourſes.

We reſort therefore to our *Axioms* of *Maturation*, in effect touched before: The firſt is, That there be uſed a *Temperate Heat*, for they are ever *Temperate Heats* that *Diſgeſt* and *Mature*, wherein we mean *Temperate*, according to the *nature* of the *Subject*; For that may be *Temperate* to *Fruits* and *Liquors*, which will not at all work upon *Metals*. The ſecond is, That the *Spirit of the Metal be quickned*, and the *Tangible parts opened*: For, without theſe two Operations, the *Spirit* of the *Metal* wrought upon, will not be able to diſgeſt the parts. The Third is, That *the Spirits do ſpread themſelves even*, and *move not ſubſultorily*; For that will make the Parts Cloſe and Plyant. And this requires a *Heat*, that doth not riſe and fall, but continue as *Equal* as may be. The Fourth is, That *no part of the Spirit be emitted, but detained*. For, if there be *emition* of *Spirit*, the Body of the Metal will be Hard and Churliſh: And this will be performed, partly by the temper of the Fire, and partly by the cloſeneſs of the Veſſel. The Fifth is, That there be *Choice made of the likelieſt and beſt prepared Metal, for the Verſion*: For that will facilitate the work. The Sixth is, That you *give Time enough for the Work*: Not to prolong hopes, (as the *Alchymiſts* do) but, indeed, to give *Nature* a convenient ſpace to work in: Theſe Principles are moſt Certain and True. We will now derive a direction of *Tryal* out of them,

them, which may (perhaps) by further meditation, be improved.

Let there be a ſmall *Furnace* made, of a *Temperate heat*; let the *heat* be ſuch as may keep the *Metal perpetually Molten*, and no more; for that, above all, importeth to the Work. For the Material, take *Silver*, which is the *Metal* that in *Nature* Symbolizeth moſt with *Gold*; Put in alſo with the *Silver*, a Tenth part of *Quick-Silver*, and a Twelfth part of *Nitre* by Weight; both theſe to Quicken and Open the Body of the *Metal*: And ſo let the Work be continued by the *ſpace of ſix Months* at leaſt. I wiſh alſo that there be, at ſome times, an Injection of ſome *Oyled* ſubſtance; ſuch as they uſe in the recovering of *Gold*, which by vexing with Separations hath been made Churliſh: And this is to lay the parts more Cloſe and Smooth, which is the main Work: For *Gold* (as we ſee) is the Cloſeſt (and therefore the Heavieſt) of *Metals*: And is likewiſe the moſt Flexible and Tenſible.

Note, That to think to make *Gold* of *Quick-Silver*, becauſe it is the heavieſt, is a thing not to be hoped; For *Quick-ſilver* will not endure the Manage of the *Fire*. Next to *Silver*, I think *Copper* were fitteſt to be the *Metal*.

Concerning the Nature of GOLD.

Gold hath theſe Natures: *Greatneſs of Weight*; *Cloſeneſs of Parts*; *Fixation*; *Pliantneſs*, or *Softneſs*; *Immunity from Ruſt*; *Colour or Tincture of Yellow*. Therefore, the ſure way (though moſt about) to make *Gold*, is to know the *Cauſes* of the ſeveral *Natures* before rehearſed, and the *Axiomes* concerning the ſame: For if a Man can make a *Metal* that hath all theſe Properties, Let Men diſpute, whether it be *Gold* or no?

CHAP. VIII.

Of Incombuſtible Flax, *or a Subſtance which will not conſume by Fire.*

HAving in the foregoing Section ſpoken of *Subterranean Lamps*, I will here ſay ſomething concerning a Subſtance which will not conſume by Fire: There was anciently a kind of Flax which the *Grecians* called *Aſbeſtinum*, the *Latines Linum vivum*: Hereof were made whole Pieces of Linnen Cloath, and Garments, which were not only not conſumed by any Fire, but being caſt into the ſame, the Soil and Filthineſs being conſum'd and burn'd away, taken out again, it became more white than any Water could waſh it. The Bodies of Emperors and Kings were buried in Sheets of this Linnen, leſt the Aſhes of their Bodies burned, ſhould mingle with the Aſhes of the wood wherewith the Body was burned. This Flax, ſaith *Pliny*, is hard to be found, and as difficult to be Woven, by reaſon of the ſhortneſs thereof; and being found, in price it equalled the moſt excellent Pearls. *Nero* is reported to have had a Linnen Garment of the ſame; but at this day it is not any where to be found. Yet I remember, (ſaith my Author *H. P.*) I had given

given me by an *Arabian* in the year 1618. a pretty quantity of a Stuff like Flax, which he bad me put into the Fire, but it consumed not: Whither it were of this Flax or no, or that Flax of *Cyprus*, which *Pondacatarre*, a Knight of *Cyprus*, who wrote an History of *Cyprus*, *Anno* 1566. brought to *Venice*, and the Fire could not consume it, I know not. Now this Flax of *Cyprus* proceedeth from no Plant, as our Flax, but from the Stone *Amiantus*, which being found in *Cyprus*, and broken with an Hammer (the Earthy Dross being purged from it) there remains fine Hairy Threads like unto Flax, which are Woven into Cloath. This Flax was seen in the House of the said *Pondacatarre* by many Men of Worth and Credit, as *Porcatio* witnesseth, *Tab.* 2. *Enneralium*. Wherefore Linnen being made of the Flax or Threads of this Stone, must needs be incumbustible.—*Constantine* the Emperor ordained that it should ever burn in *Lamps* in his Chappel, at *Rome*: This Reports *Damascus* in the Life of Pope *Sylvester*. Moreover *Ludovicus Vives*, in his Commentaries upon St. *Augustins de Civitate Dei*, *Lib.* 21. *c.* 6. saith, That he saw Lamps at *Paris* whose Lights never consumed. Also at *Lovain*, a fowl Napkin taken from the Table at a Feast, and thrown into the Fire, and being quite Red as a Coal, was taken out again, cooled, and restored to the owner, more White than if it had been washed with Water and Soap: And the like my self have heard, That a Cook dressing of a Dinner at a Nobleman's House, whose Lord was to Dine there; The Noble-Man seeing this strange Cook with a Greasie Apron and fowl Sleeves, said to him, That if the Persons that were to Dine with him should see his Beastly Accoutrements, they would loath to eat the Meat that he should dress; whereupon the Cook pulls off his Apron and Sleeves, and throws them into the Fire, which in a very short time were made Red-hot, and being taken out and cooled, became White as Snow. And now to conclude this matter, I remember that about the year 1648, or 9, I was in company at a Tavern with some Gentlemen, and one of the Company took out of his Pocket a piece of a kind of a Stone, about the bigness it was of a Wall-nut, the outside whereof was of a dirty Earthy Colour; but the inside of a bright Ash-colour, not much unlike Steel when a Bar of it is new broken; and for weight it did ponderate equal with Steel: Off of the inside of this piece of Stone, several of the Company (my self for one) did with our Knives scrape off a kind of Woolley, soft Flax, and putting it to the Candle, there burning, it became immediately Red-hot, but no way consumed or diminished, but came out of the Fire White, whereas it was in the Stone of a bright Ash-colour, as I said before.

CHAP. IX.

Some Relations concerning Subterraneous Lamps.

1. ST. *Auſtine* mentions one of them in a Temple dedicated to *Venus*, which was always expoſed to the open weather, and could never be conſumed or extinguiſhed.

2. *Puncirollus* mentions a Lamp found in his time, in the Sepulchre of *Tullia*, *Cicero*'s Daughter, which had continued there for above 1550 years; but was preſently extinguiſhed upon the admiſſion of new Air.

3. It is related of *Cedrenus*, that in *Juſtinian*'s time there was a Lamp found in a Wall at *Edeſſa*, which had remained ſo for above 500 years, there being a *Crucifix* placed by it; from whence it ſhould ſeem, that theſe *Lamps* were of uſe amongſt ſome Chriſtians.

4. *Olybius* his Lamp, which had continued burning for above 1500 years; the Story is this: A Ruſtick digging the Ground in *Patavia* in *Italy*, he diſcovered a very Ancient Monument, and in it an *Urn*, or *Earthen Pot*, in which there was another *Urn*, and in this leſſer, a *Lamp* clearly burning; on each ſide of it there were two other *Veſſels*, each of them full of a pure *Liquor*, the one of Gold, the other of Silver: And the moſt Learned of that Age coming to the Monument, affirmed the ſame to be that Perpetual Fire invented by the wonderful induſtry of the Ancient Philoſophers, which would indure ſo many years. In which Opinion they were confirmed by Verſes written in either *Urn*, which ſeemed to be of great Antiquity by their vein:

Theſe were in the Bigger Urn:

Plutoni Sacrum munus, ne attingite fores;
Ignotum eſt vobis, hoc quod in Orbe latet.
Namq; elementa gravi, clauſit digeſta labore,
Vaſe ſub hoc modico, Maximus Olibius;
Adſit facundo, cuſtos tibi copia cornu,
Ne pretium tanti, depereat laticis.

Theſe were in the Leſſer Urn.

——Abite hinc peſſimi fures,
Vos, qui voltis veſtris cum oculis emiſſiſciis;
Abite hinc, veſtro cum Mercurio *petaſato, caduce*
Atoq; Maximus, *maximo donum* Plutoni, *hoc*
Sacrum facit.

One *Matarants* had the poſſeſſion of theſe after they were taken up.

5. *Baptiſta Porta* tells us of another Lamp burning in an old Marble *Sepulchre*, belonging to ſome of the Ancient *Romans*, incloſed in a *Glaſs Vial*, found in his time, about the year 1550. in the Iſle of *Neſis*, which had been buried there before our Saviour's coming.

6. In the Tomb of *Pallos* the *Arcadian*, who was ſlain by *Turnus* in the *Trojan-Wars*, there was found another *Burning Lamp*, in the year of our Lord 1401. whence it ſhould ſeem it had continued above 2600 years, and being taken out, it did remain burning, notwithſtanding either *Wind* or *Water*, with which ſome did ſtrive to quench it; neither could it be extinguiſhed till they had ſpilt that Liquor which was in it.

7. *Ludovicus Vives*, tells us of another Lamp, that did continue burning for 1500 years, which was found a little before his time.

8. There is a relation of a certain Man, who digging ſomewhat deep in the Ground, did meet with ſomething like a Door, having a Wall on each hand of it; from which having cleared the Earth, he forced open the Door; upon this there was diſcovered a fair *Vault*, and towards the farther ſide of it, the *Statue* of a *Man* in *Armor*, ſitting by a *Table*, leaning upon his Left Arm, and holding a *Scepter* in his *Right-hand*, with a *Lamp* burning before him: The Floor of his *Vault* being ſo contrived, that upon the firſt ſtep into it, the *Statue* would erect it ſelf from his Leaning Poſture; upon the ſecond ſtep it did liſt up the *Scepter* to ſtrike: And before a Man could approach near enough to take hold of the *Lamp*, the *Statue* did *ſtrike*, and beat it to Pieces.

9. *Cambden* relates in his deſcription of *York-ſhire*, ſpeaking of the Tomb of *Conſtantius Chlorus* broken up in theſe later years, mentions ſuch a *Lamp* to be found within it.

10. In moſt of the ancient *Monuments*, there is ſome kind of *Lamp*, tho' of the ordinary ſort: But thoſe Perſons that were of greateſt *Note* and *Wiſdome*, did procure ſuch as ſhould laſt without ſupply, for ſo many *Ages* together. And,

11. *Pancirollus* tells us, that it was uſual for the *Nobles* among the *Romans*, to take ſpecial care, in their *Laſt Wills*, that they might have a *Lamp* in their Monuments.

12. St. *Auguſtine* tells us, that in the Temple of *Venus* was a *Lamp* that never went out, which he ſuppoſed to have been done, either by Art *Magical*, or by induſtry of ſome Man, who had put *Lapidem Aſbeſtum*, or the unquenchable Burning-Stone within the ſame Lamp.

Concerning theſe Lamps found burning in Graves, I wonder, (1.) How by the help of Art (for Chymiſts ſay this Oyl is made of Gold.) Gold may be reſolved into a Fatty ſubſtance! (2.) How the Flame ſhould endure ſo many years! (3.) How within the Ground, all Air being excluded.

RECREATIONS Automatical.

CHAP. I.

Of the Nature and Making of Watches, Clocks, *and other* Movements.

THE Precepts following are such as were by Sir *Jonas Moor*, Extracted out of a Manuscript of Mr. *William Oughtred*, by him made for his Sons private Use, he being of that Profession.

The *Great Wheel* whereon the *Fusie* or *String* with *Weights* are fixed, divides the Nature of the Work in any *Movement*: That is to say, All the *Wheels* and *Pinions* from that *Great Wheel* to the *Ballance* or *Fly*, only prepare the *Motion*, but the other way effect it: In pursuance whereof, two things are to be noted.

1. The *Fusie*, and how many *Turns* it hath.
2. The *Number* and *Names* of the *Wheels*, *Teeth* and *Pinions*.

Example: In a *Watch* of *Four Wheels*: Suppose the Numbers following to be the *Teeth*:

First, The *Great Wheel* [Number 55. *Teeth*] Turning the *Pinion* [Number 5.] fixed to the *second Wheel*.

Secondly, The *Second Wheel* [Number 45.] turning the *Pinion* [Number 5.] fixt to the *Contrat Wheel*.

Thirdly, The *Contrat Wheel* [Number 40.] turning the Pinion [Number 5.] fixt to the *Crown Wheel*.

Fourthly, The *Crown Wheel* [Number 17.] having Odd Teeth, working upon the Pallots of the *Ballance* [Number 2.]

Thus for *Watches* of *four Wheels*: But for *Watches* with *five Wheels*, there will be a Third Wheel before the *Contrat Wheel*.

Fifthly, The *Pinion of Report*, fix'd to the *Arbor* of the *Great Wheel* [Number 4.] which lies hid betwixt the Plates in Watches, and turns the *Hour-Wheel* [Number 36.] which carries the *Hand* about upon the *Face*; divided into 12 or 24 Hours.

For

For brevities sake, let

M Stand for *Movement*, whether *Watch* or *Clock*.
F The *Fusie*.
A The *Great VVheel*.
a The *Pinion* of *Report* on the *Arbor*.
E The *Second VVheel*.
e The *Pinion* on its *Axis*.
I The *Contrat VVheel*.
i The *Pinion* on its *Axis*.
O The *Crown-Wheel*, carrying
o Its *Pinion* on its *Axis*.
B The *Dial-VVheel* carrying the *Hand* in 12 hours.
T *Time*.
t *Turns*.
N *Notches*, or *Beats* of the *Ballance*.
Con. *Continuance*, and Length in *Time* of the Watches going.

The Work stands both in *Letters* and *Figures*, as in the *Example*.

a) *B* (*d* 4) 36 (9
e) *A* (*f* 5) 55 (11
i) *E* (*g* 5) 45 (9
o) *I* (*k* 5) 40 (8
O 17 *Crown VVheel*
2 *Pallats*.

Where every Wheel is Divided by the *Pinion* it moves from, *A* the Great Wheel, to *O* the Crown-Wheel, *viz.* 55, by 5 equal to 11, equal to *f* 5.— 45 69 5 equal to 8, equal to *k*– But *B* divided by *a* gives 9, that is, *B* by *a*, equal to *d*.

RULE I.

WHere *f* 11, *g* 9, *k* 8, *o* 17, and 2 *Pallats* Multiplyed into each other successively, produce 26928 equal to *N*, the *Notches* or *Beats* made in one Turn of the *Great VVheel*: And 26928 Multiplied by 9, is equal to 242352, the *Beats* that are made in one turn of the *Hand*, whether 12 or 24---- Lastly, Divide 242352 by 12, it gives 20196, the *Beats* in one *Hour*; and 20196 Divided by 60, gives 336.6, the *Beats* in one *Minute*. All this is plain enough, and is the foundation of the whole Work, and by it may be easily found how many *Turns* any *Wheel* or *Pinion* makes for one *Turn* of the *Fusie* or *Hour-wheel*.

RULE II.

AS the *Beats* for one *Turn* of the *Great Wheel*, or *Fusie*. 26928
Is to the *Beats* gone in one *Hour* 20196
So is the Continuance of the *Time* of the *Watches* going 16 ho.
To the Quotient of the *Hour-Wheel* divided by *a*, the *Pinion* of *Report*. } 9

These

Theſe Proportions holding, then any Three given (not of the ſame kind) you may find a Fourth. As by *Example.*

To know the Continuance of the Watches *going, that hath* 12 Turns *in the* Fuſie, *and* 26928 Beats *in one* Turn, *and* 20196 Beats *in an Hour.* Say,

As the number of *Beats* of the *Ballance* in an Hour
Is to the number of *Turns* in the *Fuſie,*
So is 12 *Turns* of the *Fuſie*
To the *Continuance of Time* of the *Watches* going.

For 20196 : 26928 :: 12 : 16.

But if it be demanded by the *Beats*, and the *Time* of the *VVatches going*, to know the *Turns* of the *Fuſie*: Then it will be,

As 26928 : 20196 : 16 : 12.

Or, if it be demanded what *Quotient* ſhall be laid upon the *Pinion* of *Report.* Then ſay,

As 16 : 12 :: 12 : 9
Or, 26928 : 20196

Note, That the fewer *Beats* are taken, the longer ſhall be the *Continuance* of the *VVatches* going at an *Equal Time.*

RULE III.

Concerning PENDULUMS.

THE *Spring* in a *Watch* drawing harder at the firſt, than at the laſt: And likewiſe in *Clocks* with *VVeights* and *Strings*, there is added the weight of the *String* gotten every moment to the *Clock VVeight*; and for that no *Motion* can be Made by hand ſo fit, but there will come ſome unequalneſs, as you may hear by the *Beats* either of *VVatch* or *Clock.* To juſten and regulate theſe inequalities, Monſieur *Hugens* invented the way of applying *Pendulums* to either, for which his name will be ever in venerable eſteem.

Pendulums whoſe *Vibrations* are of the ſame Degree and Minute, are *Equal*, or if they riſe not above a Degree; And the Squares of their *Vibrations* are in proportion to their *Lengths*: For a *Standard* or *Rule*, the aforeſaid Monſieur *Hugens* gives the length of a *Pendulum* that ſhall *Swing Seconds*, to be 881 to the *Pariſian Feet*, 864. The *Engliſh Feet* to *Paris Feet*, by a Table of Sir *Jonas Moors* are, As 1000 to 1068, Therefore, As 864 : 881 :: 1.068 : 1089 : And 1.089 Multiplyed by 3, is equal to 3.267, *i. e.* equal to *Three Feet, Three Inches, and two Tenths of an Inch.*

The late Honourable Lord *Bruncker*, and Mr. *Hook*, found the Length to be 39 Inches, and .325 Parts, which a little exceedeth the other, and it may be was juſtned by Monſieur *Hugen's* Rule for the Center of *Oſcillation*: For *Montons* Pendulum that ſhall Vibrate 132 times in a Minute, it will be found likewiſe 8.1 Inches, agreeing to 39.2 Inches, *Engliſh*: Therefore, for certain, 39.2 Inches may be called, *The Univerſal Meaſure*; and relyed upon to be the neer *Length* of a *Pendulum* that ſhall *Swing Seconds* every *Vibration*: With this *Caution* and *Rule.*

As

As the length of the *String* from the point of the Suſpenſion to the Centre of a *Round-Ball*,
Is to the *Radius*;
So is *Radius*
To a Fourth Number.

Let $\frac{2}{5}$ two Fifths of that *fourth Number*, be added to the former *Length*, for the *Length* of the *Pendulum*. Having this Standard.

The next RULE *is this.*

THat the *Lengths* of two *Pendulums*, are in Proportion to the *Squares* of their ſeveral *Vibrations*; which will be equal to the *Beats* of the *Ballance*; therefore the *Beats* that ſhall be propoſed in a *Minute* being given, to be 50; and it be demanded to give the *Length* of a *Pendulum*, The *Analogy* is,

As the Square of 50, viz. 2500
Is to the Square of 60, viz. 3600,
So is 39.2
To 56.4, the Length required: For,

As 2500 : 3600 :: 39. 2 : 56.4

And if the Length be given, to find the *Swings* or *Beats* in a *Minute*; The *Analogy* is,

As the *Altitude* given,
To the *Altitude* known,
So the *ſquare* of the *Vibrations* known,
To the *ſquare* of the *Vibrations* required,
The *ſquare Root* whereof is the *Anſwer*.

And becauſe the two middle *Terms* ſtand in all ſuch *Queſtions*, and will be always 141120; therefore divide 141120, by the *ſquare* of the *Swings* in a *minute*, it gives the length ſought: or, by the length, it gives the *ſquare* of the *Swings*.

And thus a *Swing* may be hang'd by any *Clock*, upon a Pin, ſo that it may freely *Vibrate* to Regulate the ſame Clock.

48)—4 (12
56— 7 (8
54 6 (9
21

The Numbers of the *Great Wheel* 56.
Its *Pinion* 4, turning the *Hour-wheel* 43.
The *great VWheel* turning a *Pinion* of 7, fixt to
The *Crown VWheel* 54, turning a *Pinion* of 6, fixt to
The *Ballance-wheel* 21.

The Quotients 8, 9. 21, and 2 Multiplyed into each other, produce 3024, Equal to the *Beats* in an hour: Becauſe the *great wheel* turns once in an hour.

Other-

Otherwiſe 12, 8, 9, 21, 2, Multiplyed into each other, produce 36288.— 12) 36288 (3024, and 60) 3024 (50.4 *Beats* in a *Minute*: And (as was ſhewed before) the *Length* of the *Pendulum* will be 55.5 Inches.

Fix a Weight upon a Wyre, running into a Rod, that ſhall have four Feet 7.5 Inches below the *Pin* whereon it plays; and about a Foot (or above) a Wyre beaten flat, with ſeveral holes to fit to the Top of this Rod; and to a *Pin* placed upon the *Ballance* towards the back-ſide, will regulate the *Motion* exceeding well.

For the Regulating the inequality of a *Swing*, when it may riſe ſometimes high, ſometimes lower: There are two ways, either by making the Line play betwixt *two Cheeks* of a *Cycloid*, as Monſieur *Hugens* hath directed, which may eaſily be effected to any Length of the *Pendulum*: Or elſe, by not ſuffering the *Pendulum* to *Vibrate* above an Inch from its ſettlement.

Monſieur *Hugens* in his Book of *Pendulum Clocks*, propoſeth a *VVatch* about a *Mans height* to go 30 Hours, and to have theſe *Numbers*.

D B
6——72
a 30—30 *a*

a 80—8—(10
E 48 8—(6
I–48——24(2
O 15

The Great Wheel 80, *&c.* which turns about in an Hour, and ſhews Minutes; therefore for an Hour, Multiply the Quotients 10, 6, 2, 15, 2, and they will produce 3600, the Seconds in an Hour (60 in 60) is equal to 3600, or *Beats*. Now the Third Wheel I turn about in one Minute, for 10 in 6 is equal to 60, and carries a Plate divided into 60 Seconds, and ſhews the Seconds: And upon the Arbor of the Great Wheel, is fixed a Wheel, *a*, turning another Wheel *e*, both of 30 Teeth; and both turning about in an Hour; the latter has on it a Pinion *b* of 6 Teeth, turning *B* 72 in 12 Hours. This Watch hath a Pully tyed to its Weight, by which you may pull it up, and not ſtop the Watch. The Pendulum plays between two Cheeks, part of a *Cycloid*.

The next Queſtion (ſuppoſing there be a Screw below, or above the Pendulum, to lift it up, or let it down upon a Square Braſs Rule, divided into Inches and Tenth Parts) to know how many Minutes and Seconds every Tenth Part of an Inch will make the Watch go faſter or ſlower in a Day.

I take the Pendulum which ſwings Seconds, whoſe length is 39.2, And then by the Logarithms I make the following Table.

I.	II.	III.	IV.	V.
38.7	1.587711	1.790988	60.39	9.21
38.6	1.588832	1.780378	60.31	7.26
38.9	1.589949	1.779819	60.23	5.31
39.0	1.591065	1.779261	60.15	3.36
39.1	1.592177	1.778705	60. 8	1.55
39.2	1.593286	1.778151	60.	1.11
39.3	1.594393	1.777597	59.92	1.55
39.4	1.595496	1.777046	59.85	3.36
39.5	1.596597	1.776495	59.77	5.31
39.6	1.597695	1.775996	59.70	7.26
39.7	1.598790	1.775399	59.62	9.21

The first Column in the *Table*, has in the Middle the *Length* of the *Pendulum* 39.2 Inches: *Upwards* it diminisheth one Tenth, and *downwards* increaseth one Tenth.

The Second Column, are the Logarithms of the First.

The Third Column, are half of the Logarithms of the difference of the II. taken out of the Logarithm 5.149588, which is, of the standing Number 141120 aforesaid.

The Fourth Column are the Numbers answering to the Logarithms in the III.

The Fifth Column are the *Minutes* and *Seconds* that those *Augmentations* or *Diminishings* will cause in a Day; and are obtained by Multiplying 24 in 60, which makes 1920 equal to the Minutes in one Day, by the Decimals above or under 60″. which work may be easily done, for any length of a Pendulum.

RULE IV.

Of finding out fit Numbers for the Wheels *and* Pinions.

1. ANY two Fractions whose Terms are proportional perform the same Motion;

As $\frac{9}{1}$ $\frac{36}{4}$ $\frac{45}{5}$ $\frac{63}{7}$

The upper for the Wheel,
The lower for the Pinion.

2. If it be, As one Wheel, to one Pinion,
So is the Product of many Wheels,
To the Product of many Pinions: And both will perform the same Motion.

Example.

So $\frac{1440}{28}$ is equal to $\frac{36}{28}$ in $\frac{8}{1}$ in $\frac{5}{1}$

And $\frac{36}{4}$ in $\frac{8}{7}$ in $\frac{50}{10}$ is equal to $\frac{14400}{280}$ equal to $\frac{1440}{28}$

And it matters not in what Order the Wheels and Pinions are ſet, or which Pinion ſtands under every Wheel.

3. Theſe Factors 36 in 8, *i. e.* 288, may thus be varied, *viz.* Divide them by ſuch Numbers as will Meaſure them, and Multiply the Quotients by the Alternate Diviſors: The Product of theſe two laſt Numbers ſhall be two to the Product of the Factors given. For 36 in 8 is 288: Equal to 9 in 32, equal alſo to 288.

$\frac{9}{36}$ $\frac{8}{8}$

$\frac{4}{32}$ $\frac{1}{9}$

4. If fit Numbers cannot be had by any of the Three former ways, you muſt ſeek ſome *Ratio*, as near as poſſible, in this manner.

As one of the two Numbers
Is to the other;
So is 360,
To a Fourth Number:

Divide this Fourth Number, and alſo 360, by 4, 5, 6, 7, 8, 9, 10, 12, 15, or which of them bringeth the *Quotient* neareſt to an *Integer*— As if the two Numbers be 247 and 170, (which are too great to be cut into Wheels) and yet cannot be reduced into leſs, becauſe they have no greater Common Meaſure than Unity: Say therefore,

170 : 147 :: 360 : 3114 (6 311(52 (8
147 : 170 :: 360 : 4164 360(60
311(39 (8 360(45
360(45 416(52

Wherefore, For the two Numbers 147, and 170, you may take 52 and 60—39 and 45—45 and 52.

RULE V.

THe *Diameter*, or *Circumference* of any *Wheel* being given, in Inches and hundred parts of an Inch; and the Number of Teeth it is divided into; to give the Diameter, or Circumference of a Leſſer Wheel or Pinion, with a number of Teeth given, ſhall exactly agree with the Teeth of the Greater Wheel.

Example. The Great Wheel hath One Inch Diameter, and 50 Teeth: The Leſſer Wheel or Pinion 10 Teeth, ſay,

As 7 : 22 :: 1 : 3.14
And Again
As 50 : 3.14 :: 10 : 63 Ferè

For the Circumference of the *Pinion*, whoſe Diameter will be .2 Tenths of an Inch.

RULE

RULE VI.

TO give *Numbers* to a *Watch* that ſhall have a ſwift *Train*, about 20000 *Beats* in an *Hour*, that may have *Turns* about the *Fuſie*, and go 16 Hours; and the Number of the *Crown Wheel* 17.

Say (by *RULE* II.)

As 12 : is to 16 :: So is 20000 : to 26666, the *Beats* for one turn of the *Fuſie*: And becauſe (by *RULE* I.) 26666 is equal to all the *Quotients* Multiplyed together into 17, and into 2, that *Number* being halv'd is 13333, and that again divided by 17, gives for the *Quotient* 784, which being broken into *Three Numbers*, which Multiplyed together, will be 784, or near to it; let them be 11, 9 and 8, which Multiplyed into each other, are 792: Then 792 in 17 in 2, are equal to 26928, And then ſay,

As 16 : 12 :: 26928 : 20196, the *Beats* in one Hour. Alſo,

As 16 : 12 :: 12 : 9.

Laſtly, By theſe three *Quotients* aſſured 11, 9 and 8, find out the Three *Wheels* and *Pinions*, by taking the *Pinions* as you deſire, as is done in the *Margine*. You may try ſeveral Experiments, to make the *Watch* go *Longer*, by altering the *Beats* and *Pinions* of *Report*.

36
$\frac{9}{1} \times \frac{36}{2}$
36
4) 36 (9
5) 55 (11
5) 45 (9
5) 40 (8
17
2

EXAMPLE, Of a *Clock* or *Watch* propoſed to go a *Week* (or *Eight Days*) with this *Order*: That the *Ballance Wheel*, or that which moves the *Pendulum*, may go about in a *Minute*, with an *Index* to ſhew *Seconds*: That the *Great Wheel* may go about in 12 *Hours*: And that the *Wheel* next it may go about in *One Hour* to ſhew *Minutes*.

Firſt, Find how many *Seconds* there are in 12 *Hours*, by Multiplying 12 in 60, equal to 43200, and theſe are the *Beats* that ſhall be in one turn of the *Great Wheel*. Theſe are double, becauſe there are two *Swings* to one *Tooth* of the *Ballance Wheel*; The half of 43200 is 21600; Now the *Ballance Wheel* muſt needs be 30; Divide 21600 by 30, the *Quotient* will be 720, which is to be broke into Three *Quotients*, whereof the firſt muſt needs be 12 for the *Teeth* of the *Great Wheel*; Divide 720 by it, the *Quotient* is 60 for the two *Quotients* remaining, which may be either 10 and 6, or 5 and 12, or 8 and $7\frac{1}{2}$, which laſt let ſtand, and then the Work will ſtand as in the *Margine*, thus: And the *Pinions* to be taken as you pleaſe, to be all 8, the *Wheels* muſt be 96, 64, and 60; ſo then the great *Wheel* will go about in 12 hours; the *ſecond Wheel* in an *Hour*, and the *Ballance Wheel* in a *Minute*, as was deſired.

8) 96 (12
8) 64 (8
8) 60 ($7\frac{1}{2}$
30
10.140
8.128
8.120
72.

RULE VII.

Of giving particular *Motion* to any *Movement*.

THe Number of a *Motion* is the proportion that it bears to one *Turn* of the *Hour Wheel*, or the *Pinion* of *Report*, from whither soever it be taken, which Proportion being broken into *Two* or *Three Quotients*, will shew the *Wheels* and *Pinions*, as if you took it for the *Beats* of the *Ballance*.

A NOTE Concerning *Time*. That which is ordinarily called *The Hour of the Day*: You are to consider this in the length of *Days*, which are twofold, distinguished only by the *Revolutions* of the *Earth* or *Sun*. The First is the *Syderial Day*, where any fixt point or points of the *Earth* in the same *Meridian* or *Azimuth*, returns from any *Star* to the same again. The Second is the *Solar Day*, where the same Meridian of the *Earth* returns from the *Sun* to the same again: Neither of these two are the true *Equinoctial day*: Indeed, the *Syderial* is insensibly the same, if it be but for some small space of *Time*, the Difference being only some *Fourths* and *Fifths* of a *degree* slower in a *Day*; but the *Solar* is notably *Longer* than the other, *viz.* By 3 Minutes, 56 Seconds, 53 Thirds, 19 Fourths of *Time* in a *Day*; and from hence the Length of an *Hour* is generally accounted. Therefore to fit the *Pendulum* of a *Watch* or *Clock* to this *Solar Day* and Hour:

I. By the *Revolution* of a *Fixed Star* to the some point again, after one or more *Revolutions*, (which you must curiously *Observe* by fixing your Eye to a Point:) If the Motion for one *Revolution* want 3 Minutes 56 Seconds of 24 hours; or for Two 7 Minutes 43 Seconds; for Three 40 Minutes 39 Seconds, &c. Then doth your *Watch* or *Clock* go true to the *Equall* or *Middle Motion* of the Sun; if otherwise, the *Pendulum* must be altered to make it go so.

II. By a *Sun Dial*, which though it be made never so exact, and your *Motion* so too, yet there will be a considerable difference after some *Dayes*, nay, even in one *Day*: All which falls out by reason of the *Inequality* of *Natural Days*. And this inequality is now at last setled and demonstrated by Mr. *William Flamsted*: A Table of his for this Equation for every Day in the year, I have here inserted: And although it be not the very same every year, yet this (being for the year 1693) will sufficiently serve for any other.

Of the following TABLE of Equation of Time *and its Use; How hereby to Regulate a true made* Pendulum Clock or Watch, *shewing how much it ought to differ from the same* Sun Dial.

THe following *Table* shews the *Equation* of *Time* in *Minutes* and *Seconds* (as the Letters M *and* S, at the head of each *Column* denotes) for every day of the year, having these Words in the middle of each *Column*, *Watch too Fast*, or *Watch too Slow*: And is of excellent Use, as appears by this which follows out of my loving friend Mr. *George Parker* in his *English Mercury* for the Year 1693.

Not-

Notwithſtanding, all perſons generally conclude, that the Sun by his Motion ſhews the *apparent time* of the *Day*, which is true, and that a *South* Sun always makes *Noon*, or 12 a *Clock*; yet this *Southing* of the Sun, or the ſpace of time from one Noon to the Next, is not equal, but longer or ſhorter, according to the *Suns* poſition in his *Elipſis*; ſo that it is impoſſible for any *Clock*, *Watch* or *Movement* whatſoever, to keep *Time* therewith. For this Reaſon, many perſons that are furniſhed with excellent *Movements* many times blame the Maker of them, when the *Errors* lie where they are not capable of diſcerning: For if a *Clock* truly *Rectified*, be ſet by the *Sun*, and kept going a Year, and at the Years end, agree exactly again with the *Sun*; yet in the interim, it would be fouud, at ſeveral times, to deviate conſiderably; and be ſometimes too *Faſt*, and at other times too *Slow*. The Reaſon of this *Difference* is: The *Clock* meaſures *Time equally*, but the *Sun unequally*; the Quantity of this *Difference* is, as before expreſt: As thus.

Admit on the firſt of *February* I would ſet a *Clock* to ſhew the *equal time* of the *Day*, I look in the following *Table* againſt *February* the *Firſt*, and there I find 14 Minutes, 48 Seconds, with the Words, *Watch too Faſt*, which informs me, that I muſt ſet my *Clock* ſo much faſter than the *Time* given by a *Sun-Dial*, and then, being kept going, it will by the Firſt of *March* be but 10 Minutes *too Faſt*; and on *April* the Third, the *Clock* and *Sun* will agree together exactly: Afterwards, the *Clock* will begin to be *Slower* than the *Sun*, as you may ſee by the *Table*.

A Table of Æquation of Time, for the Regulating of Pendulum Clocks or Watches.

	January		Febr.		March		April		May		June		July		August		Sept.		October		Novem.		Decem.	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S
1	9	4	14	48	10	4	0	W. too F. 45	4	10	1	W. too Slow. 2	4	45	4	28	3	53	13	16	15	21	5	35
2	9	26	14	46	9	47	0	20	4	11	0	50	4	53	4	18	4	14	13	32	15	13	5	6
3	9	48	14	44	9	30	0	12	4	12	0	37	5	0	4	8	4	34	13	46	15	3	4	38
4	10	10	14	41	9	13	0	3	4	13	0	23	5	7	3	57	4	55	13	59	14	53	4	9
5	10	31	14	37	8	55	0	* 18	4	12	0	13	5	13	3	46	5	15	14	11	14	41	3	Watch 40
6	10	50	14	32	8	37	0	Watch 33	4	11	0	* 0	5	18	3	37	5	36	14	23	14	29	3	too Slow. 10
7	11	9	14	27	8	19	0	too 48	4	10	0	13	5	24	3	Watch 21	5	56	14	34	14	17	2	40
8	11	27	14	21	8	1	1	Watch too 2	4	8	0	26	5	29	3	Watch 8	6	17	14	44	14	3	2	10
9	11	45	14	15	7	Watch 43	1	too 16	4	6	0	39	5	33	2	too 55	6	Watch 38	14	54	13	Watch 49	1	too Slow. 40
10	12	2	14	7	7	Watch 25	1	too 28	4	3	0	52	5	36	2	too 46	6	Watch 58	15	4	13	Watch 34	1	Watch 11
11	12	Watch 18	13	Watch 59	7	too 0	1	Slow. 41	4	too 0	1	4	5	Watch 39	2	Fast. 27	7	too 19	15	Watch 13	13	too 17	0	41
12	12	Watch 34	13	Watch 50	6	too Fast. 47	1	54	3	Watch too 56	1	17	5	Watch too 42	2	13	7	too Slow. 39	15	Watch too 22	13	too Slow. 0	0	11
13	12	Watch too 47	13	Watch too 41	6	Fast. 28	2	6	3	too 41	1	30	5	too 44	1	58	7	too Slow. 59	15	too 28	12	too Slow. 43	0	* 19
14	13	too 2	13	too 21	6	too Fast. 10	2	16	3	too 46	1	Watch too 43	5	too 45	1	42	8	too Slow. 19	15	too 34	12	too Slow. 24	0	40
15	13	too 15	13	too 21	5	too Fast. 51	2	too 27	3	too 40	1	Watch too 56	5	too 46	1	too 26	8	too Slow. 38	15	too 40	12	too Slow. 5	1	too Slow. 19
16	13	Fast. 27	13	Fast. 10	5	too Fast. 32	2	too 38	3	Slow. 34	2	too Fast. 8	5	Fast. 46	1	too 9	8	too Slow. 58	15	Slow. 45	11	too Slow. 45	1	too Slow. 49
17	13	Fast. 38	12	Fast. 59	5	too Fast. 14	2	too 48	3	too Slow. 28	2	too Fast. 20	5	too Fast. 45	0	too 52	9	too Slow. 18	15	too Slow. 50	11	too Slow. 25	2	too Slow. 18
18	13	too Fast. 48	12	too Fast. 47	4	too Fast. 55	2	too 55	3	too Slow. 21	2	too Fast. 32	5	too Fast. 44	0	too 35	9	too Slow. 37	15	too Slow. 53	11	too Slow. 4	2	too Slow. 47
19	13	too Fast. 58	12	too Fast. 35	4	too Fast. 36	3	too 6	3	too Slow. 13	2	too Fast. 44	5	too Fast. 42	0	too 17	9	too Slow. 57	15	too Slow. 56	10	too Slow. 42	3	too Slow. 16
20	14	too Fast. 7	12	too Fast. 22	4	too Fast. 17	3	too 14	3	too Slow. 5	2	too Fast. 56	5	too Fast. 40	0	* too 1	10	too Slow. 16	15	too Slow. 58	10	too Slow. 26	3	too Slow. 45
21	14	too Fast. 15	12	too Fast. 8	3	too Fast. 58	3	too 22	2	too Slow. 16	3	too Fast. 7	5	too Fast. 38	0	too 18	10	too Slow. 34	15	too Slow. 59	9	too Slow. 56	4	too Fast. 14
22	14	too Fast. 22	11	too Fast. 54	3	too Fast. 40	3	too 30	2	too Slow. 48	3	too Fast. 19	5	too Fast. 34	0	too 36	10	too Slow. 52	16	too 1	9	too Slow. 32	4	too Fast. 42
23	14	too Fast. 48	11	too Fast. 49	3	too Fast. 22	3	too 37	2	too Slow. 39	3	too Fast. 30	5	too Fast. 30	0	too 55	11	too Slow. 19	16	too 0	9	too Slow. 8	5	too Fast. 9
24	14	too Fast. 34	11	too Fast. 24	3	too Fast. 3	3	too 43	2	too Slow. 29	3	too Fast. 41	5	too Fast. 26	1	too Watch 14	11	too Slow. 28	15	too Slow. 59	8	too Slow. 43	5	too Slow. 36
25	14	too Fast. 38	11	too Fast. 9	2	too Fast. 45	3	too 39	2	too Slow. 19	3	too Fast. 51	5	too Fast. 20	1	too Watch 34	11	too Slow. 45	15	too Slow. 57	8	too Slow. 17	6	too Slow. 4
26	14	too Fast. 42	10	too Fast. 51	2	too Fast. 25	3	too 53	2	too Slow. 9	4	too Fast. 1	5	too Fast. 14	1	too too 53	12	too Slow. 2	15	too Slow. 55	7	too Slow. 51	6	too Slow. 30
27	14	too Fast. 45	10	too Fast. 38	2	too Fast. 9	3	too 57	1	too Slow. 59	4	too Fast. 11	5	too Fast. 8	2	too 12	12	too Slow. 18	15	too Slow. 51	7	too Slow. 23	6	too Slow. 57
28	14	too Fast. 47	10	too Fast. 21	1	too Fast. 51	4	too 1	1	too Slow. 48	4	too Fast. 26	5	too Fast. 2	2	too 32	12	too Slow. 33	15	too Slow. 47	6	too Slow. 59	7	too Slow. 23
29	14	too Fast. 48			1	too Fast. 34	4	too 4	1	too Slow. 37	4	too Fast. 29	4	too Fast. 54	2	too 52	12	too Slow. 49	15	too Slow. 41	6	too Slow. 31	7	too Slow. 48
30	14	too Fast. 49			1	too Fast. 17	4	too 7	1	too Slow. 25	4	too Fast. 37	4	too Fast. 46	3	too 13	13	too Slow. 4	15	too Slow. 35	6	too Slow. 3	8	too Slow. 12
31	14	too Fast. 49			1	too Fast. 1			1	too Slow. 14			4	too Fast. 37	3	too 33			15	too Slow. 27			8	too Slow. 35

Thos

Thus by knowing the *Equal Time*, you may at any time (by help of this *Table*) know the *True* or *Apparent Time*: For, Look againſt the *Day* of any *Month*, and find the *Æquation*, which if it be *Watch too faſt*; then ſubſtract that Days *Æquation* from the *time* given by the *Clock*, and it will be the *Apparent Time*: And if it be *Watch too ſlow*, then, contrarily, Add the *Æquation* to the *Time* ſignified by the *Clock*, and you have your deſire.

But in the right managing of a *Clock* or *Watch*, it ought, always, to be examined by the ſame *Sun-Dial* it was *ſet by*, and at, or near, the *Hour* too, for there is often difference in *Sun-Dials*: And withal, not to Set the *Clock* by the *Sun* early in the *Morning*, or late in the *Afternoon*; for then the ſhadow on the *Dial* is not the true *Time* of the *Day*, in regard the *Sun* hath then conſiderable *Refraction*, which makes the *Sun* appear higher in *Altitude* than really he is, and the nearer the *Horizon* the *Greater*; So that the *Time* given by the *Sun* is *later* in the *Morning* then truth, and the contrary in the *Afternoon*. Therefore, To obſerve the true *Time* of the *Day*, let it be always near *Noon*, if it be a *South*, or an *Horizontal Dial* that you make uſe of: But if it be a *Quadrant* or other *Inſtrumental Dial*, which give the *Hour* by dependance on the *Sun's Altitude*, it ought to be done with great niſity, and never after *Ten* in the *Forenoon*, nor before *Two* in the *Afternoon*; in regard the *Sun* between the Hours of 10 and 2, varies not much in *Altitude*.

CHAP.

RECREATIONS
Hiſtorical.

Of the Meaſures, and Proportions of the Members of Man's Body.

Procœme.

MAN (ſaith the Philoſopher) is the *Meaſure* of all things:
Firſt, Becauſe he is the moſt *perfect Body*.
Secondly, For that (in effect) our uſual *Meaſures*, have taken their *denominations* from *Humane Bodies*, As the *Foot*, the *Inch*, the *Cubit*, the *Pace*, *&c.*

Thirdly, Becauſe the Symetry, and Concordancy of the *Parts* of *Man's Body* is ſo admirable; That all *Magnificent Buildings*, as of *Temples*, *Columns*, *&c.* are in ſome Meaſure, Faſhioned and Compoſed after *Man's Proportion*.

Fourthly, For that *Architects* do reſemble the ſeveral *Members* of a *Corinthean* and other Columns, to the *Members* of *Mens Bodies*: As *Vitruvius* in his Third Book: And *Albertus Durens* hath writ a whole Book of the *Meaſures* of *Man's Body*, from *Head* to *Foot*; of which ſome follow:

1. The *Height* of *Man*, is equal to the diſtance, from one end of the *Middle Finger* to the *other*, the *Arms* being extended to their full length.

2. If the *Legs* and *Arms* of a *Man*, be extended *Salteir* wiſe, or as it is uſually termed making a St. *Andrew's Croſs*; If one foot of a pair of *Compaſſes* were placed upon the *Navel* of the *Belly*, and the other turned about, it would juſtly touch the *Toes* and *Fingers* of the *Man* ſo extended.

3. The breadth of a *Man* from one ſide to the other, the *Breaſt*, the *Head*, and the *Neck*, make the ſixth part of all the Body.

4. The *Length* of the *Face* and *Hand* are equal.

5. The *Thickneſs* of the *Body*, taken from the *Belly* to the *Back*, is the *Ninth* part of the whole Body.

6. The length of the *Brow*, the length of the *Noſe*, the ſpace between the *Noſe* and the *Chin*, the length of the *Ears*, the greatneſs of the *Thumb*; are all of them equal one to the other.

But

But beſides theſe Symetries of the *Members* of *Mens Bodies*, I will here inſert,

The Deſcription *and* Meaſures *of all the Principal* Parts *of* Mans Body, *as they are ſet down by* Leon Baptiſta Alberti, *Reduced to all our* Engliſh Meaſures; *In* Feet, *and Hundred parts of a* Foot.

The height from the Ground to the

	F.	100 Parts
The *Inſtep* of the *Foot*,	0	30
Ankle-Bone, on the out-ſide of the *Leg*,	0	22
Ankle-Bone, on the in ſide of the *Leg*,	0	31
Receſs which is under the *Calf of the Leg*,	0	85
Receſs which is under the *Relieve* of the *Knee Bone* within,	1	43
Muſcle on the out-ſide of the *Knee*,	1	70
Buttocks and *Teſticles*,	2	69
Os-Sacrum,	3	00
Joynt of the *Hips*,	3	11
Navel,	3	60
Waſt,	3	79
Tets, and *Blade-Bone* of the *Stomach*,	4	35
Part of the *Throat* where the *Weezle Pipe* beginneth,	5	00
Knot of the *Neck*, where the *Head* is ſet on,	5	10
Chin,	5	20
Ear,	5	50
Roots of the *Hairs* in the *Forehead*,	5	90
Middle Finger of a *Hand* that hangs down,	2	30
Joynt of the *Wriſt* of the ſaid *Hand*,	3	00
Joynt of the *Elbow* of the ſaid *Hand*,	3	35
Higheſt Angle of the *Shoulder*,	5	18

The Amplitude *or* Largeneſs *of the* Parts, *and* Meaſures *from the* Right-hand *to the* Left.

The

		100 Parts
Greateſt breadth of the *Foot*,	0	42
Greateſt breadth of the *Heel*,	0	23
Breadth of the fulleſt part beneath the jettings out of the *Ankle Bones*,	0	24
Receſs, or faling in above the *Ankles*,	0	15
Receſs of the *Mid Leg*, under the *Muſcle* or *Calf*,	0	25
Greateſt thickneſs of the *Calf*,	0	35
Falling in under the *Relieve* of the *Knee Bone*,	0	35
Greateſt breadth of the *Knee-Bone*,	0	40
Falling in of the *Thigh*, above the *Knee*,	0	35
Breadth of the middle or biggeſt part of the *Thigh*,	0	35
Greateſt breadth among the *Muſcles* in the joynt of the *Thigh*,	1	11
Greateſt breadth between the two *Flanks*, above the Joynts of the *Thigh*,	0	00
Breadth of the largeſt part of the *Breaſt*, beneath the *Arm-Pits*,	1	15

		F.	100 Parts
The	Breadth of the largeſt part between the *Shoulders*,	1	50
	Beneath the *Neck*,	0	00
	Breadth between the *Cheeks*,	0	48
	Breadth of the *Palm* of the *Hand*,	0	00

The Breadth *and* Thickneſs *of the* Arms, *differ according to the ſeveral motions thereof; but the moſt common are theſe following:*

The	Breadth of the *Arm* at the *Wriſt*,	0	23
	Breadth of the *Brawny* part of the *Arm*, under the *Elbow*,	0	32
	Breadth of the *Brawny* part of the *Arm* above, between the *Elbow* and the *Shoulder*,	0	40

The Thickneſs *from the* Fore part *to the* Hinder-part.

The Length from the *Great Toe*, to the *Heel*,		1	00
The Thickneſs from the *Inſtep*, to the *Angle* or Corner of the *Heel*,		0	43
The Falling in of the *Inſtep*,		0	30
From the falling in under the *Calf*, to the middle of the *Shin*,		0	36
The out-ſide of the *Calf* of the *Leg*,		0	40
The out-ſide of the *Pan* of the *Knee*,		0	40
The thickneſs of the biggeſt part of the *Thigh*,		0	60
From the *Genitals* to the higheſt Riſing of the *Buttocks*,		0	75
From the *Navel* to the *Reins*,		0	70
The thickneſs of the *Waſte*,		0	66
From the *Teats*, to the higheſt riſing of the *Reins* of the *Back*,		0	73
From the *Wheezle pipe*, to the *knot* or *joynture* of the *Neck*,		0	40
From the *Forehead*, to the hinder part of the *Head*,		0	64
From the *Forehead* to the *hole* of the *Ear*,		0	00
The thickneſs of the *Arm* at the *Wriſt*,		0	00
The thickneſs of the *Brawn* of the *Arm* under the *Elbow*,		0	00
The thickneſs of the *Brawn* of the *Arm*, between the *Elbow* and the *Shoulder*,		0	00
The greateſt thickneſs of the *Hand*,		0	00
The thickneſs of the *Shoulders*,		0	34

By means of theſe Meaſures it may eaſily be computed what proportions all the parts and Members of the Body, have one by one to the whole Length of the Body; and what Agreement and Symetry they have among themſelves; as alſo how they vary or differ one from another; which things are moſt fit to be known to *Painters* and *Carvers* eſpecially: Nor were it from the purpoſe to particulariſe how the *Parts* vary and alter, according to the ſeveral geſtures incident to humane *Bodies*, as, whither they be *ſitting*, or *inclining* to this or that *ſide*. But we ſhall leave the more curious diſquiſition into theſe things, to the diligence and induſtry of our *Artiſt*.

CHAP.

CHAP. I.

Of Men *or* Giants, *of Prodigious Statures.*

HAving (before in this Book) given you a perfect account of the Dimensions of *Mans Body*, according to every Part and *Member* thereof, that is, of such a Body as the generality of *Men* and *Women* are, not extraordinary *Tall*, *Thick*, or *Dwarfish*, but of a middle and comely Stature; I shall now give account of some *Monstrous Giants*, which *History* makes mention of:

1. We read in the 3*d Chapter* of *Deutrinomy*, of a *Giant* called *Ogge*, of the Town of *Rabath*; who had a *Bed* of *Iron* which was Nine Cubits long, and Four Cubits broad.

2. In the 17th *Chapter* of the 1st Book of the *Kings*, there is mention of *Goliah*, whose height was a *Palm* of six Inches, which is more than Nine of our *English* Feet; He was Armed from Head to Foot, whose *Curnat*, *Launce* and other *Armor* which he wore, did Weigh, of our *Weight*, at the least 500 *l.*

3. In the time of the *Grecian* Wars, after a great overflowing of the *Rivers*, *Salinus* reports, That there was found upon the Sands the *Carcase* of a *Man*, whose *length* was 33 *Cubits*; (which in our measure is 49 Foot and a half) a prodigious Carcase! For (according to the foregoing Proportions) his *Face* should be *five Foot* in length.

4. *Pliny* reports, That after an *Earthquake*, a Mountain being cleaven thereby, in it was found a *Body* standing upright, which was 46 Cubits high; some report it to be the Body of *Orion*, but whose Body soever it was, it must be Monstrous; for what can be thought of a *Hand* to be seven Foot, and his *Nose* two Foot and a half long.

5. *Plutarch* reports, in the Life of *Sentorius*, That in *Timgy*, a Marative Town; That because *Sentorious* could not believe what he had heard reported, caused a *Sepulchre* to be opened, and found a *Body* therein which contained 60 Cubits in *Length*; according to which proportion, he should be 15 of our Feet in *Breadth*, his *Face* 9 Foot long, his *Thumb* 3 Foot; which is neer as big as the *Colloßus* at *Rhodes*.

6. It is reported by *Symphoris Campeßus*, That at the foot of a Mountain near *Trepane*, in opening the foundation of a House, they found a *Cave* in which was found a *Giant*; which held in his hand a great *Post*, like the *Maßt* of a *Ship*; and handling it, it moldred all into Dust, except the *Bones* which remained, and were of so vast a bigness, that the *Head* would hold 5 Quarters of Corn; by which proportion, his *Length* should be 300 Foot, the length of his Face 30 *Foot*, and his *Nose* 10 Foot.

7. *Josephus Ancosta*, in his *Italian History* reports, That at *Peru* was found the *Bones* of a *Giant* 18 Foot high: And other Histories are full of the Description of *Giants* of 9, 10 and 12 Foot high.

8. In the 58 *Olympiad*, by the admonition of the *Oracle*, the Body of *Oreſtes* was formed at *Tegaa* by the *Spartans*, and we underſtand (ſaith *Solinus*) that the juſt length of it was 7 Cubits, that is, of our Meaſure 10 Foot and a half.

9. In the Senate Houſe at *Lucerne*, in the year 1584, I was ſhewed (ſaith my Author *) the fragments of ſome Bones of a prodigious greatneſs; They were found in a Cave near the Monaſtery of *Reiden*, under an old Oak which the Wind had then blown down: When I had conſidered them (ſaith he) though they were waſted, ſpunged, and light, I obſerved that they anſwered (though the Skull was not there) to the Body of a Man, and wrote upon each of them what they were, as the loweſt Bone of the Thumb, a Cheek Tooth, the Shoulder Blades, a Heel-bone, and many others, all which differed nothing from the Bones of a humane Body. Theſe Bones I compared with a Skeleton of my own, and cauſed an entire Skeleton to be drawn of ſuch greatneſs, as all thoſe Bones would have made, if they had been whole and together; and it amounted to full 19 Foot in height.

* *Plat. obſ. Med. l. 1. p. 584, 585.*

10. *Walter Parſons* an *Engliſh-man*, born in *Staffordſhire*, he was put Apprentice to a Smith, and grew ſo tall, that a hole was made for him in the ground to ſtand in up to the Knees, to make him adequate to his fellow Workmen: He was afterwards Porter to K. *James* the Firſt. He would make nothing to take two of the talleſt Yeomen of the Guard (like the Gizard and Liver) under his Arms at once, and order them as he pleaſed.

11. *William Evans* born in *Monmouthſhire*, and may juſtly be count the Giant of our Age, for he was full 7 Foot and a half in height: He was Porter to King *Charles* the firſt, and ſucceeded *William Parſons* in his Place, and exceeded him in the height two Inches; but not ſo proportionable in all the parts of his Body as *Parſons* was.

CHAP. II.

Of Dwarfs *and* Pigmies; *or* Men *and* Women *of lower then ordinary ſtature.*

1. *Julia*, the Neice of *Auguſtus*, had a little Dwarfiſh Fellow called *Canopus*, whom ſhe much eſteemed, he was not above 2 foot and 5 inches in height; and *Andromeda*, a freed *Maid* of *Julia*, was of the ſame height.

2. Two Gentlemen, both Knights of *Rome*, as *Marcus Varro* reporteth, *viz. Marius Maximus* and *Marcus Tullius*, were either of them but 2 Cubits high, that is 3 foot.

3. *Nicepholus* in his Eccleſiaſtical Hiſtory, ſaith, I ſaw one *John de Eſtrix* of *Mechlen*, when he was brought through *Baſil* to the Duke of *Parma*, then in *Flanders*, *Anno* 1592; He was 35 years of Age, he had a long Beard, was perfect and ſtreight in all his Limbs, and was but 3 foot high; he could not go up Stairs, much leſs could he get upon a Form

Form, but was always lift up by a Servant: He was skilled in Three Tongues, ingenious and induſtrious, and with him (a while) I played at Tables.

4. There was a Dwarf at *Wartemberg*, at the *Nuptials* of the Duke of *Bavaria*: He was Armed Cap-a-Pee, Girt with a ſhort Sword, and a ſhort Spear in his hand; He was put into a Pie, which was ſet upon the Table, at laſt, raiſing the Lid, he ſtepped out, drew his ſword, and after the manner of a Fencer, traverſed about the Table.

5. *Marcus Antonius* had one *Siſyphus* a Dwarf, of a vivid wit, and yet was not full Two foot high.

6. I ſaw (ſaith my Author*) in the year 1610, one *John Ducker*, an *Engliſh* Man, (whoſe Picture I have by me, drawn at full length) he was about 40 years of Age; he had a long Beard, and was only 2 foot and a half high; of ſtreight and thick Limbs, and well proportioned. *Plater. Obſ. Lib. 3. p. 582.

7. *Auguſtus Cæſar* exhibited in his Plays one *Lucius*, a Young Man; he was not full 2 foot high; and (ſaith *Raviſius*) he weighed not full 17 Pounds; and yet he had a great and ſtrong Voice.

8. *Characus* was a man of exceeding ſmall ſtature, yet was he the Wiſeſt Councellor that was about *Saladine*, that great Conqueror of the *Eaſt*.

9. In the year 1306 *Uladiſlaus Cabitalis* (called the *Pigmy* King of *Poland*) Reigned, and Fought more Battles, and obtained more glorious Victories therein, than any of his Long Shanked Predeceſſors. [*Vertue refuſeth no Stature*] But, commonly, Vaſt Bodies, and extraordinary Statures, have dull, ſottiſh and leaden Spirits.

10. *Cardan* reports, That he ſaw a Man in *Italy*, at full Age, not above a foot and half high carryed about in a Parrots Cage.

11. There was a *French* Man, of the Country of *Lamoſin*, of about the ſame height, with a formal Beard, who was ſhewn in a Cage for Money, at the end whereof was a little Hutch into which he retired; and when the Aſſembly was full, he would come out and play upon an Inſtrument.

CHAP. III.

Of Monſters, and other Perſons, as have made entrance into the World in a different manner from the reſt of Mankind.

I. *Bucanon* relates of a Monſter which had only one Body below the Navel, but above, two different ones; when any part below the Navel was hurt, both the Bodies above participated of the Pain; if above, that Body only felt it that was hurt. Theſe two upper Bodies would ſometimes quarrel, and one Dying, the other Pined away by degrees: It lived 28 years, and could ſpeak ſeveral Languages, and was taught to play on a *Muſical* Inſtrument.

II. In

II. In the year 1538. there was Born one who grew up to the Stature of a Man; he was double as to his Head and Shoulders, ſo that one Face ſtood oppoſite to the other: Both were of a likeneſs, reſembling each other both in Beard and Eyes: They had both the ſame Appetite, and both Hungred alike: The Voyce of both was almoſt the ſame; and both Loved the ſame Wife.

III. *Johannes Naborowsky*, a Noble *Polonian*, had ſeen in his Country of *Baſil*, two little Fiſhes without Scales, which were brought forth by a Woman, and as ſoon as they came out of her Womb, did Swim in Water as other Fiſhes.

IV. Not long ſince in *Elſingorn* lived a Woman of good quality, who by her account drew near to the time of her delivery, and ſo provided all things neceſſary: But in her laſt Month, her big Belly ſeemed to be much fallen. Her time of Travel being come, and the uſual Pains of Labour going before, ſhe was delivered of a Creature very like unto a Dormouſe of the greater ſize, which (to the amaſement of the Women then preſent) with great celerity ſought out, and found a hole in the Chamber into which it crept, and was never ſeen after.

V. In the year of our Lord 1639, at *Norway* we read of a marvelous Example of a Woman, who having often before been delivered of Humane Births; and again Big, and after ſtrong labour was delivered of Two *Eggs:* This Womans name was *Anna*, the daughter of *Amundus*, and Wife to *Gudbrandus Erlandſonius*, who already had been Mother of Eleven Children. This *Anna*, in the year 1639, *Apr.* 17. and in the Evening of the ſame day, was delivered of an Egg, in all reſpects like that of a Hen, which being broken by the Women then preſent, they found that in the Yolk and White thereof there was no difference, but anſwered directly to a common Egg. Upon the 18th day of *April.* about noon, in the preſence of the ſame perſons, ſhe was delivered of another Egg, in all reſpects like the former. This ſecond Egg was ſent to Dr. *Olaus Wormius*, to the Univerſity, in whoſe Study it is reſerved to be ſeen of ſuch as ſhall deſire it.

VI. In the year of our Lord 1576, upon the 27th day of *December*, One *Ann Tromperin* about 30 years of Age, was delivered of a Boy and two Serpents. This Woman (ſaith *Caſpar Banbinus*, in his Hiſtory of *France*) told me upon her faith, that in the ſummer before, in an extream hot day, ſhe had drunk of a Spring in the Grove called *Brudetholk*, adjoyning to *Baſil*, where ſhe ſuſpected that ſhe had drank of the ſperm of Serpents; ſhe grew ſo big, that ſhe was fain to carry her Belly in a Swathling-band The Child was very Lean, the Serpents were each of them an Ell in length, and thick as the Arm of an Infant, both which, alive as they were, buried by the Midwife in the Church-Yard of St. *Elizabeth.*

VII. The Concubine of Pope *Nicholas* the third was delivered of a Monſter which reſembled a Bear. *Martin* the 4th, his ſucceſſor, entertained the ſame Lady, and fearing that ſhe ſhould bring forth other Bear-whelps; he cauſed all the Painted or Carved Bears about his Palace to be

be expunged or removed: For this Pope was not ignorant how the ſhapes and Pictures which are conceived in a Womans imagination at the time of conception, do remain imprinted in the Body of that which is conceived.

VIII. At *Tertoghenboſch*, a City in *Brabant*, upon a ſolemn Feſtival, ſome of the *Citizens* diſguiſed themſelves inſeveral habits, ſome like Angels, ſome like Devils, to augment the ſport. One of theſe (who acted a Devils part, ran home to his houſe in his Devils ſhape, took his Wife, threw her upon a Bed, ſaying, he would get a young Devil upon her; he was not deceived; for of that copulation, there was born a Child, ſuch as the wicked Spirit is Painted, which at his coming into the World, began to run and skip up and down the Chamber.

IX. In the Little Town of *Quiero*, amongſt the *Subalpines*, upon the 17th day of *January* 1578, An honeſt Matron was there delivered of a Child which had upon its head 5 horns, like to thoſe of a *Ram*: From the upper part of his Forehead there hung backwards a long piece of Fleſh, that covered moſt part of its Back, in form like a Womans Headlace: About its Neck was a double row of Fleſh like the Collar of a Horſe. At the ends of its Fingers were Claws, like to thoſe Tallons of Birds of Prey. Its Knees were in the hinder parts of its Legs: Its Right Leg and Foot were of a ſhining Red Colour; and the reſt of its Body all Swarthy: It came into the World with ſuch a great Cry, that it affrighted the Midwife and the reſt of the Women there preſent—This Monſter was ſent for by *Subalpinus* to his Court to be ſeen of him and his Courtiers.

X. In *Leſina*, the biggeſt *Iſland* in the *Adriatick Sea*, there was born a Monſter: Below the middle part whereof, there was but one Body; and above the middle there were Two Living Souls; each ſeparated from the other, with ſeveral Members; their Heads being both of one bigneſs, but different in *Phiſiognomy*: The Belly of the one joyned to the poſterior parts of the other; and their Faces looked both one way, as if the one had carried the other on his Back; for he that was behind would often lay his hands about the Neck of the other. Their Eyes were exceeding big, and their Hands, Thighs and Legs greater than thoſe of an Infant of three times their age, which was but 36 days: The Excrements of both iſſued forth at the ſame place: Their Feet were round and cloven, like thoſe of a Camel: They received their Food with an inſatiable deſire, and continually mourn'd with a pitiful noiſe: When one Slept the other waked: The Mother bought that Birth with the loſs of her life, and the Monſters died ſoon after. This was ſhewn to *Lithgoe* in his Travels by a *Venetian*, the Governour of that *Iſland*.

XI. When *Fulvius Flaccus* was Conſul, there was born in *Rome* of a Maid Servant, a Monſter: It had Four Feet, Four hands, Four Eyes, Four Ears, and Two Members of virility.

XII. A Woman in *Prague* being with child was cauſed to hold a Calf while it was kill'd, and ſtanding by while it was opened, at the falling of the Bowels ſhe felt a commotion within her: and ſoon after

was

was delivered of a Boy, whoſe Liver, Inteſtines, Stomach and Spleen, with the greateſt part of the Meſentery hung out beyond the Navel. It lived but a few Hours.

XIII. At *Cracovia*, (ſaith *Lycoſthines*) in the year 1543, there was born of Noble Parents, a *Monſter* with Flaming and Shining Eyes; the Mouth and Noſtrils were like thoſe of an Ox, it had long Horns, and a Back hairy like a Dogs: It had the Faces of Apes in the Breaſt where the Tets ſhould ſtand: It had the heads of Dogs upon the two Elbows, and at the whirl bones of each Knee looking forward: It was Splay footed and Handed: The Feet were like Swans Feet: It had a Tail turned upwards, that was crooked backwards, and about half an Ell Long. It lived but Four hours from the birth of it; and neer its death it uttered theſe Words; *Watch, For the Lord your God comes.*

XIV. At St. *Lawrence* in the *Weſt Indies* in the year 1573, there was a Monſter born, Which (beſides the horrible deformity of its Mouth, Ears and Noſe; had two horns on the head, like thoſe of young Goats; long hair on the body; a fleſhy Girdle about the middle, double; from whence hung a piece of Fleſh like a Purſe, and a Bell of Fleſh in his left hand like thoſe the *Indians* uſe when they dance; White Boots of Fleſh on his Legs doubled down. In brief (ſaith my Author, Dr. *Henry More*, in his Immortality of the Soul, P. 173.) The whole Shape was horrid and diabolical; and conceived to proceed from ſome Fright the mother had taken from the Antick Dances of the *Indians*; amongſt whom the Devil himſelf does not fail, ſometimes, to appear.

XV. Upon the 17th of *October* 1637, At *Boſton* in New *England*, one Miſtreſs *Dyer* was delivered of a Monſter which had no head; the Face was on the Breaſt; the Ears like Apes, grew upon the Shoulders; the Eyes and Mouth ſtood far out; the Noſe hooking upwards; the Breaſt and Back full of Prickles; the Navel and Belly where the Hyps ſhould have been; inſtead of Toes, on each foot it had three Claws; upon the Back it had two great holes like Mouths; above the Eyes it had Four horns; It was of the Feminine Sex. The Father and Mother of this Monſter were both great Familiſts, ſaith my Author, *Clark's Miror*, pag. 249.

As there have been ſeveral Monſters brought into the World; So alſo have there been brought perſons into the World (although not Monſtrous, yet) in a different manner from the generality of mankind.

XVI. *Zoroaſtres* Laughed the ſame day he was born; his Brain alſo did ſo evidently pant and beat, that it would bear up the hands of thoſe that laid them upon his head: An evident preſage (ſaith *Plato*) of the great Learning he at length attained unto.

XVII. *Nero Cæſar*, the Emperour, came into the World with his Feet formoſt: Who all the time of his Reign was a very enemy to mankind.

XVIII. Some Children are brought into the World with Teeth in their heads; As M. *Curius*, and *On Papyrius Carbo*; both of them great men and honourable Perſonages.

XIX. Some

XIX. Some are cut out of their Mothers Wombs; As *Scipio Affricanus* the first, *Julius Cæsar*, *Manillius*, *Macduff* Earl of Fife, And *Edward* the Sixth of *England*.

XX. In the year 959. *Buchardus* Earl of *Limsgow*, *Beuchorn*, *Montfort*, and *Abbot* of *Sangal*, was vulgarly called Unborn, because he was cut out of his Mothers Womb.

XXI. One *Cornelius Gemma*, a *German*, says, that himself had cut out of the Wombs of Six several Women, Six living Children.

XXII. *Pliny* in his Natural History, speaks of a Child in *Sagnotum* (in that year it was sacked by *Hanibal*) which so soon as it was come out of the Mothers Womb, presently returned into it again.

XXIII. *Johannes Dubravius* relates of *Lewis* the Second King of *Hungary* and *Bohemia*, that there were Four things in him remarkable for haste, *viz.* (1) That he became Great in a short time: (2) That he had a Beard too soon: (3) That he had White hares before he was 17 years of Age: And (4) That he was born into the World without any of that Skin which they call *Epidermis*: He died *Anno* 1526. in the 29*th* year of his Age.

XXIV. When *Spinola* Besieged the City of *Bergopsonna*, a Woman near her time of Travel went out to draw water, at which time she was taken off in the middle by a Cannon Bullet; so that the lower part of of her fell into the Water; such as were by, ran to her, and saw there a Child moving it self in the Bowels of the Mother: They drew it forth, and soon after brought it to *Antwerp*, where the *Infanta Isabella*, caused it to be Baptized; and gave it the name of *Albertus Ambrosius*, One of her Fathers Captains.

XXV. *Enecho Arista*, the first King of *Navar* being dead, *Garsias* his Son succeeded, who being one day in the Village of *Larumbe*, was surprised by some *Moorish* Robbers, assaulted and slain: they Wounded *Urracha* his Queen in the Belly with a *Launce*: The Thieves put to flight, the Queen at the Wound was delivered of a Son and dyed; the Child was safe, and was named *Sancius Garsia*: He was well educated, proved a Valliant Man, and succeeded his Father in the Kingdom.

XXVI. The Wife of one *Simon Kneuter* of *Wessemberg* went with Child to the ninth Month, and then falling into Travel she dyed Undelivered; those that were by doubted not but that the Child was dead also; they disposed of the Mother as is usual; but some few hours after, they heard a Cry; they ran, and then found the Mother dead indeed, but delivered of a Daughter, that was in good health, and lay at her feet. *Salmuth* saith that he hath seen Three several Women, who have been dead in Travel; and yet, after Death, delivered of the Children they went with.

To conclude these two last Paragraphs, I shall give an instance in one Person participating of both, viz.

XXVII. There never was in any man a greater uniformity of Body and Mind (ſaith my Author Sir *Rich. Baker*) than there was in *Richard* the Third, King of *England*, for both of them were equally deformed—Of Body he was but Low, *Crook Back'd* and *Hook-Shouldred*; *Splay-Footed* and *Goggle Ey'd*; His face Little and Round; his *Complexion* Swarthy; His left Arm (from his Birth) Dry and Withered—He was born a Monſter in Nature, having all his Teeth, Hair on his head, and Nails on his Fingers and Toes. And juſt ſuch were the qualities of his Mind.

XXVIII. *Offa*, the Son of *Waſmond*, King of the *Weſt Saxons*, was Tall of Stature, and of a good conſtitution of Body. He was Blind till Seven years of Age, and then could See: He was alſo Dumb till Thirty years of Age, and then ſpake.

CHAP. IV.

Of the length of Age which men lived in former times, ſhortly after the Creation, and of others of later date.

IT hath been a *Queſtion*, what manner of *Years* thoſe were, our Fore-Fathers are ſaid to have lived, as 700, 800, and 900 years, as *Methuſula*, ſince years have been taken diverſly. Some have imagined they could not be our *Anni Solares*, as we account our years, as being a thing incredible, that the date of a mans life ſhould extend it ſelf to that length, far beyond the Age of the eldeſt Oake. Some account (as *Pliny* reporteth) every *Summer a Year*, and *Winter* another: As alſo the *Arcadians* counted their year by Three Months; others again by the courſe of the *Moon*, as the *Egyptians*; ſo ſome of them are reported to have lived 1000 years. There are thoſe therefore that imagine thoſe years mentioned *Gen.* V. To be underſtood to conſiſt of 36 days, Ten whereof make but one *Salar year*, hereby reckoning that who are in the Scripture ſaid to have lived 900 years, lived of our years but 90, every hundred of thoſe Monthly years amounting but to Ten of ours. So they believed all that ſpace of time which is contained in one year to have been anciently divided into Ten parts, and every part taken for a year, and every one of theſe ten parts to have had *Senarium quadratum*, becauſe in Six days God finiſhed his Work of the Creation; all which number multiplyed by Ten, make juſt 12 Months. But theſe concluſions fall out to be moſt abſurd, for if we conſider what will follow thereupon, *Canaan*, who begat children when he was 70 years of age, ſhould have begotten them when he was but Seven: Beſides, if we make a year but of 36 days, of what length muſt the Month be? Surely no more then Three days: And then, How can that place in *Gen.* VII. 11. Where it is ſaid, *The Flood began the 27th day of the ſecond Month*? And how will it agree where it is ſaid, *Gen.* VIII. 4. *The Ark reſted upon the Mountains of Ararat the 27th day of the ſecond Month*?

Let

Let us therefore certainly believe the years then to have been all one with ours, and that which is ſpoken of the great Age of thoſe Fathers, not much to differ from the computation of our years. For, it is again ſaid, *Gen.* 8. 5. That *The Mountains appeared upon the firſt day of the Tenth Month*; wherefore the Year conſiſted of many Months: And that we may not think the Month conſiſted but of Three days, obſerve, the Twenty ſeventh day is mentioned.

Now, If you would know the Reaſon, why the Fathers then lived ſo long, know that there are Two Cauſes; Firſt, the *Final*; Secondly, the *Efficient*.

The *Final* Cauſes were, (1.) Increaſing the World with People, whereby it might be repleniſhed, which they could not do but by living a long time. (2.) Arts were to be invented, for they are not found but by long experience. (3.) The Worſhip of God was to be delivered by Tradition; for, as yet, the Written Word was not: But that could not be in ſuch a variety of People, except thoſe who received it from God had not been long liv'd.

The *Efficient* Cauſes of their length of Life were, (1.) The ſingular Bleſſings of God mentioned, *Gen.* 30.20. *I am thy Life, and the Length of thy days.* (2.) The nearneſs of Time to the Creation, when the Bodies of Men were of a ſingular and a moſt perfect Conſtitution, Soundneſs, and ſtate of Health. (3.) The Diet and Feeding was far more wholeſome before the Flood, than ſince. (4.) The Wits and Invention were more accurate and ſubtil in ſearching and finding out the Nature and Qualities of all Things whereof they had need to the ſuſtenance of Life, than ours are in theſe days: And for theſe Reaſons, it may be ſuppoſed their Lives were extended to that length.

Neither may we wonder at it, ſince Heathen Writers teſtify, That even in their times, ſome Thouſands of years after the Flood, many lived 200, others 300 years: Of ſome of which, as alſo of others of later date (even to this preſent Age) I ſhall give the Reader an Account:

I. *Hellanicus* (cited by *Pliny*) reports, That in *Ætolia*, many lived till they were 200 years of Age, which *Damaſtes* confirmeth, while he maketh mention of one *Pictoreus* amongſt them, who lived ſtrong and able of Body, till he had fulfilled 300 years.

II. Sir *Walter Raleigh*, in his Diſcovery of *Guinea*, reports, That the King of *Aromaia*, being 110 years of Age, came in a Morning on foot to him from his Houſe, which was 14 *Engliſh* Miles, and returned on foot the ſame day.

III. *Buchanan* in his *Scotch* Hiſtory, ſpeaks of one *Lawrence*, who lived in one of the *Orcadean Iſlands*, Married a Wife after he was above 100 years of Age, and that when he was 120 years old, he doubted not to go a Fiſhing in a Rough and Tempeſtuous Sea.

IV. *Sigiſmundus Polcaſtrus*, Phyſician at *Padua*, Read there 50 years; in his old Age he buried four Sons in a ſhort time; at 70 years of Age he Married again, and by this ſecond Wife had three Sons; the eldeſt of which, called *Antonius*, he ſaw dignified with a *Degree* in both *Laws*:

Jerome, another of his Sons; had his Cap ſet upon his Head by the Hand of his Aged Father, who trembled, and wept for joy; not long after the old Man died, Aged 94 years.

V. *Fœlix Platerus* ſaith, That in *Anno* 1572, his Father *Thomas Platerus* buried his Wife, which was *Fœlix*'s Mother, and at the 73 year of his Age, Married a ſecond Wife, and within the compaſs of 10 years had ſix Children by her, two Sons and four Daughters; the youngeſt of the Daughters was born in the 81 year of his Age, and he died in the 83 year of his Age. Now that which is memorable between two of his Sons is: I *Fœlix* was born *Anno* 1536, and my Brother *Thomas* 1574, the diſtance 38 years; and yet this Brother of mine, (to whom (for Age) I might have been Grand-father) is all Grey, and ſeems elder than my ſelf.

VI. *M. Valerius Corvinus*, lived 100 years, between whoſe firſt and ſixth Conſulſhip there was 47 years; yet was he ſufficient, not only for the moſt important matters of the Commonwealth, but alſo for the exacteſt cultures of his Fields.

VII. *Mitellus* was of the like Age, and was very old when created *Pontiffe*: For 22 years he had the ordering of the Ceremonies: His Tongue never faultered in ſolemn Prayers; nor his Hand tremble in offering the Sacrifices.

VIII. *Nicholaus Leonicenus*, was in the 96 year of his Age, when *Langius* heard him at *Ferrara*, where he had taught more than 70 years.

IX. *Maſſaniſſa* was the King of *Numidia* 60 years together, and excelled all other Men in ſtrength. *Cicero* ſaith, That for no Cold or Rain he could be induced to cover his Head; He would continue ſtanding in one and the ſame place, not moving a foot till all about him were weary; and when he was to tranſact any buſineſs ſitting in his Throne, he would perſiſt oftentimes the whole day, without turning his Body on this or the other ſide for a more eaſeful Poſture: He would Lead his Army Day and Night: After the 86 year of his Age he begat a Son, whoſe name was *Methimnatus*, and died in the 91 year of his Age.

X. *Xenophilus*, the *Pythagorian* Philoſopher, and of great Learning was 90 years of Age, and (as *Ariſtoxenus* ſaith) died free of all thoſe incommodities that attend upon humane Life.

XI. *Lemnius* tells us of one at *Stockholm*, who at the Age of 100, Married a Wife of 30, and begat Children of her: He looked ſo freſh, that thoſe that knew him not, deemed him to be not above 50.

XII. *Iſocrates*, in the 94th year of his Age put forth his Book *Panathenaicus*, and lived 15 years after it, in which time he was ſufficient for any work he undertook, both in Judgment, Strength and Memory.

XIII. The

XIII. The Men of the County of *Cornwall* in *England* are more hardy and ſtrong, alſo very Healthy and Long Liv'd, 80 and 90 years is ordinary: To inſtance in ſome of them:——One *Polzew* lived 130 years, and a Kinſman of his 112.----One *Beancham* 106.----And one *Brown* an *Iriſh man*, but a *Corniſh Beggar*, who lived 120 years; upon whom a Gentleman of this County made this Epitaph:

Here Brown *the* quondam *Beggar lies*,
Who counted (*by his tale*)
Some Sixſcore Winters and above,
Such Virtue is in Ale:
Ale *was his* Meat, *his* Drink, *his* Cloth,
Ale *did his death reprieve*;
And could he ſtill have drank his Ale,
He had been ſtill alive.

In one Pariſh of this County in the Reign of Q. *Elizabeth*, there died in 14 Weeks ſpace four People, whoſe Ages added together made 340 years.---And farther, One Mr. *Chamond* who lived at *Stratton*, was Uncle, and Great-Uncle to at the leaſt 300 Perſons.

XIV. One Mr. *Macklane*, Parſon of *Lesbury* in the County of *Northumberland*, who died about the year of our Lord 1658, did in the 1656 renew his youth; ſo that (though 40 years before he could not read without Spectacles, being 116 years of Age) he could then read the ſmalleſt Print without them: His Hair, which before he had loſt, came again as a Childs.

XV. In the Pariſh of *Alderbury* in the County of *Shropſhire*, lived one *Thomas Parre*, who was 152 years old, who about two years before he died, was ſent for up to *London* to *White-hall*, by K. *Charles* the Firſt, aud died there *Anno Dom.* 1635.

XVI. One *James Sands* of *Harborn* in the County of *Staffordſhire*, lived 140 years, his Wife 120: He out-lived Five Leaſes of 21 years apiece, made unto him after his Marriage.

CHAP. V.

An Account of the firſt Authors *of divers famous* Inventions.

I. THE firſt Invention of Printing is attributed by *Peter Ramus* to one *John Fauſt* a *Magnutine*; for he tells us, that he had in his keeping, a Copy of *Tully's* Offices Printed upon Parchment, with this Inſcription at the end of it, *viz. The excellent Work of* Marcus Tullius, *I* John Fauſt *a Citizen of* Mentz, *happily imprinted, not with Writing Ink,*

or

or Braſs Pen, but with an excellent Art, by the help of Peter Gernesham, *my Servant:* Finiſhed it was in the year 1466, the 4th of *February*. Of this Book there are divers Printed Copies ſeen, by ſeveral, for *Paſquier* ſaith he had one: *Sadmuth* ſaith, there is one in the Publick Library of *Auſburg*; another in *Emanuel Colledge* in *Cambridge*; another in the Publick Library at *Oxford*: And another of them is (or lately was) in the poſſeſſion of my worthy Friend, Dr. *Francis Barnard* in *London*.--*Polider Virgil*, from the Report of the *Megamines* themſelves, attribute the Invention of this Art to *John Gutenberg* a Knight, dwelling at *Mentz*, *Anno* 1440, and with him agree divers Learned Men; But the forementioned *Faus* (from him) was the firſt that made proof thereof in Printing of Books, and by what is before ſaid, *Tully*'s Offices was the firſt Book Printed that we have any Record of.---It was firſt brought into *England*, and practiſed in *London* by one *William Caxto* Mercer, in the year 1471. As concerning the Printing uſed in *China* above 1600 years before, it is quite different from what we uſe now in *Europe*: For whatſoever they Print, is firſt cut out in Wood, and Inked over with a Ball of Cloath, upon which the Paper being laid, they rowl over with a wooden Rowler covered with a ſoft Woollen Cloath: which is no other than as Card-Makers Print their Cards; and thoſe that make Patterns for Women and Children to Work by do.

II. Guns, Sir *Walter Raleigh* will have them to be found out by the *Indians*: ---*Petrach* and *Vulturius* will have it to be the Invention of *Archimedes*: But the Common (and moſt received) Opinion is, That it was firſt found out by a Monk of *Germany*; who, by chance, a ſpark of Fire falling into a Pot of *Nitre*, which he had prepared for ſome Chymical Experiment, it cauſed it to fly upwards: He thereupon made a Compoſition, which he encloſed in an Inſtrument of Braſs or Iron; and putting Fire to it, found his concluſion to take effect. The firſt Publick uſe of Guns (ſays *Magius*) was about the year 1380, or 1400 (ſays *Ramus*) at a Battel between the *Genoways* and the *Venetians* at *Cledia Faſſa*; where the *Venetians* ſo galled their Enemies, that they ſaw themſelves wounded and ſlain, and knew not by what means.

III. The Mariners Compaſs, an admirable Invention, of which the Time when, and the Author who, is uncertain. Dr. *Gilbers* of *Oxford*, who hath writ a Large and Learned Diſcourſe of the *Load-ſtone*, ſeems to be of Opinion, that *Paulus Venetus* brought the Invention from the *Chineſes*.--*Oſorius* refers it to *Gama* a *Portugal*.----*Goropius Becanus* attributes it to the *Germans*, becauſe the 32 Points of the Compaſs receive their names from the *Dutch* in all Languages.----But *Blondus*, who is therein followed by *Pancirollus*, will not have *Italy* loſe the praiſe thereof, telling us, that about the year 1300, it was found out at *Melphis*, a City in the Kingdom of *Naples*, but *Blandus* names him not; and the other, *Poncirollus* ſays, he is not known: Yet *Salmuth*, out of *Ciezus* and *Gomara*, confidently attributes it to *Flavius*, and ſo doth *Du Bartus*; whoſe Verſes upon this matter (as they are Tranſlated by *Silveſter*) are,

We're not ſo much to Ceres *bound for Bread,*
Neither to Baccus, *for his Cluſters Red*;

As

As Siegnior Flavio *to thy witty Trial,*
For first inventing of the Sea-mans Dial:
The Use of th' Needle turning in the same;
(Divine device! O admirable Frame!)
Whereby through th'Ocean, in the darkest Night,
Our hugest Carracks are conducted right:
Whereby we're steer'd with Trouchman, *Guide and Lamp,*
To search all Corners of the Watry Camp:
Whereby a Ship that Stormy Heavens have whield
Near (in one Night) unto the other World,
Knows where She is, and in his Chart descries
What Degrees thence the Æquinoctial lies,

IV. Sailing Coaches were the Invention of *Simon Stevinius* in the *Netherlands*. Of one of these Coaches *Peireskius* made tryal of its swiftness in the year 1606, (after his Victory at *Newport*) he put himself, together with *Francis Mendoza* his Prisoner, into one of them at *Scheveling*, and within two hours they arrived at *Putten*, which is above 40 Miles.

V. Dice, Ball, Cards, Tables, Chess, Draughts, and such like Games, were first invented by the *Lydians*.

VI. The *Phenicians* are said to be the first Builders of Ships; They first invented open Vessels.—The *Ægyptians*, Ships with Decks; and Gallies with two Banks of Oars on a side.—Great Ships of Burthen, were first made by the *Cyprians*.—Cock-Boats and Skiffs by the *Illyrians* and *Libinnians*.—Brigantines by the *Rhodians*.----Frigats or Light Barks by the *Cyrenians*.---- Men of War, by the *Pamphilians*.---- As for Tackle, The *Bœetians* invented the Oar---*Dædalus* of *Crete*, Masts and Sails--- *Anacharsis*, Grapling Hooks---The *Tuscans* Anchors---*Typhis*, the Rudder, Helm, and Art of Steering.

VII. The *Sicilians* (saith *Pliny*) were anciently famous for Invention, as Hour-Glasses, called *Clepsira:* Military Engines, brought to great perfection by *Archimedes*---*Palamedes* first instituted Centinels in an Army, and the Watch-word.---*Pentheselea*, Queen of the *Amazons*, first invented the Battel-Axe.

VIII. The Inhabitants of *Sidon* are said to be the first makers of Glass; the Materials for the Work being first brought over from the Sands of a River running not far from *Ptolomais*, and only made Fusible in this City. The Mystery of making of Glass here in *England* was brought over by one *Benault*, a Foreign Bishop, in the year 662.

IX. To the *Flemings* is attributed the making of Cloath; Arras-Hangings; Dornix and Tapestry, Musical Instruments, Clocks, Watches, Coaches, Chariots, Painting in Oil; Nealing upon Glass, *&c.* The first that brought Cloathing into *England* was K. *Edward* the 3*d*, by Transporting some Families of Artificers from *Gaunt* hither.

X. Bra-

X. *Brachygraphy*, or the Art of Writing Short by Characters, is ſaid, by Dion, to be invented by *Mecænas*, the great favorite of *Auguſtus Cæſar*, *ad celeritatem ſcribendi*; for the ſpeedy diſpatch of Writing.

XI. The Baking and Boyling of Sugar, as it is now uſed, is not above 200 years old, and the Refining thereof more new. It was firſt found out by the *Venetians*.

XII. Of Paper; That which was firſt in Uſe, was the Invention of the *Ægyptians*; for on the Banks of the River *Nilus*, grew thoſe ſedgy Weeds called *Popyri*, which have ſince given the Name to Paper. By means of this Invention *Ptolomy Philadelphus* was inabled to make his excellent Library at *Alexandria*. After this (theſe ſedgy leaves being prohibited to be tranſported out of *Ægypt*) *Attalus*, King of *Pergamus*, invented the uſe of Parchment, made of the Skins of Sheep and Calves, from the Materials called *Membrana* and *Pergamena*, from the Place where it was invented. The convenience hereof was ſuch, that in ſhort time the *Ægyptian* Paper was worn out of uſe, in place whereof ſucceeded the Paper we now uſe made of Rags; The Author of which excellent Invention is not left by our Anceſtors to Poſterity: But the Lord *Bacon* reckons it amongſt the ſingularities of Art, although it derives its Pedigree from the Dung-hill.

XIII. Of all the Inventions and Productions of Humane Wit, that of *Writing* is the moſt admirable and Uſeful: For by means thereof, a man may Copy out his very thoughts; utter his mind without opening his Mouth; and ſignifie his pleaſure at any diſtance howſoever remote; and all this by the help of 24. Letters, and in ſome *Alphabets* fewer: By the Various joyning and combining of which, all words utterable or imaginable may be framed: For the ſeveral varieties or changes of theſe 24 Letters, as *Clavius* hath computed (and it is eaſie to do, though laborious) to amount to.

5852616738497664000.

So that all things that are, or were, or can be imagined to be, may be expreſſed and ſignified by help of this marvelous *Alphabet*. This Miracle of Inventions hath loſt its Maſter, for it is ſet down by *Thomas Read* among the *Inventa Adeſpota*, And thus ſung by him.

Quiſquis erat, mervit ſenii tranſcendere metas,
Et fata neſcire modum, qui myſtica primus
Senſa animi docuit, Magicis ſignare Figuris.

In Engliſh *thus:*

Who er'e he was, that firſt did ſhew the way,
T' expreſs by ſuch like *Magick* Marks our Minds;
Deſerv'd reprieve unto a longer day,
Than Fate to Mortals moſtly hath aſſign'd.

XIV. Notwithſtanding the Invention of Letters is attributed to the *Phœnicians*, As *Sandys* in Chriſts Paſſion, *Act.* 1.

Phœnicians, who did firſt produce
To Mortals, *Letters*, and their Uſe.

The

The *Phœnicians* inhabited between the Great Sea and *Galile* (ſo called of *Phœnix* their King, the fifth in deſcent from *Jupiter* :) honoured for the invention of *Letters* : As *Lucan*, Lib. 3.

> *Phœnicians* firſt expreſt (if Fame be true,)
> The fix'd Voice in rude *Figures*. *Memphis* knew
> Not yet how Stream-lov'd *Biblus* to prepare :
> But *Birds* and *Beaſts*, carv'd out in ſtone, declare
> Their *Hierogliphick* Wiſdom.—

And theſe Letters *Cadmus*, the Son of *Agenor*, communicated to the *Grecians*.

XV. The firſt Invention of the Artificial Sphere or Globe is not evidently know ; ſome think (with *Pliny*) that it was fouud out by *Atlas*, and carryed into *Greece* by *Hercules* : Others have aſcribed it to *Anaximander Mileſius* : Some to *Muſæus*, as *Diogines Laertius* : And ſome to other Authors, amongſt whom *Architas Tarentius* is not forgotten. But all theſe were outſtript by *Archimedes* the *Syracuſan Mathematician*, who flouriſhed *Anno Mundi* 3739, and before the Nativity of Chriſt 209 years : It was he that compoſed a Sphere of tranſparent Glaſs : Of which you have a deſcription in this Treatiſe. Sect. Par. 3.

Of ſuch like Spheres *Peter Ramus* ſayes he ſaw two at *Paris*, yet not of Glaſs but of Iron ; the one of which *Ruellius* the *Phyſician* brought from the *Spoyls* of *Sicily* : The other *Orontius* the *Mathematician* recovered from the *German* Wars.

XVI. *Bells* are imagined to have been invented in the year of our Redemption 400, by *Paulinus* Biſhop of *Nola*, a Town in *Campania*, where *Auguſtus* died : They were called by the Name of *Campana*, becauſe they were invented in *Campania* ; and the Leſſer Bells *Nola*, from the Place where they were made—The Uſe of Bells is very great, for by the benefit of them the Hours of the time of the Day or Night are heard a far off, whether we lye in our Beds, be abroad in the Fields, or journying on the way, although the Sun be obſcured by Clouds. Moreover, Bells call us to Divine Service : They call for help in time of Fire or other dangers or Mutinies : They call Magiſtrates of Cities to their Common Halls, Judges to the Bench, Scholars in Univerſities to Congregations and Diſputations : And in a Word, they help us in all publick actions.—And indeed *Paulinus* that Holy and Religious Biſhop, did rather reform the abuſe of Bells than invent them; and taught them to call Chriſtians to Church to ſerve God, whereas in former times their chief uſe was (as they then imagined) to chaſe away Devils and evil Spirits : they hindred alſo Magical Inchantments, as *Tibullus* notes, when he thus writes ;

> *Cantus & è curru Lunam deducere tentat,*
> *Et faceret ſi non æra repulſa ſonent.*

For they believed, that by the tingling of Braſs, that the ſound of Magical Verſes ſhould be hindred from coming up to the Moon ; and when ſhe was moved with theſe Verſes, this Sound relieved her : in

which ſenſe may be taken that of *Statius Papinius*, *Thebaid.* 6.

——Attenitis quoties avellitur æſtris,
Soli s opaca ſoror, procul anxiliantia gentes
Aera crepant, fruſtaque timent——

Bells are rung many times in Thunder, to reverberate the infectious Air; the like doth Great Ordnance ſhot off, as well in Thunder as in ſoultry and cloſe hot Weather——The *Laconians* when their King dyed, uſed to beat upon Kettles, inſtead of Ringing of Bells. The *Africans* (eſpecially thoſe that are *Preſter Johns* Subjects) have Bells made of Stone. The *Jews* at Funerals uſed playing upon Pipes, as it may be gathered out of *Matthew* 9. 18, 23. Which cuſtom it ſeemed the *Romans* borrowed from the *Jews*, as appeareth by *Ovid*, 1 Triſt.

Tibia funeribus convenit iſta meis.

The Little Bell, which we commonly call the Saints Bell, *John Pierius* uſeth for an *Hieroglyphick*, Teaching Preachers of Gods Word, that to the ſound of their Voice, they ſhould lead their Lives accordingly; elſe like the Bell, While they call upon others, themſelves are deaf and ſtupid; in alluſion whereunto, *Beza* hath this excellent *Epigram.*

Aera gravi cunctos, veluti Campana ſonore,
Ipſa licet penitus ſunt ſibi ſurda, cient:
Sic es recta docens alios, perverſa ſequnte,
Quique alijs ſapiens, non ſapis ipſe tibi.

And ſo much concerning the Invention and uſe of Bells.

XVII. *Lucius Papyrius* was the firſt that ſet up a Sun Dial in *Rome*, which being only of uſe when the Sun ſhined——And the hourly meaſure of Time was firſt found out by *Scippio Maſſala*; whereas before his time the *Romans* knew no diſtinction in the time of the Day, than the *Morning Noon* and *Evening*; But afterwards the ſaid Conſul. *M. Val. Maſſala*, beautified a Columb with a Dial neer the *Roſtra*, as *Varro* relateth.

XVIII. *Doxius*, the Son of *Cælius*, is ſaid to be the firſt that built a houſe in *Athens* who taking his pattern from the Neſts of the Swallows, began the way of making of Houſes with Clay; whereas, before, men dwelt in Caves and Caverns of the Earth.

XIX. The firſt that ever began to erect *Obeliſqs*, and conſecrated them to the *Sun*, they having emboſſed or ingraven upon them certain Characters and Figures, which were the *Egyptians Hieroglyphicks*, wherein a great part of their beſt Learning was contained; was *Mitres*, once King of *Egypt*, who held his Court in the Royal City of *Heliopolis*, the City of the *Sun*: And it is ſaid he was admoniſhed in a Viſion or Dream ſo to do: Some of theſe *Obeliſqs* were Stones cut out of the Solid Rock, and very large, ſome of them have been on every ſide 7 foot and a half ſquare, at the end and in length 100 foot, as was that of *Ramiſes*, once King of *Egypt*.

XX. Con-

XX. Concerning the Invention, of the *Teleſcopick Inſtrument*, called the *Micrometer*, there are ſeveral Competetors; For, *Monuſ. Petit*, Surveyor of the Fortifications in *France* was the firſt that publiſhed to the World the rough draught thereof in *March* 1667. After him, *Monuſ. Azout*, another ingenious *Frenchman*, publiſhed a Tract concerning the exact meaſuration of the Planets diameters, wherein he ſeems to challenge the invention of this Inſtrument to himſelf and *Monus Picard*: But laſt of all, a candid *Engliſh-man* of our own, Mr. *Richard Townley*, does vindicate the firſt contrivance hereof to its true and original Author; One Mr. *Gaſcoigne*, an *Engliſh* Gentleman, who was killed in King *Charles* I. Service, wherein Mr. *Townley* aſſerts, that Mr. *Gaſcoigne* made and uſed this Inſtrument before the Civil Wars in *England*; and that Mr. *Townley* had then in his cuſtody two or three of thoſe Inſtruments firſt deviſed by Mr. *Gaſcoigne*, to which Mr. *Townley* himſelf had added ſome conſiderable improvements.

XXI. Concerning *Optick Glaſſes*, it is evident, by what is ſaid relating to ſuch Glaſſes either in *Plantus* or *Pliny*, is little to the diſcovery of the real inventor of them; but rather that the Ancients were wholly ignorant of them; Wherefore, we muſt neceſſarily allow their invention due to the Modern Age of the World: And where to fix it we ſhall now enquire.

One *Monuſ. Monage*, a learned and ingenious *French-man*, in his *Origini della Lingua Italiana*, Commenting on the Word *Occhiali Galalæi*, diſcourſes there of the time of the invention of *Spectacles*, and there relates, that *Monuſ. de Cange*, had told him of a *Greek Poem*, the Manuſcript whereof is now in the *French* Kings Library, wherein the *Poet* (who lived *Anno* 1150) jeſting on the Phyſicians of thoſe Times, ſays of them, in *French*, *Qu'ils tatent le Poux*, &c. That they obſerved the Excrements of their Patients with a Glaſs: But *Mounſ. Monage* is of opinion that this was rather a tranſparent *Glaſs* whelmed over the Veſſel, more for the relief of their Noſe againſt the *Stench*, than to help their Eyes.

Whether *Spectacles* were of this antiquity, as 1150 or not, is by what is already ſaid, uncertain; but about the 13th Centrury they were known and uſed: For in a Letter of *Signior Redi*, Dated *Anno* 1313. He ſays, *I find my ſelf ſo preſſed by Age, that I can neither read nor* write *without thoſe* Glaſſes *they call* Spectacles, *lately invented, to the great advantage of poor Old* Men, *when their Sight grows weak.*

Curſca in his *Italian Dictionary*, makes this remark on the word *Occhiale.* That Frier *Jordan* in a Sermon of his preached at *Piſa*, An. 1305, tells his Auditory, *That it is not* 20 *years ſince the Art of making* Spectacles *was found out; and is one of the moſt neceſſary Inventions in the World.*

About the ſame time, *viz.* 1305, *Bernard Gorden*, a famous Phyſician of *Montpelier*, ſpeaking and commending of an excellent *Eye-Salve*, concludes: That, *If this or the like will not do, They muſt make uſe of* Spectacles.

From what hath been ſaid we may conclude, that *Spectacles* were known and uſed in the 13th Century, and not much before, but who the happy man was, that was the firſt inventor, Frier *Spina* makes the faireſt Challenge to the Invention: But thoſe *Monkiſh* men (as *Jordan* amongſt the reſt) had this invention amongſt themſelves before it was

publick: and that they all had the first hint thereof from our Countrey-man Fryer *Roger Bacon*; of whom as followeth:

This Learned Fryer *Bacon*, who died *Anno* 1292, and lies buried at *Oxford*, did well understand all sorts of *Optick Glasses*, as plainly appears by his Book of *Perspective*; And that he not only understood the Effects of single *Plain*, *Concave* and *Convex* Glasses; but knew likewise the way of combining them, so as to compose some such *Instrument* as our *Telescope*; And this appears plainly by his own words in the foresaid Book, Part III. Dis. 2. Chap. 3.---*Greater Wenders than all these are performed by Refracted Vision; For, thereby it is easily made appear, That the Greatest Object may be represented as very Little, and contrarily---And so likewise, The most distant Objects as just at hand, and contrarily.---Hereby also may we bring, the Sun, Moon and Stars down here below in Appearance, &c.* And in another place, *viz.* in his Epistle *ad Parisiensem*, of the *Secrets of Art* and *Nature*, Chap. 5. he says,---*Glasses, or Diaphanous Bodies, may be so formed, that the most remote Objects may appear as just at hand, and contrarily; So that we may read the smallest Letters at an incredible distance, and may number things though never so small, and may make the Stars appear as near as we please:* And this I think is sufficient to prove him to be the first Inventor of *Optick Glasses*:---Yet notwithstanding the incredible things which this Man performed, (as may be read of him in Dr. *Plott's Nat. Hist.* of *Oxfordshire*, Ch. 9. Sect. 2, 3.) yet was he Persecuted by the Ignorant Malicious Fryers of his Order, as practising *Magick* and *Necromancy*, for which they cast him into Prison, and there detained him for a long time, some say to his Death, in the 78th year of his Age; in which time 'tis said, no one was admitted to speak with him; and that all his *Writings*, *Books* and *Instruments* were seized and burnt. And thus leaving Fryer *Bacon* as the Inventor of *Single Convex* and *Concave Glasses*; I shall give an account of some other Inventors of the *Telescope* as now made and used.

Borellus has written a small Tract on this Subject, *De vero Telescopii Inventore*, wherein (*Chap.* 12.) he seems to give the Invention to *Zacharias Joannides* of *Middleburg* in *Zealand*, *Anno* 1590. Also he names *Johannes Lipperboy*, or *La Prey*, *Anno* 1609, a Dutchman also; whom *Saturnus* calls *Lippersein.*

Adrianus Metius, Mathematical Professor at *Franequer* says, his Brother *Jacobus Metius*, an Alkmaer, was certainly the first inventor of the *Telescope.* And if we believe the *Italians*, we shall have the Honour of inventing this Instrument conferred on the incomperable *Galileo:* But he himself (in his *Nuncius Sidereus*) confesses that the first intimation he received of this Instrument, was, that a *Dutchman* had then lately made one; which set him (*Galileo*) upon the thought how to effect it, which he successfully discovered by the consideration of *Refraction*, and found, that a *Concave* and a *Convex-Glass* rightly adapted, would perform what he only heard in general of the *Dutch* Invention:--- But certainly, the first publick notice of this Contrivance, came from some of the forementioned *Dutch-men*, and therefore is the Instrument deservedly call'd *Tubus Batavus:* Although we must confess, at the same time, that *Galileo*, *Anno* 1610, did first apply this curious Instrument to *Celestial Ob-*

ser-

servations, and had then made such wonderful discoveries in the Heavens thereby, that all his Philosophical Successors have ever since attempted to climb higher, by lengthening their Ladder, and advancing this Instrument by many Degrees.

XXII. Of the *Microscope*: *Franciscus Fontana*, in his Book Entituled, *Cœlestium Terrestriumque Rerum*, challenges to himself the invention of the *Double Microscope*; 'Tis true, that he was the first that published *Microscopical Observations* of some few Bodies, in the year 1618. After him *Borellus*, *Anno* 1650; Next, Dr. *Power*, a Learned *Englishman* 1664. All these went no farther then Verbal Description: But the Learned and Ingenious Mr. *Hook*, in his *Micrographia*, Printed *Anno* 1665. hath presented to the World curious and lively *Schemes* of several *Creatures*, and other curious Enquiries. And the Learned Dr. *Grew*, and the excellent *Bononian* Philosopher, *Marcellus Malphigius*, have laboured most successfully in the *Anatomy of Plants* by the *Microscope*: The last Author that has professedly treated of *Microscopick Observations*, is *Johan. Fran. Griendelius*, in his *Micrographia nova Norimberg*, 1687. wherein he hath taken a great deal of pains in giving the Genuine representations of his *Objects* as Magnified: And (to conclude this matter) says my Author, *Will. Molyneux* of *Dublin* Esq, "I have been often "delighted with the curious appearance of many Objects seen through "the *Microscope*; But none ever surprised me more, than the visible *Circulation of the Blood* in *Water-Newts* (*Lacerta aquatica*) to be seen as "plainly as Water running in a River, and proportionably much more "rapid.

XXIII. *Of such Cœlestial Discoveries as have in this our Age been made by the help of Telescopes.*

AS to the first Inventors of *Concave* and *Convex Optick Glasses*, and how they came first to be adapted to *Telescopes*, I have in this Book made enquiry into; wherein I at last conclude *Galileo* to be deservedly reputed the first that raised up this *Gigantick Instrument*, that ventures to climb Heaven, and from thence brings down the Stars.

And now to recreate my Reader, I shall here recite some such discoveries as have been made in the *Heavens* in this our Age, by help of these *Telescopes*: And I shall begin my Discourse with the remotest Heavens, namely with the *Fixed Stars*, and from them gradually descend down to this our *Terrestrial Ball* by us inhabited. And,

I. *Of the* FIXED STARS.

THat Whitish Band or *Zone*, commonly known and called by the name *Galaxia*, or the *Milkey Way*, that so irregularly incompasses a great scope in the *Heavens*, and of which the Ancients could give no tolerable Account, is found by the *Telescope* to be no other, than an heap of very minute *Stars* thickly set together; which, by their *Great Distance*, *Smallness* and *Closeness*, appear to the naked Eye, as one united whitish *Cloud*.

Like-

Likewiſe thoſe Stars which we call *Nebulus*, as the *Head* of *Orion*, the *Crib* in *Cancer*, *&c.* are found to be a *Congeries* of ſmall Stars cloſely ſet together, but eaſily diſtinguiſhable by the *Teleſcope*.

The *Pleiades*, or *Seven Stars* (though ſcarce more than *ſix* appear) are found by an ordinary Glaſs, to be nigh Forty: And in that one *Conſtellation* of *Orion*, the *Teleſcope* diſcovers more *Stars* than the naked Eye can number in all the *Heavens*.

II. *Of* SATURN.

THis *Planet* by his ſlow Motion takes State upon him, as carrying about him ſomething more weighty than ordinary; But our ſhort ſight perceives nothing thereof, only a plain Round Globe, as the reſt of the *Chorus* Dancing round the Sun: All his Equipage and Attendants are hid from our view, till ſurveyed more cloſely by the *Teleſcope*; by which you may perceive a mighty *Ring* parallel to the *Equator*, Bright as the *Planets* own *Face*, encompaſſing round his Body; very Thin, and ſeparated in all appearance on all ſides from his Globe: Sometimes appearing *broader*, ſometimes *narrower*; and ſometimes almoſt *vaniſhing*: Then again returning by a regular *Period*, and reſuming by degrees its former ſhape; which again by degrees it loſes according to its own *Periodical Motion*.

But this is not all his *Equipage*; For, beſides this *Throne* of *Light*, this Majeſtick Planet is attended by a *Guard* of *Five Satillets*, that follow his *Motion*, and *Dance* round him continually in a Circle; And theſe were firſt diſcovered by *Galileo* in *October* 1610, but his *Glaſſes* were too ſhort to give the true ſhape of this Planet: But when the *Teleſcope* was farther advanced by *Chriſtopher Hugenius*, to 12 Foot in length, and afterwards, *Anno* 1656, he doubled that length, and then Publiſhed a Treatiſe entituled *Syſtema Saturnum*, in which the *Satillet* which he diſcovered is the fourth from *Saturn*, of which he there gives us the *Epochæ* and *Theorie* of its Motion.

The other four *Satillets* were ſince diſcovered by Monſieur *Caſſini*, in this Order, The *Third* and *Fifth* were firſt ſeen by him *Anno* 1671, 72, and 73, by a 17 Foot Glaſs: But the innermoſt, or *Firſt* and the *Second* were not ſeen by him till the year 1684, at which time having procured Glaſſes of an extraordinary Length, as 80, 100, 150, and 200 Feet; the vaſt *Diſtance* and *ſmallneſs* of theſe *Planets* could no longer conceal them from his ſight: And ſince he hath made *Tables* of the *Motions* of all *Saturns Satillets*, together with their *Diſtances* from *Saturn* correſpondent to their *Periodical Times*.

III. *Of* JUPITER.

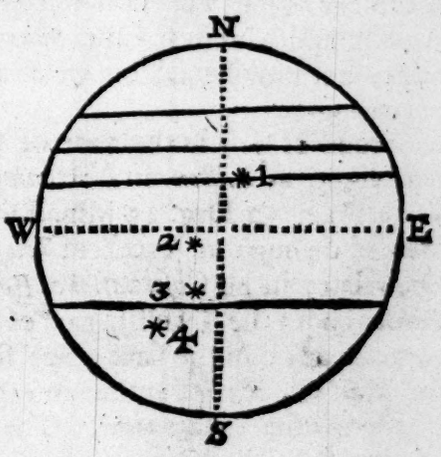
JUPITER next preſents himſelf leſs incumbred than *Saturn*, yet not wanting a Courtly *Train*; For though his *Guards* are but *Four* in Number, yet their *ſize* and *brightneſs* ſhew their *ſtrength*; and their *quick Motion* round about him ſhews their *diligence*.

Galileo was certainly the Firſt that diſcovered theſe *Satillets*, *Jan.* 7. *Anno* 1610, and from that time to this, no more could ever be diſcovered about him.

These *Satillets* are easily seen by a Three-foot *Glass*: Ever since the time of their first discovery, many curious *Astronomers* have attended their *Motion* round *Jupiter* with a diligent Eye, and have found, that sometimes falling into the Shadow of *Jupiter's* Body, they disappear; and thence emerging, they again become visible: Sometimes they are hid behind the very *Globe* of their *Great Lord*; and sometimes being just in his *Face*, his splendor overcomes theirs, and they become invisible, as a glimmering Lamp between the Eye and Sun.

John Alfons. Borellus has Published a Tract of the *Theoricks* of these *Satillets*, *Anno* 1666; but none have laboured more to reduce the Observations of these little Planets to something of *Use* and *Advantage* to the World, than the two Peerless Astronomers of this present Age, *viz. Cassini* and *Flamsteed*.

Besides these Four *Little Moons* about *Jupiter*, the *Telescope* discovers other Remarkables even in his *Body*: As first, His Face is not all of a *Colour*; but there are in it *Brighter* and *Darker Parts*; and these are drawn a-thwart him, like broad *Zones* or *Belts*, almost parallel to the *Ecliptick*, as is expressed in this Figure:



IV. *MARS.*

MARS offers himself next, who trusting in his own Strength, is attended by no Guards: But the prying *Telescope* discovers in his *Face*, *Scars*, *Spots* and *Ruggedness*.

By these *Spots* the accute *Cassine* has determined that he turns on his own Axis once in 24 hours and 40 minutes, though others assign his *Revolution* performed in just half that Time.

The *Increase* and *Decrease* in *Light* like our *Moon*, is very visible in this *Planet* Mars; who in his *Quadratures* with the *Sun*, and his *Perigeon*, may be seen almost *bissected*, but never *Corniculated* or *Falcated*, as the other Inferiors do: But this *Bissection*, and *Increase* and *Decrease* in Light, in regard of the great distance above the Sun) cannot be seen in *Saturn* and *Jupiter*.

V. *Of*

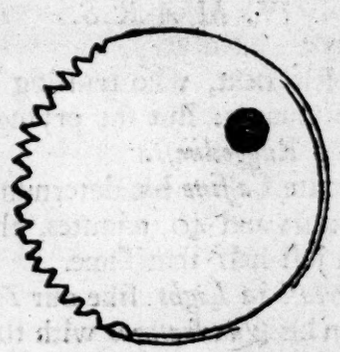
V. *Of the* SUN.

THe *Sun* in the next place preſents it ſelf, in whoſe bright *Face*, we can hardly expect to find *Dark Spots*, yet ſuch there are, and frequent too, of which, and nothing elſe, *Schinerus* hath writ a large Book in Folio: And *Hevelius* in his *Selenography* hath many *Obſervations* of theſe *Maculæ*; as alſo, of ſome *Brighter Spots* in the *Sun*, called *Faculæ*.

The way of *Obſerving* them (in ſhort) is, Either by admitting the Light of the Sun through a *Teleſcope* upon a ſheet of *White Paper*, in a *Dark Room*; Or by Arming the *Eye* with a ſmall thin *Glaſs* Smoaked over with a *Torch*, *Lamp* or *Candle*, and with it looking through a *Teleſcope* at the Suns Body.

The only Diſcovery that has been made by theſe *Maculæ* is, That the Sun revolves round his own Axes, in the ſpace of about 25 days and 6 hours. And here it is to be noted, That for theſe ſeveral years paſt, the appearance of theſe *Maculæ* has been much more rare, than when *Galileo* (who firſt diſcovered them) *Schinerus*, and *Hevelius*, *&c.* lived: Who attended their Obſervations, *viz.* between 50 and 60 yerrs ago: About which time one ſhould ſeldom ſee the *Suns Face* free from one or more *Black Patches*; but now he ſeldom wears any, One in *Five* or *Seven* years hardly appearing.

Upon *Tueſday* the ſecond of *July* in the year of Chriſt 1651, about Eight of the Clock at Night, at *Eaſton* in *Northamptonſhire*, under the Elevation of the *North* Pole, 52 Deg. 15 Min. Dr. *John Twyſden* ſaw in the *Body* of the *Sun*, (through an excellent *Teleſcope*, whoſe Glaſſes are very clean) as he relates in his *Obſervationes Eclipſium*, a very *dark Round ſpot*; in *Diameter* about the Twelfth part of the Suns *Diameter*, which to his ſight appeared ſtill in the ſame place for a matter of *Nine* or *Ten Minutes*; tho' thin Clouds oftentime interpoſed and hindred him from the ſight of the *Sun* for a ſhort time. The *Left* Margine of the *Sun* was very *uneven*, and *Tooth'd* in the manner of a *Saw*, as in this *Figure*.



He conceived it was one of thoſe Spots which *Galileus*, *Hevelius*, *Schinerus* and others have obſerved. *I could not* (ſays he) *ſuſpect* Mercury *in that place, by reaſon the lateſt* Tables *give him near* 5 *deg. of* South Latitude; *though in* Longitude *he be not far diſtant from the* Sun: *As by the ſeveral* Calculations *following may appear:*

Locus

		Longit. ☿ S.	D.	M.	So.	Latitude ☿ S.	D.	M.	So.	Longitude ☉ S.	D.	M.	So.
Locus Mercurii ex Tab.	Origani,	69	8	58	0	4	10			69	19	19	16
	Argol. Ephem.	69	9	41	0	3	17			69	19	42	27
	Eichſtad. Eph.	69	19	8		4	05			69	19	45	10
	Lansbergii,	69	15	48	40	3	49	14		69	20	04	03
	Britannicis,	69	18	55	01	4	51						
	Vincentii Wing,	69	19	12	41	4	53						

Whatever this Spot *was, of this I am certain, It was never my fortune ſince to ſee the like* Spot, *nor the* Margine *of the* Sun *ſo uneven, though I have often tryed.*

VI. *Of* VENUS *and* MERCURY.

THE Brighteſt *Planet* in the Heavens; *Venus* fears not ſometimes even at Noon-day to diſplay her *Beauty*; She performs her Courſe alone, and free from all other Attendants.

MERCURY, Wit and Quickneſs ſecure him, therefore he hath no Train; but generally ſhelters himſelf under the Beams of his Potent Lord the *Sun*.

Notwithſtanding, Both theſe *Inferior Planets* are found by the *Teleſcope* to *Increaſe* and *Decreaſe*, as our *Moon:* For, ſometimes they appear *Corniculated*, ſometimes *Falcated*, ſometimes *Gibbous*, and ſometimes *Full*; on, or near their *Conjunctions* with the *Sun:* By this laſt *Phænomanon* it is manifeſt they move about the *Sun*; ſometimes farther from us, ſometimes nigher to us than *He*; and conſequently the *Ptolomaick Hypotheſis* is evidently diſconſentanious to modern *Obſervations*.

VII. *Of the* MOON.

WE are now arrived at home, to contemplate our Neighbour the *Moon:* We may properly call her *Our Moon*, as making *Her* the *Centre* of her *Periodical Motion:* For, as the *Satellites* about *Saturn* and *Jupiter* move about *Them*; ſo moves the *Moon* as a *Satellite* about our *Earth*.

Galileo with his *Teleſcope* firſt diſcovered great *Ruggedneſs* in the *Moons Face*; after him *Langrenus* attempted to draw her *Picture:* But *Hevelius* has accurately performed that Work; in whoſe *Schemes* we may ſee the *Moons* Countenance diſtinguiſhed in an admirable difference of *Parts*, both for *Shape* and *Colour*. We may there ſee greater parts that reſemble our *Seas*, *Lakes*, *Rivers*, *Iſlands Peninſulaes* and *Continents*: Other leſſer *Spots* that reſemble our *Mountains*, *Hills* and *Valleys*: Thoſe of the *Greater* parts ſomething Obſcure, may we reckon as *Seas*, and *Lakes*; and the *Brighter* we may account *Land*: For ſo does our *Earth* appear, when from a diſtant *Height* we look upon a mixture of *Land* and *Water* enlightned by the *Sun*; Of the ſmaller *Spots*, thoſe that are brighteſt and ſhine, are *Mountains* and *Rocks*: And the *Darker* parts encompaſſing them we may eſteem as *Valleys*.

Now it is manifeſt by the *Teleſcope*, That ſome parts of the *Moon* are much higher than others ; as that ſome parts of our *Earth* are higher then others. For, If you look upon the *Moon* about the *Quarter* days, we may plainly ſee the *Edge* towards the *Dark Part*, *broken* and *cragged* ; and many little *Bright Spots* that are clearly ſeparated from the reſt of the enlightned *Part* : Which is an evident proof, that theſe are the high Tops of *Emenencies*, which receive the *Suns* Light before the parts below them are enlightned. Moreover, The *Moons* Spots caſt their ſhadows oppoſite to the *Sun*, that is, to the *Eaſtward*, whilſt the Moon is *Increaſing*, and to the *Weſtward* on her Decreaſe.

Now for the better diſtinguiſhing theſe *Spots*, and making them more uſeful in the *Obſervation* of *Lunar Eclipſes*, there are *Names* impoſed upon them by Authors ; *Hevelius aſſigns* to them the *Names* of *Places* here on *Earth* ; *Grimaldus* and *Ricciolus*, gives to them the *Names* of famous *Mathematicians* and *Aſtronomers* ; to the great advancement of *Geography* and *Navigation*, in ſetling the *Longitudes* of Places.

Moreover, By theſe *Spots* the *Moon* is diſcovered to have various *librating* Motions, from *Eaſt* to *Weſt*, and from *Weſt* to *Eaſt* ; alſo from *North* to *South*, and from *South* to *North*.

MUSEUM BRITANNICUM

ALGE-

Arts and Sciences MATHEMATICAL Profeſſed and Taught by the AUTHOR.

Arithmetick,
- In Whole Numbers, and Fractions.
- In Decimals, and by Logarithms.
- Inſtrumentally, by Decimal Scales, Napiers Bones: And to extract the Square and Cube Roots by Inſpection.

Geometrie:
- The Principles thereof with the { Practice, and Demonſtration.

Aſtronomie:
- Theorical and Practical.
- The Deſcription of the Circles of the Sphere.
- The Uſe of the Globes, { Celeſtial, and Terreſtrial.
- To project the Sphere *in Plano* upon any Circle, { Right, or Oblique.

And upon theſe Foundations, the following Superſtructures.

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- *Longimetria*, or the Menſuration of { Heights, Depths, Diſtances, } as of { Trees, Towers, &c. Mines, Wells, Deſcents, &c. Churches, Towers, &c.
- *Planometria*, or the Menſuration of { Board, Glaſs, Pavement, Tiling, &c. } Or any other Superficies.
- *Stereometria*, or the Menſuration of { Timber, growing or ſquared. Stone, regular or irregular. Cask, commonly called Gaging.
- *Geodeſia*, or the Meaſuring of Land divers ways, and by ſeveral Inſtruments; to draw the Plot of a whole Mannor or Lordſhip; to caſt up the Content thereof; and to beautify the ſame with all neceſſary Ornaments thereunto belonging.

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- The Application thereof, in the ſolution of Problems in { Geometry. Aſtronomy. Geography. Navigation. Fortification. Dialling, &c.

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- The Principles thereof, and the manner of Sailing by { The Plain Sea-Chart. *Mercator's* Chart. The Arch of a great Circle.

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- *Arithmetically*, by the Tables of { Sines. Tangents. Logarithms.
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We should be injurious to him, if we did not further inlarge; by telling the World how much it is obliged for his Pains in exciting the Learned to publish their Works, and in acting the Part of an *Ingenious Obstetrix* at the Press, in correcting and in drawing of *Schemes*; So that he hath been Instrumental in furnishing the World with the many learned *Mathematical Books* here lately published

(for which, his chief Reward hitherto hath been to obtain from the Learne
the Title of *Merſennus Anglicanus*) and many more may be expected, if moderat
Encouragements towards Printing ſuch Works, and Leiſure for ſuch an Affair b
not impeded through the neceſſary Avocations for a livelyhood, and though it b
beſides my Deſign, yet I cannot but digreſs in giving *him* and *others like minde*
(which are very rare to be found) their due commendations, in promoting the lau
dable Deſign of getting *Learned Men* to impart their *Labours* to be Printed; and
exciting others to encourage the ſame, as being of ſingular Uſe and advantage t
the *Republick* of Learning; through the want whereof many Learned Men
Works of much worth have been loſt, ſuppreſſed or long delayed. As thoſe o
Maurolycus, *Abbot of Meſſina*, a large Catalogue whereof is to be ſeen at the En
of *his Opuſcula*, but by the Care of the learned Mathematician *Alphonſus Borelli*
ſome of them have been publiſhed in this Age, ninety years after the Author
Death; as his *Apollonius* at *Meſſina* in 1656. containing the ſubſtance of the fou
firſt Books of the *Apollonius* of *Commandinus*, and two more Additional Books o
Maurolycus, and all in leſs Room, and at a much cheaper price. And now by th
like Diligence the ſaid *Borellius* is publiſhing *Maurolycus* his *Archimedes* in *Latin*, re
puted *a Good one*, after we have been long tired with the *Common Latin bad one.*

Through want of ſuch care the many learned Works of *Vernalion* of *Naple*
Maſter to *Joſephus Auria*, have not hitherto come to light, as his Commentaries u
on all *Archimedes*, *Apollonius*, *Serenus*, *Euclid*, *Ptolemæus*, and divers others of th
Ancients; which is much to be lamented; ſeeing he was, according to the Teſt
mony of the ſaid *Auria*, *Vir*, *Divinitate quadam Ingenii Ornatus.*

Nor thoſe of the learned *Bernardinus Baldus*, *Abbot of Guaſtalla*, mentioned
the End of his Comment on *Ariſtotle's Mechanicks*, amongſt which are two Vo
lums of the *Lives of Mathematicians*, whereof *Bartholinus* in his Preface to th
Edition of the Optick Fragments of Heliodorus Lariſſæus, Printed at *Paris* 165
gives an honourable *Elogium.*

Varenius could find no Stationer or Printer in *Holland* to undertake his *Treati*
of Conicks and curved Lines. See the Preface to his *Hiſtory of Japan.*

If Sir *Charles Cavendiſh* deceaſed, Brother to the preſent *Duke of Newcaſtle*, ha
not (as 'tis credibly reported) given liberally toward the Printing of *Mydorgius'*
four firſt Books of *Conicks*, they had never come to publick view; the four laſt,
likewiſe thoſe *of Paſchal the Younger*, yet remaining unprinted upon the ſa
Accompt, of whom *Merſennus* gives this Cenſure, *quòd Unicâ Propoſitione U*
verſaliſſimâ, quadringentis Corrolariis armatâ, totum Apollonium complexus eſt. T
Manuſcript as yet remaining unprinted (as I am informed) in the Hands, or at t
Diſpoſal of *Monſieur Du Prez* a Bookſeller in *Paris*; the want whereof is the mo
conſiderable in regard the Author (beſides the ordinary method) treats of *the Con*
Sections, as projected from leſſer Circles of the Sphere.

Eraſmus Bartholinus well known by his Additions to the ſecond *Volume of D*
Cartes, as it is commonly termed; could find none to undertake the Printing
De Beaunes Treatiſe De Angulo ſolido, and other *Treatiſes* both of that *Auth*
and his own.

Jungius his *Phoranomicks*, and *Treatiſe De locis Planis*, *&c.* and other *Algeb*
cal Tractates; have remained at *Hamborough* above ten years ſince the Autho
Death unprinted, for want of due Encouragement: Albeit a *Jeſuit*, who wi
his Life, makes him, although a *Phyſician*, equal in Mathematical Knowledge
Des Cartes.

On the like Reaſons we may conceive we want the many learned *Algebrai*
Wo

Works of our famous Countryman Mr. *Thomas Harriot*, (and of Mr. *Warner*, into those Hands they fell) who is esteemed by some of the most knowing Persons alive to have been much Superiour to all that ever writ; and, that equivalent to what of his might have been forty or fifty years since known, is not readily to be expected.

For want of the like Encouragement, we have lost that most excellent Piece of the incomparable *Vieta*, his *Harmonicon Cœleste*; as likewise the Remains of *Alexander Anderson* the *Scot*, as his *Conicks*, *Stereometria Solidorum*, & *Triangul. Sphæric.* the want whereof *Guldinus* much bewails and excites the ingenious to enquire after them.

On the same Accompt the Remains of *Griembergerus*, as his *Conicks*, *Dialling*, and *Projections* of the Sphere have not come to light; and for the very same Reason the second Tome of *Galilæus* in *English* doth, and is like to remain, unprinted. And for the same Reason the *English* Tongue is barren of Mathematical (and divers other) Books in respect of the *French* or *Latin*, there being little or nothing yet extant of *Catoptricks*, *Dioptricks*, *Staticks*, *Mechanicks*, &c. and not much of Perspective and other Kinds, the number of Students in these Sciences, as yet scarce being sufficient to take off 500 Books, or so small an Impression as may give a Bookseller a Prospect of moderate gain in a competent time by his undertaking.

With the like *Remora* in *France*, have met the Works of the Excellent *Monsieur Fermat*, *viz. Euclidis Porismata restituta*; his *Treatise De Locis Planis Solidis Libris & ad Superficiem*, and his *Treatise De Contactibus Sphæricis*.

As also the Remains of the much knowing *Lalovera*, as his *Geometrical Dictionary*, *sive Explicatio vocum Geometricarum*, four Books *Problematum Illustrium*, four Books *Problematum Physico-Mathematicorum*, and a *Collection of Letters* between him and the *learned*, containing the *Solution of many Problems of great Curiosity and Difficulty*, which seeing they are not like to get Printed there, they have written over to know if they would be undertaken here; promising to send the *MS. Copies*. And having hinted thus much at the Instance of this *Ingenious* and *Industrious Person*, to the *Curious*, and *Generously learned*; I come now again further to acquaint the Reader, that we have more particularly obtained from him an Accompt of two of his own *Designs*, relative to the *Sphere* and *Astronomy*.

The one of *Geometrical Dialling*, whereby *reflex Dyalling* is rendred *Geometrical*, *and reduced to a Method of Calculation*; so that if a *Glass* were placed at *Random*, and Lines drawn on the *Plain* by chance, by either Method, *Points* might be found in the said *Lines* which joyned should be the *Hour Lines*; and the like when the *Glass* is so placed, that the Hour Lines may be drawn in that Part of the Room which is most capable of them; and as a *Corollary* of this Doctrine a *Dial* for any *Latitude* may be suddainly divided from a Line of *Tangents* parallel to any Line proposed, and that without any *Calculation* for the *Horary Divisions*.

The other a *Treatise of Projections of the Sphere*, and *concerning Spherical Trigonometry*; in which many extraordinary Cases will be solved; those *Proportions* mentioned before in the *Narrative* concerning Mr. *William Oughtred*, several wayes more easily demonstrated, and all Spherical Triangles measured by a new Method (not by him formerly insisted on) after the manner of *Plain Triangles*; all which perchance may be handled in some little *Tractates* concerning the Use of *Prints* of several other *Instruments* designed to be cut, pasted, and varnished, as before mentioned; *viz.* the *Analemma*, the *double Horizontal Dial*, the *Logarithmical Serpentine Line*, Prints *of Logarithmical Rulers*; whereof if there be three, they may be placed as to lie still all Day, and as fast as the Height of the Sun is given, shall find either the *Hour* or *Azimuth* universally by bare *Inspection*.

Collins (John) Mathematician
531. n. 20 (3)

To the READER.

COURTEOUS READER,

IN *my Youth-hood I was employed in Clerkship under, and received some* Mathematical *Knowledge from Mr.* John Marr, *one of the Clerks of the* Kitchin *to His present Majesty, when* Prince of Wales; *the said Mr.* Marr *being very eminent for his* Mathematical *Knowledge, some testimony whereof may be evinced, from those excellent* Dyals, *wherewith the Gardens of our late Soveraign were adorned.*

Our Intestine Miseries increasing, I lost that Imployment, and went Seven Years to Sea, most part of it in an English *Merchant-man, become a Man of Warr in the* Venetian *Service against the* Turks, *in which having leisure, I applyed part of my Studies to* Mathematicks, *and* Merchants-Accompts, *and upon my return, fell to the Practice thereof, and afterwards profest* Writing, Merchants-Accompts, *and some parts of the* Mathematicks; *and having drawn up some Books of Accompts, and divers loose Questions, for the Instruction of my* Scholars, *in the Year* 1652. *I committed them to the Press, under the name of* An Introduction to Merchants-Accompts, *they were again re-printed in the Year* 166⅔ *without Alteration, and in* 1666. *the Fire consumed the greatest part of the Impression: The* Stationer *hath since often urged me to alter and enlarge the same for another* Edition, *presuming that I that have been much and lately concerned in great and publick* Accompts, *might alter the same to good advantage, and perchance, others may be of the same mind; but I concurr not, finding that my said long experience, hath not at all advanced my Knowledge in good Methods of* Accompts, *though I confess I understand the nature and Intrigues of bad ones much better than I did. Whereupon I have consented, and advised the* Stationer *to re-print the same, with the Addition of two more* Accompts *than were formerly extant, and was the more willing to do it at this time, for two Reasons: The first is,*

1. *That I might have an opportunity of returning my most humble and hearty Thanks (which I hereby do) to the Right Honourable* ANTHONY *Earl of* Shaftsbury, *Lord President of His Majesties Council for Trade, and Plantations; who upon the Information of the most worthy Sir* Robert Murray *deceased, that Accomptants might be presumed to understand the general affairs of Trade, and what were the causes, and what might be the remedies of the decay thereof, and*

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in Particular, that my self had drawn up some Papers and Proposals about the same, which were presented to his Lordship; they were not only kindly received, but his Lordships favour promised, and his endeavour used to promote me to a Clerkship in His Majesties former Council of Trade, which at the Motion of Sir Robert Murray, *had its event in His Majesties late Council of Plantations, and wherein his Lordships approbation was not wanting.*

Moreover, during his Lordships High Chancellorship, *his Lordship was pleased to nominate me in divers References concerning Suits depending in* Chancery, *about intricate Accompts, to assist in the Stating thereof; which hath not been without some Emolument to my self, and to the shortning of the Charge of the parties concerned. And here I must not omit to mention (to his Lordships great renown) that it fell to my* Province, *to transcribe a Schedule of about* 80 *Sheets of Paper, intended, and I believe principally (if not wholly) composed, by his Lordship, for the regulation of* Chancery *affairs, the speedy dispatch and Abridgement of the charge of Suits there depending; all which to pretermit, were to wear the badge of Ingratitude, and to veyl the well know worth of the said most sagacious and Honourable Peer.*

2. *That I might take the opportunity of acquainting those who are, or may be concerned, That I spend part of my time in Stating* Merchants-Accompts, Ship-Accompts, *or any Controversal Intricate* Accompts, *and in computing of* Interest, *the Valuation or forbearance of* Leases, Mortgages, *and* Annuities; *and am to be heard of at Mr.* Robert Horn's *the* Stationer, *at the entrance on the* Royal Exchange, *for whom this Book was Printed.*

And now as to the ensuing Work, I have this to say, That when I first Composed it, I observed a want of such Questions, that might not tire out the patience of the Reader *or* Student, *there being none but* Journals *and* Leidgers *of other Authors in Print, seeming to some like a Labyrinth of a more than sufficient length, and the which, when traced thorough, might possibly leave the Reader unskilful in these shorter Questions; some whereof have had their rise from real Affairs, which have happened much of the same kind. And now if the Reader be still unexperienced, and desirous to reap some benefit thereby, let me advise him to write out the* Journals *of some of these* Accompts, *omitting the Relative Numbers in the Margin, of the which see Section the seventh following, and are not to be made till the Parcel is entirely Posted, which that he may be enabled to do, he may use the ensuing Directions; but if this be thought too laborious, he may prick and examin over the said* Questions, *the* Accompt *it self gives Instruction for* Ballancing *the* Ballance *of each* Accompt, *being entred out of the* Leidger *at the end of the* Journal *parcels.*

Divers Accomptants have made the business of Simple *and* Compound Interest *and* Annuities, *the subject of their Discourses: And I think it not improper to tell the Reader, that in* 1664. *I published a Paper, consisting of a quarter of a Sheet and fitted for a Letter-case, Intituled,* The Doctrin of Decimal Arithmetick, Simple Interest, *&c. as also of* Compound Interest and Annuities, *generally performed for any time of Payment or Rate of* Interest, *by help of any particular Table of the forbearance of one pound Principal.*

The which having been censured by some that understood it not, I think it no amiss, a little better to inform them, That the Table therein used, is a Table of

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To the READER.

ompound Intereſt, at 6 per Centum per Annum, *for ſome Dayes, as well as it is uſual) for ſome Years, the which to the ſaid, or any other* Ratio, *may be uſed by ſeveral Methods, without any Extractions of* Roots, *or aid of* Logarithms, *yea any* Logarithm *in the* Canon *may be eaſily computed thereby, and the converſe; alſo any Number of mean* Proportionals *found between two Extreams given.*

The which Table and Precepts, removing the neceſſity of uſing a large Table of Logarithms, *or of thoſe multitudes of Tables, fitted for the ſeveral kinds of* Annuity Problems, *at ſeveral Rates of* Intereſt, *and ſeveral term of Payment, having been of conſtant and ſingular uſe to my ſelf, and believing it will be of no leſs to others, I intend, (God Willing) to reprint, to be fitted as before, to be a* Vade-mecum *in a Letter-caſe, though with a larger Table, (which will remove ſome Operations hitherto neceſſary, to ſupply the want thereof) and beſides an Explication thereof apart, illuſtrated with variety of Examples: And hope alſo hereafter to have ſome time to alter, amend, and re-print other Treatiſes of my own, as my Book of* Quadrants and Dyalling, *but moſt eſpecially my Treatiſe of* Navigation, *Intituled*, The Mariners Plain Scale new plained; *the rather, in regard His preſent Majeſty, to His Immortal Praiſe, hath been pleaſed to ſettle a Maintainance for* 40 Hoſpital *Boys, and for a Learned* Lecturer *to teach them* Navigation; *on the Advancement whereof, the ſplendor of the Government and the Trade and ſafety of this Nation doth ſo much depend, that any that love their Country cannot but be zealous for, as particularly is*

Thy Friend and Well-wiſher,

JOHN COLLINS.